CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-20

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The number of polynomials having zeroes as -2 and 5 is [1]

(a) 1 (b) 2
(c) 3 (d) more than 3

Ans : (d) more than 3

There may be infinite number of polynomials having zeroes as -2 and 5.

Option (d) is correct.

2. If α and β are roots of the equation $2x^2+3\sqrt{3}\,x-6=0$, then the value of $\alpha^2+\beta^2$ is [1]

(a) $\frac{3}{4}$ (b) $\frac{51}{4}$
(c) $\frac{33}{4}$ (d) $\frac{39}{4}$

Ans : (b) $\frac{51}{4}$

$$\alpha+\beta = -\frac{3\sqrt{3}}{2}$$

and $$\alpha\beta = -\frac{6}{2} = -3$$

So, $$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$$
$$= \left(-\frac{3\sqrt{3}}{2}\right)^2 - 2\times(-3)$$
$$= \frac{27}{4}+6 = \frac{51}{4}$$

3. If the equation $2x^2-5x+(k+3)=0$ has equal roots, then the value of k is [1]

(a) $\frac{9}{8}$ (b) $-\frac{9}{8}$
(c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

Ans : (c) $\frac{1}{8}$

$$b^2-4ac = 0 \quad \text{(For equal roots)}$$
$$(-5)^2-4\times2\times(k+3) = 0$$
$$25-8k-24 = 0$$
$$1-8k = 0$$
$$k = \frac{1}{8}$$

4. The famous mathematician associated with finding the sum of first 100 natural numbers is [1]

(a) Pythagoras (b) Newton
(c) Gauss (d) Euclid

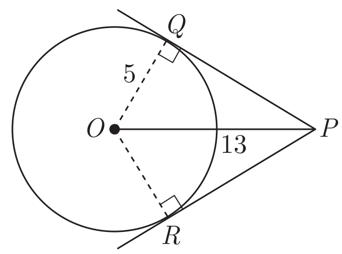
Ans : (c) Gauss

5. From a point which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral $PQOR$ is [1]

(a) 60 cm^2 (b) 65 cm^2
(c) 30 cm^2 (d) 32.5 cm^2

Ans : (a) 60 cm^2

In $\Delta\,OPQ$,



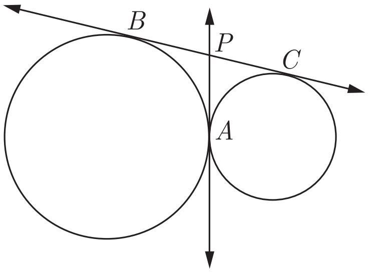
$$PQ^2 = OP^2-OQ^2$$
$$PQ^2 = 13^2-5^2$$
$$PQ = 12\text{ cm}$$

Area of quadrilateral $PQOR$

$$= 2\times\text{area of }\Delta\,OPQ$$
$$= 2\times\frac{1}{2}\times12\times5$$
$$= 60\text{ cm}^2$$

Option (a) is correct.

6. In the given figure, two circles touch each other at A. BC and AP are common tangents to these circles. If $BP=3.8\text{ cm}$, then the length of BC is equal to [1]



(a) 7.6 cm (b) 1.9 cm
(c) 11.4 cm (d) 5.7 cm

Ans : (a) 7.6 cm

$$PA = PB$$

(Length of tangents from an external point)

$$PA = 3.8\,\text{cm}$$

Again, $PC = PA$

$$PC = 3.8\,\text{cm}$$

$$BC = BP + PC$$

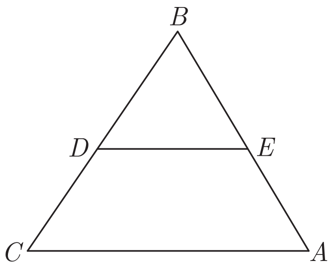
$$= 3.8 + 3.8 = 7.6\,\text{cm}$$

Option (a) is correct.

7. If ABC and BDE are two equilateral triangles such that D is mid-point of BC, then the ratio of areas of triangles ABC and BDE is [1]

(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

Ans : (d) 4 : 1



ΔABC and ΔBDE are two equilateral triangles

$$\Delta ABC \sim \Delta BDE$$

So, $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta BDE)} = \left(\dfrac{BD}{BC}\right)^2 = \left(\dfrac{2BC}{BC}\right)^2$

(D is mid-point of BC)

$$= \frac{4}{1}$$

Option (d) is correct.

8. If the sum of the circumferences of two circles with diameters d_1 and d_2 is equal to the circumference of a circle of diameter d, then [1]

(a) $d_1^2 + d_2^2 = d^2$ (b) $d_1 + d_2 = d$
(c) $d_1 + d_2 > d$ (d) $d_1 + d_2 < d$

Ans : (b) $d_1 + d_2 = d$

$$2\pi\frac{d}{2} = 2\pi\frac{d_1}{2} + 2\pi\frac{d_2}{2} \quad \text{(given)}$$

$$d = d_1 + d_2$$

Option (b) is correct.

9. The area of the circle that can be inscribed in a square of side 6 cm is [1]

(a) $36\pi\,\text{cm}^2$ (b) $18\pi\,\text{cm}^2$
(c) $12\pi\,\text{cm}^2$ (d) $9\pi\,\text{cm}^2$

Ans : (d) $9\pi\,\text{cm}^2$

Diameter of the circle= Length of side of the square

$$2r = 6$$

$$r = 3\,\text{cm}$$

$$\text{Area of circle} = \pi \times 3^2 = 9\pi\,\text{cm}^2$$

Option (d) is correct.

10. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ$ is [1]

(a) 0 (b) 1
(c) 2 (d) $\frac{1}{2}$

Ans : (b) 1

$\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 44^\circ \tan 45^\circ \tan 46^\circ \ldots\ldots \tan 88^\circ \tan 89^\circ$

$$= \tan 1^\circ \tan 2^\circ \ldots\ldots \tan 44^\circ 1 \cot 44^\circ \ldots\ldots \cot 2^\circ \cot 1^\circ$$

$$(\tan(90^\circ - \theta) = \cot\theta)$$

$$= 1 \quad (\tan\theta \cot\theta = 1)$$

Option (b) is correct.

(Q.11-Q.15) Fill in the blanks.

11. A shuttle cock used for playing badminton has the shape of the combination of .... [1]

Ans : hemisphere and frustum of a cone

12. The value of $\dfrac{\sin 39^\circ}{\cos 51^\circ} + \dfrac{\sec 47^\circ}{\text{cosec}\, 43^\circ}$ is .... [1]

Ans : 2

$$\frac{\sin 39^\circ}{\cos 51^\circ} + \frac{\sec 47^\circ}{\text{cosec}\, 43^\circ} = \frac{\sin 39^\circ}{\sin 39^\circ} + \frac{\sec 47^\circ}{\sec 47^\circ}$$

$$= 1 + 1 = 2$$

or

The value of $(\sec^2\theta - 1)(1 - \text{cosec}^2\theta)$ is ....

Ans : -1

$$(\sec^2\theta - 1)(1 - \text{cosec}^2\theta) = \tan^2\theta \times (-\cot^2\theta)$$

$$= -1$$

13. The value of $\sqrt{3}\,\text{cosec}\, 60^\circ - \sec 60^\circ$ is .... [1]

Ans : 0

$$\sqrt{3}\,\text{cosec}\, 60^\circ - \sec 60^\circ = \sqrt{3} \cdot \frac{2}{\sqrt{3}} - 2 = 0$$

14. The common point of a tangent to a circle and the circle is called .... [1]

Ans : point of contact

15. A quadratic equation $ax^2 + bx + c = 0$ with rational coefficients $a \neq 0$ has real and distinct rational roots if its discriminant is .... [1]

Ans : a perfect square number of a rational number

(Q.16-Q.20) Answer the following

16. Find the product of HCF and LCM of two numbers 50 and 20. [1]

Ans :

$$\text{HCF} \times \text{LCM} = \text{Product of two numbers}$$
$$\text{HCF} \times \text{LCM} = 50 \times 20 = 1000$$

17. Is the system of linear equations $2x+3y-9=0$ and $4x+6y-18=0$ consistent? Justify your answer. [1]

Ans :

Given, $2x+3y-9 = 0$

and $4x+6y-18 = 0$

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$
$$\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$$
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

System of linear equations has infinite solutions.
Hence, system is consistent.

or

What is the common difference of an AP in which $a_{21}-a_7 = 84$?

Ans :

Let the first term and common difference of AP be a and d respectively,

given, $a_{21}-a_7 = 87$

$$a+20d-(a+6d) = 84$$
$$14d = 84$$
$$d = 6$$

Hence, common difference of AP is 6.

18. If $\Delta ABC \sim \Delta PQR$ and $\angle A = 32^\circ$, $\angle R = 65^\circ$, then find $\angle Q$. [1]

Ans :

Given, $\Delta ABC \sim \Delta PQR$

$$\angle P = \angle A$$
$$\angle P = 32^\circ \qquad (\angle A = 32^\circ, \text{given})$$

In ΔPQR,

$$\angle P + \angle Q + \angle R = 180^\circ$$
$$(\text{sum of angles of a } \Delta = 180^\circ)$$
$$32^\circ + \angle Q + \angle 65^\circ = 180^\circ$$
$$\angle Q = 180^\circ - 97^\circ$$
$$\angle Q = 83^\circ$$

19. What is the probability of getting neither prime nor composite number, when an unbiased die is tossed? [1]

Ans :

Total number of possible outcomes = 6
Favourable outcome = 1 (neither prime nor composite)
Number of favourable outcomes = 1

$$P(\text{neither prime nor composite}) = \frac{1}{6}$$

20. Following table shows sale of shoes in a store during one month: [1]

Size of shoe	3	4	5	6	7	8
Number of pairs sold	4	18	25	12	5	1

Find the modal size of the shoes sold.

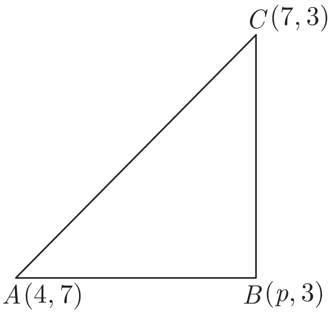
Ans :

$$\text{Maximum number of pairs sold} = 25 \text{ (size 5)}$$
$$\text{Modal size of shoes} = 5$$

Section B

21. The points $A(4,7)$, $B(p,3)$ and $C(7,3)$ are the vertices of a right triangle, right angled at B. Find the value of p. [2]

Ans :



Given, that ΔABC is right angled at B.
By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
$$(7-4)^2 + (3-7)^2 = (p-4)^2 + (3-7)^2 + (7-p)^2 + (3-3)^2$$
(using distance formula)
$$9+16 = p^2 - 8p + 16 + 16 + 49 - 14p + p^2 + 0$$
$$2p^2 - 22p + 56 = 0$$
$$p^2 - 11p + 28 = 0$$
$$(p-4)(p-7) = 0$$
$$p = 4, 7$$

But, $p \neq 7$

Because if $p = 7$

Then, $BC = 0$

Hence, $p = 4$

or

ABC is a triangle and $G(4,3)$ is the centroid of the triangle. If A, B and C are the points $(1,3)$, $(4,b)$ and $(a,1)$ respectively, find the values of a and b. Also, find the length of side BC.

Ans :

The centroid G of ΔABC is $G\left(\frac{1+4+a}{3}, \frac{3+b+1}{3}\right)$

i.e. $G\left(\frac{5+a}{3}, \frac{4+b}{3}\right)$

But G is given (4, 3)

$$\frac{5+a}{3} = 4$$

and $\frac{4+b}{3} = 3$

$$a = 7 \text{ and } b = 5$$

So, coordinates of B and C are (4, 5) and (7, 1) respectively

$$BC = \sqrt{(7-4)^2 + (1-5)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$BC = 5 \text{ units}$$

22. Show that any positive even integer can be written in the form $6q$, $6q+2$ or $6q+4$, where q is an integer. [2]

Ans :

Let a be any positive even integer.
Applying Euclid's division lemma with divisor 6, we get

$$a = 6q, 6q+1, 6q+2, 6q+3, 6q+4 \text{ or } 6q+5$$

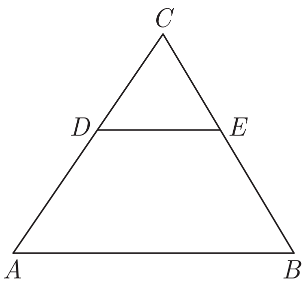
where, q is any whole number.

But $6q+1, 6q+3$ and $6q+5$ are odd integers and a is an even integer, therefore a cannot be of the form $6q+1, 6q+3$ or $6q+5$.

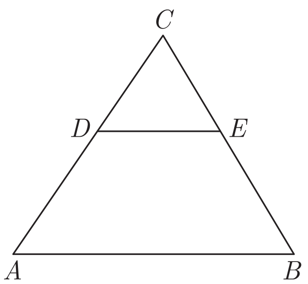
Hence, a is of the form $6q, 6q+2$ or $6q+4$, where q is some whole number.

or

23. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE \parallel AB$. [2]



Ans :



Given, $\angle A = \angle B$

$$CB = CA$$

(sides opposite equal angles in a triangle are equal)

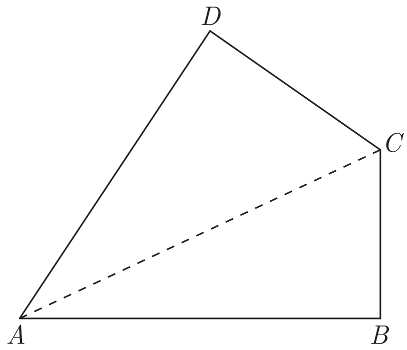
$$CE + BE = CD + AD$$

but, $AD = BE$ (given)

$$CE = CD$$
$$\frac{CE}{BE} = \frac{CD}{AD}$$
$$DE \parallel AB \quad \text{(converse of B.P.T.)}$$

24. In a quadrilateral $ABCD$, $\angle B = 90°$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90°$. [2]

Ans :



Given, in a quadrilateral $ABCD$,

$$\angle B = 90°$$

and $AD^2 = AB^2 + BC^2 + CD^2$

We need to prove that,

$$\angle ACD = 90°$$

Join AC. In ΔABC,

$$\angle B = 90°$$

By Pythagoras theorem,

$$AB^2 + BC^2 = AC^2 \quad \text{...(i)}$$

Given that $AD^2 = (AB^2 + BC^2) + CD^2$

$$= AC^2 + CD^2 \quad \text{[using (i)]}$$

Thus, in ΔACD,

$$AD^2 = AC^2 + CD^2,$$

therefore by the converse of Pythagoras theorem,

$$\angle ACD = 90°$$

25. If $\sqrt{3}\tan\theta = 3\sin\theta$, then find the value of $\sin^2\theta - \cos^2\theta$. [2]

Ans :

Given, $\sqrt{3}\tan\theta = 3\sin\theta$

$$\sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$$

$$1 = \sqrt{3}\cos\theta$$

or $\sin\theta = 0$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

or $\theta = 0$

When, $\cos\theta = \frac{1}{\sqrt{3}}$

$$\sin^2\theta - \cos^2\theta = (1-\cos^2\theta) - \cos^2\theta$$
$$= 1 - 2\cos^2\theta$$
$$= 1 - 2\left(\frac{1}{\sqrt{3}}\right) = 1 - \frac{2}{3} = \frac{1}{3}$$

Where, $\theta = 0$;

$$\sin^2\theta - \cos^2\theta = \sin^2 0 - \cos^2 0$$
$$= 0^2 - 1^2 = 0 - 1 = -1$$

or

Prove that :
$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Ans :

$$\text{LHS} = (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$$
$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A}$$

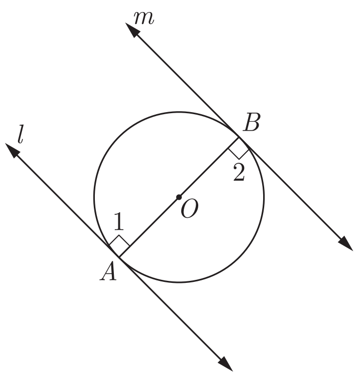
$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A}$$

$$= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A}$$

$$= \frac{2\sin A \cos A}{\sin A \cos A} = 2 = \text{RHS}$$

26. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [2]

Ans :



Let l and m be the tangents drawn at the end points A and B of a diameter AB of a circle with centre O. We need to show that l and m are parallel. Mark the angles as shown in the given figure.
Since, the tangent at a point to a circle is perpendicular to the radius through the point of contact, $AB \perp l$ and $BA \perp m$ $\angle 1 = 90^\circ$ and $\angle 2 = 90^\circ$ $\angle 1 = \angle 2$. But $\angle 1$ and $\angle 2$ form a pair of alternate angles, therefore, $l \parallel m$.

Section C

27. Prove that $\sqrt[3]{6}$ is an irrational number. [3]

Ans :

Suppose that $\sqrt[3]{6}$ is a rational number, then $\sqrt[3]{6} = \frac{p}{q}$, where p, q are both integers, $q \neq 0$ and p, q have no common factors (except 1)

$$6 = \left(\frac{p}{q}\right)^3$$

$$p^3 = 6q^3 \quad \text{...(i)}$$

As 2 divides $6q^3$, so 2 divides p^3 but 2 is prime 2 divides p
Let, $p = 2k$, where k is some integer.
Substituting this value of p in equation (i), we get

$$(2k)^3 = 6q^3$$

$$8k^3 = 6q^3$$

$$4k^3 = 3q^3$$

As 2 divides $4k^3$, so 2 divides $3q^3$ but 2 is prime 2 divides 3 or 2 divides q^3
But 2 does not divide 3, therefore, 2 divides q^3 but 2 is prime 2 divides q
Thus, p and q have a common factor 2. This contradicts that p and q have no common factors (except 1).

Hence, our supposition is wrong. Therefore, $\sqrt[3]{6}$ is an irrational number.

or

Use Euclid's division algorithm to find HCF of 441, 567, 693.

Ans :

First, we find HCF of 441 and 567.
Using Euclid's division algorithm, we have

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0$$

Thus,last non-zero remainder $= 63$

$$\text{HCF } (441, 567) = 63$$

Now, we find HCF of 63 and 693.
By using Euclid's division algorithm, we have

$$693 = 63 \times 11 + 0$$

So, last non-zero remainder $= 63$

$$\text{HCF } (63, 693) = 63$$

Hence, HCF of 441, 567, 693 $= 63$

28. A man earns ₹ 600 per month more than his wife. One-tenth of man's salary and one-sixth of wife's salary amount to ₹ 1500, which is saved every month. Find their salaries. [3]

Ans :

Let the salary of man be ₹ x and that of his wife's be ₹ y, then

according to given, $x - y = 600$ ...(i)

and $\frac{1}{10}x + \frac{1}{6}y = 1500$

$$3x + 5y = 45000 \quad \text{...(ii)}$$

Multiplying equation (i) by 5, we get

$$5x - 5y = 3000 \quad \text{...(iii)}$$

Adding equation (ii) and (iii), we get

$$8x = 48000$$

$$x = 6000$$

Substitute the $x = 6000$ in equation (i), we get

$$6000 - y = 600$$

$$y = 5400$$

Hence, the salary of man is ₹ 6000 and salary of his wife is ₹ 5400.

29. If the sum of first m terms of an AP is the same as the sum of its first n terms $(m \neq n)$, show that the sum of its first $(m + n)$ terms is zero. [3]

Ans :

Let a be the first term and d be the common difference of the AP.

Given, $S_m = S_n$

$$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$$

$$2am + m(m-1)d = 2an + n(n-1)d$$

$$2a(m-n) + (m^2 - n^2 - m + n)d = 0$$

$$2a(m-n)+(m-n)(m+n)-1(m-n)d = 0$$
$$(m-n)[2a+(m+n-1)d] = 0$$

but $m \neq n$

i.e. $m-n \neq 0$

$$2a+(m+n-1)d = 0 \quad ...(i)$$

Now, sum of first $(m+n)$ terms of the AP

$$= \frac{m+n}{2}[2a+(m+n-1)d]$$
$$= \frac{m+n}{2} \times 0 = 0$$

Hence, the sum of the first $(m+n)$ terms of the AP is zero.

30. If $\sin\theta = \frac{1}{2}$, then show that $3\cos\theta - 4\cos^3\theta = 0$. [3]

Ans :

Given, $\sin\theta = \frac{1}{2}$

$$\sin\theta = \sin 30^\circ$$
$$\theta = 30^\circ$$

Now, $3\cos\theta - 4\cos^3\theta = 3\cos 30^\circ - 4\cos^3 30^\circ$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$
$$= \frac{3\sqrt{3}}{2} - \frac{4 \times 3\sqrt{3}}{8}$$
$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

31. The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of distances travelled by their tips in 24 hours. (Use $\pi = 3.14$) [3]

Ans :

Long hand completes one round in 1 hour, so it will complete 24 rounds in 24 hours.

Distance travelled by the tip of long hand in 24 hours.

$$= (24 \times 2\pi \times 6)\text{ cm} = 288\text{ cm}$$

Short hand completes one round in 12 hour, it will complete 2 rounds in 24 hours.

Distance travelled by the tip of short hand in 24 hours.

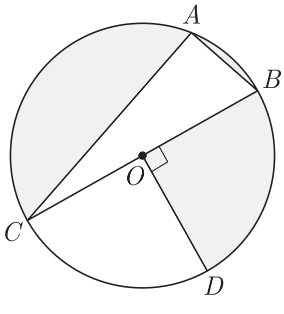
$$= (2 \times 2\pi \times 4)\text{ cm} = 16\pi\text{ cm}$$

The sum of distances travelled by the tips of both hands in 24 hours.

$$= (288\pi + 16\pi)\text{ cm} = (340 \times 3.14)\text{ cm}$$
$$= 954.56\text{ cm}$$

or

In the given figure, O is the centre of a circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90^\circ$. Find the area of the shaded region. (Use $\pi = 3.14$)



Ans :

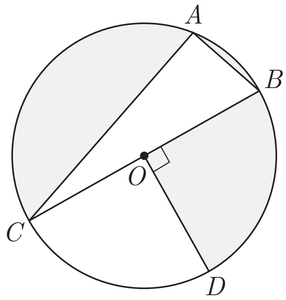
In ΔABC,

Given, $AC = 24$ cm

$$AB = 7\text{ cm}$$
$$\angle BAC = 90^\circ$$

(angle in semicircle is right angle)

In right angled ΔABC, by Pythagoras theorem,



$$BC^2 = AC^2 + AB^2$$
$$BC^2 = (24)^2 + (7)^2$$
$$BC^2 = 576 + 49 = 625$$
$$BC = 25\text{ cm}$$

Radius of circle with centre

$$O = OB = \frac{BC}{2} = \frac{25}{2}\text{ cm}$$

Now, area of shaded region

$$= \text{area of circle} - [\text{ar}(\Delta ABC) + \text{ar(quadrant)}]$$
$$= \pi \times \left(\frac{25}{2}\right)^2 - \left[\frac{1}{2} \times 24 \times 7 + \frac{1}{4} \times \pi \times \left(\frac{25}{2}\right)^2\right]$$
$$= 3.14 \times \frac{25}{2} \times \frac{25}{2} - \left[84 + \frac{1}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}\right]$$
$$= 490.625 - [84 + 122.656]$$
$$= 490.625 - 205.656 = 283.968 = 284\text{ cm}^2$$

32. If 65% of the population have black eyes, 25% have brown eyes and the remaining have blue eyes, what is the probability that a person selected at random has [3]

(i) blue eyes

(i) brown or black eyes

(iii) neither blue nor brown eyes?

Ans :

Given 65% of the population have black eyes, 25% have brown eyes and the remaining i.e. $(100-65-25)\%$ i.e. 10% have blue eyes. Let the total number of people be 100, then 65 have black eyes, 25 have brown eyes and 10 have blue eyes.

Thus, the sample space of the experiment has 100 equally likely outcomes.

(i) Let the event be 'have blue eyes'. As 10 people have blue eyes, so the number of favourable outcomes to the event 'have blue eyes' = 10.

$$P(\text{blue eyes}) = \frac{10}{100} = \frac{1}{10}$$

(ii) Let the event be 'have brown or black eyes'. The number of persons who have brown or black eyes $= 25 + 65 = 90$.

So, the number of favourable outcomes to the event 'have brown or black eyes' = 90.

$$P(\text{brown or black eyes}) = \frac{90}{100} = \frac{9}{10}$$

(iii) The event 'have neither blue nor brown eyes' mean have black eyes.
The number of persons who have neither blue nor brown eyes

$= \text{number of persons who have black eyes}$

$= 65$

So, the number of favourable outcomes to the event 'have brown or black eyes' $= 65$

$$P(\text{neither blue nor brown eyes}) = \frac{65}{100} = \frac{13}{20}$$

or

A child's game has 8 triangles of which 3 are blue and rest are red and 10 squares of which 6 are blue and rest are red. One piece is lost. Find the probability that the lost piece is a
(i) triangle or square
(ii) square of blue colour
(iii) triangle of red colour.

Ans :

Total number of pieces in the game $= 8 + 10 = 18$
As one piece is lost at random means each piece is to be lost equally likely. So the random experiment has 18 equally likely outcomes.

(i) The number of triangle or squares

$= 8 + 10$

$$P(\text{triangle or square}) = \frac{18}{18} = 1$$

(ii) Number of blue coloured squares $= 6$

$$P(\text{square of blue colour}) = \frac{6}{18} = \frac{1}{3}$$

(iii) Number of triangles of red colour

$= \text{total number of triangles} - \text{triangles of blue colour}$

$= 8 - 3 = 5$

$$P(\text{triangle of red colour}) = \frac{5}{18}$$

33. The median class of a frequency distribution is 125-145. The frequency of the median class and the cumulative frequency of the class preceding the median class are 20 and 22 respectively. Find the sum of the frequencies, if the median is 137. [3]

Ans :

Here, median class is $125 - 145$.

So, l = lower limit of median class $= 125$

h = size (width) of each class $= 20$

f = frequency of median class $= 20$

$c.f.$ = cumulative frequency of the class preceding median class $= 22$

Let n be the sum of frequencies, median $= 137$ (given)

We know that,

$$\text{Median} = l + \frac{\frac{n}{2} - c.f.}{f} \times h$$

$$137 = 125 + \frac{\frac{n}{2} - 22}{20} \times 20$$

$$12 = \frac{n}{2} - 22$$

$$\frac{n}{2} = 34$$

$$n = 68$$

Hence, the sum of frequencies $= 68$.

34. Find the mode of the given distribution: [3]]

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	12	22	8	6

Ans :

Class interval	Frequency
0-10 10-20	2 $12 = f_0$
$l = 20 - 30$	$22 = f_1$ Modal class
30-40 40-50	$8 = f_2$ 6

Here, the distribution is continuous and classes are of equal size.
As the class $20 - 30$ has maximum frequency, so modal class is $20 - 30$.

Here, $l = 20,\ h = 10,\ f_1 = 22,\ f_0 = 12,\ f_2 = 8$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 20 + \frac{22 - 12}{2 \times 22 - 12 - 8} \times 10$$

$$= 20 + \frac{10}{24} \times 10$$

$$= 20 + \frac{25}{6}$$

$$= 20 + 4.17 = 24.17$$

Hence, the mode of the given distribution $= 24.17$

Section D

35. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of form $a, a+b, a+2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial. [4]

Ans :

Given polynomial is $x^3 - 6x^2 + 3x + 10$.
Zeroes are $a, a+b, a+2b$ (given)

$$\text{Sum of the zeroes} = a + (a+b) + (a+2b)$$

$$= \frac{-\text{Coeff. of } x^2}{\text{Coeff. of } x^3}$$

$$= \frac{-(-6)}{1}$$

$$3a + 3b = 6$$

$$a + b = 2 \quad \text{...(i)}$$

$$\text{Product of the zeroes} = a(a+b)(a+2b)$$

$$= \frac{\text{Constant teram}}{\text{Coeff. of } x^3}$$

$$= \frac{-10}{1}$$

$$a \times 2 \times (a + 2b) = -10$$

$$a^2 + 2ab = -5 \quad \text{...(ii)}$$

From equation (i),

We have, $b = 2 - a$

Putting, $b = 2 - a$ in equation (ii), we get

$$a^2 + 2a(2 - a) = -5$$

$$a^2 + 4a - 2a^2 = -5$$

$$a^2 - 4a - 5 = 0$$

$$(a - 5)(a + 1) = 0$$

$$a = 5 \text{ or } a = -1$$

When, $a = 5,\ b = 2 - 5 = -3$

When, $a = -1,\ b = 2 - (-1) = 3$

Hence, $a = 5,\ b = -3$

or $a = -1,\ b = 3$

Now, the zeroes are 5, $5 + (-3)$, $5 + 2 \times (-3)$ i.e. 5, 2, -1

or -1, $-1 + 3$, $-1 + 2 \times 3$ i.e. -1, 2, 5

Hence, the zeroes are -1, 2, 5.

or

When a polynomial $p(x)$ is divided by $(x-1)$, the remainder is 5 and when it is divided by $(x-2)$, the remainder is 7. Find the remainder when $p(x)$ is divided by $(x-1)(x-2)$.

Ans :

Given, when $p(x)$ is divided by $(x-1)$, then remainder is 5

$$p(1) = 5 \quad \text{...(i)}$$

Also, when $p(x)$ is divided by $(x-2)$, then remainder is 7

$$p(2) = 7 \quad \text{...(ii)}$$

We have to find the remainder when $p(x)$ is divided by $(x-1)(x-2)$.

We know that when a polynomial is divided by another polynomial of degree 2, then the remainder can be a polynomial of degree 1 i.e. $ax + b$.

By division algorithm,

let $p(x) = (x-1)(x-2) \times q(x) + r(x)$

where, $q(x)$ is quotient and $r(x)$ is remainder

i.e. $p(x) = (x-1)(x-2) \times q(x) + (ax + b)$

$$p(1) = (1-1)(1-2) \times q(1) + (a + b)$$

$$5 = 0 + a + b$$

$$a + b = 5 \quad \text{...(iii) (using (i))}$$

Also, $p(2) = (2-1)(2-2) \times q(2) + (2a + b)$

$$7 = 0 + 2a + b \quad \text{(using (ii))}$$

$$2a + b = 7 \quad \text{...(iv)}$$

Subtracting (i) from (ii), we get

$$a = 2$$

Putting, $a = 2$ in equation (i), We get,

$$2 + b = 5$$

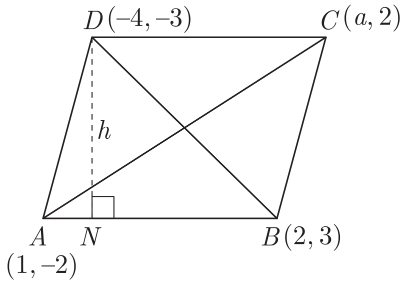
$$b = 3$$

Remainder $= 2x + 3$

36. If the points $A(1, -2)$, $B(2, 3)$, $C(a, 2)$ and $D(-4, -3)$ form a parallelogram, find the value of a and the height of the parallelogram taking AB as base. [4]

Ans :

As the points $A(1, -2), B(2, 3), C(a, 2)$ and $D(-4, -3)$ form a parallelogram, its diagonals AC and BD bisect each other i.e. they have same mid-point.



Mid-point of AC is $\left(\frac{1+a}{2}, \frac{-2+2}{2}\right)$ i.e. $\left(\frac{a+1}{2}, 0\right)$ and

mid-point of BD is $\left(\frac{2+(-4)}{2}, \frac{3+(-3)}{2}\right)$ i.e. $(-1, 0)$.

So, $\frac{a+1}{2} = -1$

$$a + 1 = -2$$

$$a = -3$$

Let h be the height of the parallelogram $ABCD$ taking AB as base, then the height of ΔABD is also h.

Area of ΔABD

$$= \frac{1}{2}\left|1(3+3) + 2(-3+2) + (-4)(-2-3)\right|$$

$$= \frac{1}{2}\left|6 - 2 + 20\right| = \frac{1}{2}\left|24\right| = 12$$

Distance $AB = \sqrt{(2-1)^2 + (3+2)^2}$

$$= \sqrt{1 + 25} = \sqrt{26}$$

Now area of $\Delta ABD = \frac{1}{2} \times AB \times h$

$$12 = \frac{1}{2} \times \sqrt{26} \times h$$

$$h = \frac{24}{\sqrt{26}} = \frac{24}{26} \times \sqrt{26}$$

$$= \frac{12}{13}\sqrt{26}$$

Hence, the value of a is -3 and the height of parallelogram $ABCD$ taking AB as base $= \frac{12\sqrt{26}}{13}$ units.

37. Find the positive value(s) of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both have real roots. [4]

Ans :

Given, the equation $x^2 + kx + 64 = 0$ has real roots

its discriminant ≥ 0

$$k^2 - 4 \times 1 \times 64 \geq 0$$

$$k^2 - 256 \geq 0$$

$$(k + 16)(k - 16) \geq 0$$

$$k \leq -16$$

or $k \geq 16$ ...(i)

Also the equation $x^2 - 8 + k = 0$ has real roots

its discriminant ≥ 0

$$(-8)^2 - 4 \times 1 \times k \geq 0$$

$$64 - 4k \geq 0$$

$$16 - k \geq 0$$

$$16 \geq k$$

$$k \leq 16 \quad \text{...(ii)}$$

The positive value of k which satisfies both (i) and (ii) $k = 16$.

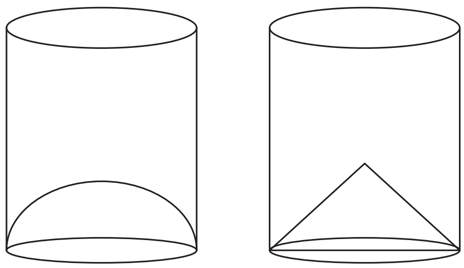
Hence, the positive value of k for which both the given equations have real roots is 16.

38. A milkman was serving his customers using two types of mugs A and B of inner diameter 5 cm and height 10 cm. The mug 'A' has hemispherical raised bottom and mug 'B' has conical raised bottom of height 1.5 cm as shown in given figure. [4]

He decided to serve the customers in 'B' type of mugs.

(i) Find the volume of the mugs of both type.

(ii) Which mathematical concept is used in the above problem?



Mug A Mug B

Ans :

Given, diameter of mugs $= 5$ cm

radius of mugs $= \frac{5}{2}$ cm

and height of mugs $= 10$ cm

(i) Volume of mug of type 'A'

$$= \text{Volume of cylinder} - \text{Volume of hemisphere}$$

$$= \pi r^2 h - \frac{2}{3}\pi r^3$$

$$= 3.14 \times 2.5 \times 2.5 \times 10 - \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$$

$$= 196.25 - 32.71 = 163.54 \text{ cm}^3$$

Volume of mug of type 'B'

$$= \text{Volume of cylinder} - \text{Volume of cone}$$

$$= \pi r^2 h - \frac{1}{3}\pi r^2 H$$

$$= 3.14 \times 2.5 \times 2.5 \times 10 - \frac{1}{3} \times 3.14 \times 2.5 \times 2.5 \times 1.5$$

(height of cone $= 1.5$ cm, given)

$$= 196.25 - 9.81 = 186.44 \text{ cm}^3$$

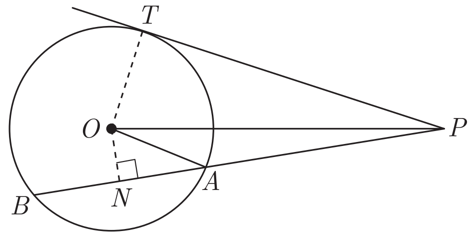
(ii) The mathematical concept used in the above problem is volume of combined solids (mensuration).

39. In the given figure, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular to the chord AB. Prove that : [4]

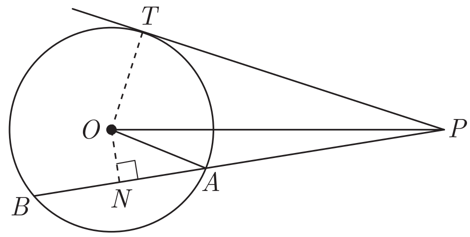
(i) $PA \cdot PB = PN^2 - AN^2$

(ii) $PN^2 - AN^2 = OP^2 - OT^2$

(iii) $PA \cdot PB = PT^2$



Ans :



(i) Since, $ON \perp AB$, N bisects AB,

so, $BN = AN$

$$PA \cdot PB = (PN - AN) \cdot (PN + BN) \text{ (from figure)}$$

$$= (PN - AN) \cdot (PN + AN) \quad (BN = AN)$$

$$PN^2 - AN^2$$

(ii) ΔONP,

$$\angle ONP = 90^\circ \quad (ON \perp NP)$$

By Pythagoras theorem,

$$ON^2 + AN^2 = OA^2$$

Now, $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$

$$= OP^2 - (ON^2 + AN^2)$$

$$= OP^2 - OA^2 = OP^2 - OT^2$$

($OT = OA$, being radius)

(iii) In ΔOTP,

$$\angle OTP = 90^\circ \text{ (tangent is } \perp \text{ to a radius)}$$

By Pythagoras theorem,

$$PT^2 = OP^2 - OT^2$$

Now, from part (i), we have

$$PA \cdot PB = PN^2 - AN^2$$

$$= OP^2 - OT^2 \quad \text{(using part (ii))}$$

$$= PT^2$$

or

Let ABC be a right triangle in which $AB = 3$ cm, $BC = 4$ cm and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. The circle through B, C and D is drawn. Construct the tangents from A to this circle.

Ans :

Steps of construction :

(i) Draw $BC = 4$ cm.

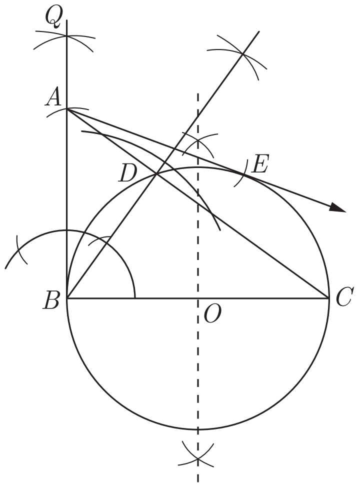
(ii) At B, construct $\angle QBC = 90^\circ$

(iii) From BQ, cut off $BA = 3$ cm.

(iv) Join AC, we get ΔABC with the given data.

(v) From B, draw BD perpendicular to AC.

(vi) Draw a circle with BC as diameter. This circle will pass through the point D because $\angle BDC = 90^\circ$.



(vii) As $AB \perp BC$, so AB is a tangent to the circle at B and its length $= 3$ cm.

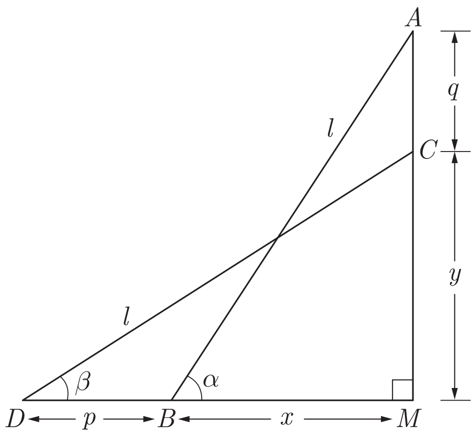
(viii) Taking A as centre and radius 3 cm (AB), draw an arc to cut the circle at E.

(ix) Join AE, then AE is a tangent to the circle at point E.

40. A ladder rests against a vertical wall at an inclination α to the horizontal. Its foot is pulled away from the wall through a distance p so that its upper end slides a distance q down the wall and then the ladder makes an angle β to the horizontal. Show that $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$. [4]

Ans :

Let AB be the original position of the ladder such that the top A of the ladder is on the wall AM and foot B is on the ground. When the foot of the ladder is pulled away from the wall by a distance p, then its top slides down on the wall be distance q. Let CD be the new position of the ladder so that $DB = p$ and $AC = q$.



Length of ladder $= AB = CD = l$ (say)

Let, $BM = x$ and $CM = y$

Given, $\angle ABM = \alpha$ and $\angle CDM = \beta$

From right angled ΔABM, we get

$$\sin\alpha = \frac{y+q}{l} \quad ...(i)$$

and $$\cos\alpha = \frac{x}{l} \quad ...(ii)$$

From right angled ΔCDM, we get

$$\sin\beta = \frac{y}{l} \quad ...(iii)$$

and $$\cos\beta = \frac{x+p}{l} \quad ...(iv)$$

Subtracting (iii) from (i), we get

$$\sin\alpha - \sin\beta = \frac{y+q}{l} - \frac{y}{l}$$

$$\sin\alpha - \sin\beta = \frac{q}{l} \quad ...(v)$$

Subtracting equation (ii) from (iv), we get

$$\cos\beta - \cos\alpha = \frac{x+p}{l} - \frac{x}{l}$$

$$\cos\beta - \cos\alpha = \frac{p}{l} \quad ...(vi)$$

Dividing (vi) by (v), we get

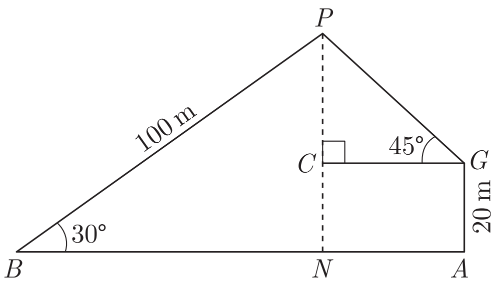
$$\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha}, \text{ as required.}$$

or

A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 330°. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45°. If the boy and the girl are on opposite sides of the bird, find the distance of the bird from the girl.

Ans :

Let the bird be at the point P. The boy is at the point B on the ground and the girl is at the point G on the roof, 20 m above the ground.



Given, $BP = 100$ m and

$AG = 20$ m

From right-angled ΔPBN,

$$\sin 30^\circ = \frac{NP}{BP}$$

$$\frac{1}{2} = \frac{NP}{100}$$

$$NP = 50 \text{ m}$$

$$CP = NP - NC = NP - AG$$

$$= 50 \text{ m} - 20 \text{ m} = 30 \text{ m}$$

From right-angled ΔPCG,

$$\sin 45^\circ = \frac{CP}{GP}$$

$$\frac{1}{\sqrt{2}} = \frac{30}{GP}$$

$$GP = 30\sqrt{2}$$

Hence, the distance of the bird from the girl $= 30\sqrt{2}$ m.