CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-19

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. If the points A$(1,2)$, O$(0,0)$ and B(a,b) are collinear, then [1]

(a) $a=b$ (b) $a=2b$
(c) $2a=b$ (d) $a=-b$

Ans : (c) $2a=b$

$$\frac{1}{2}|1(0-b)+0+a(2-0)| = 0$$
$$\frac{b}{2}+\frac{2a}{2} = 0$$
$$2a = b$$

2. The point which divides the line segment joining the points (7,-6) and (3,4) in the ratio 1:2 internally lies in the [1]

(a) Ist quadrant (b) IInd quadrant
(c) III quadrant (d) IVth quadrant

Ans : (d) IVth quadrant

Point is $\left(\frac{1\times 3+2\times 7}{1+2},\frac{1\times 4+2\times(-6)}{1+2}\right)$

i.e. $\left(\frac{17}{3},-\frac{8}{3}\right)$, which lies in IV quadrant

3. n^2-1 is divisible by 8, if n is [1]

(a) an integer (b) a natural number
(c) an odd integer (d) an even integer

Ans : (c) an odd integer

$$n^2-1 = 8\lambda$$
$$n^2 = 8\lambda+1$$

n^2 is odd and n is also odd

4. The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is [1]

(a) 27 (b) 72
(c) 63 (d) 36

Ans : (d) 36

Let the number be $10x+y$, then

$$x+y = 9$$

Also, $$10x+y+27 = 10y+x$$
$$y-x = 3$$

Solving equation (i) and (ii),

we get $x=3,\ y=6$

The number $= 10\times 3+6 = 36$

5. Two APs have the same common difference. The first term of one AP is -1 and that of the other is -8, Then the difference between their 4th term is [1]

(a) -1 (b) -8
(c) 7 (d) -9

Ans : (c) 7

$$[(-1+3d)-(-8+3d)] = 7$$

6. In triangles ABC and DEF, $\angle B=\angle E$, $\angle F=\angle C$ and $AB=3DE$, then the two triangles are [1]

(a) congruent but not similar
(b) similar but not congruent
(c) neither congruent not similar
(d) congruent as well as similar

Ans : (b) similar but not congruent

$$\angle B = \angle E \text{ and } \angle C = \angle F$$

So, by AA similarity,

$$\Delta ABC \sim \Delta DEF$$

Also, $$AB = 3DE \text{ (given)}$$
$$AB \neq DE$$

So triangles are not congruent
Hence, triangles are similar but not congruent.

7. If $\Delta ABC\sim\Delta PQR$, area of $\Delta ABC=81\text{cm}^2$, area of $\Delta PQR=144\text{cm}^2$ and $QR=6$ cm, then the length of BC is [1]

(a) 4 cm (b) 4.5 cm
(c) 9 cm (d) 12 cm

Ans : (b) 4.5 cm

$$\frac{area\ of\ \Delta ABC}{area\ of\ \Delta PQR} = \left(\frac{BC}{QR}\right)^2$$

$$\frac{81}{144} = \left(\frac{BC}{6}\right)^2$$

$$\frac{BC}{6} = \frac{9}{12}$$

$$BC = \frac{9}{2} = 4.5\text{ cm}$$

8. If $P(A)$ denotes the probability of an event A, then [1]

(a) $P(A) = 0$ (b) $P(A) > 0$
(c) $0 \le P(A) \le 1$ (d) $-1 \le P(A) \le 1$

Ans : (c) $0 \le P(A) \le 1$

9. If a fair die is rolled once, then the probability of getting an even number or a number greater than 4 is [1]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

Ans : (d) $\frac{2}{3}$

Total number of outcomes $= 6$
Favourable outcomes are 2,4,5,6, these are 4 in number

Required probability $= \frac{4}{6} = \frac{2}{3}$

10. The probability of getting a bad egg in a lot of 400 eggs is 0.035. The number of bad eggs in the lot is [1]

(a) 7 (b) 14
(c) 21 (d) 28

Ans : (b) 14

$$P(\text{bad egg}) = \frac{\text{Number of bad eggs}}{\text{Total number of eggs}}$$

$$0.035 = \frac{\text{Number of bad eggs}}{400}$$

Number of bad eggs $= 14$

(Q.11-Q.15) Fill in the blanks.

11. If the lines represented by $3x + 2y + 5 = 0$ and $kx - 6y + 4 = 0$ are parallel, then $k =$.......... [1]

Ans :

$$\frac{3}{k} = \frac{2}{-6}$$

$$k = -9$$

or

The solution of the pair of linear equations $x + 2y = 5$ and $2x - y = 5$ is..........

Ans :

$$x + 2y = 5 \quad ...(i)$$

$$4x - 2y = 10 \quad ...(ii)$$

After adding equation (i) and (ii), we get

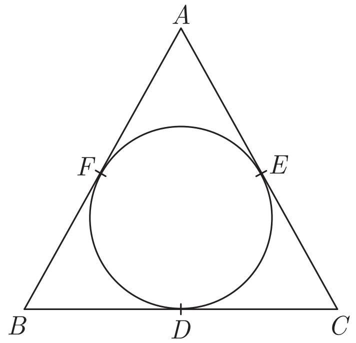
$$5x = 15 \Rightarrow x = 3$$

From equation (i), we get

$$3 + 2y = 5 \Rightarrow 2y = 2 \Rightarrow y = 1$$

$$x = 3,\ y = 1$$

12. If the given figure, sides BC,CA and AB of ΔABC touch a circle at point D,E and F respectively. If $BD = 4$ cm, $DC = 3$ cm and $CA = 8$ cm, then the length of side AB is.......... [1]



Ans :

Here, $CE = CD$

(Length of tangents from an external point)

$CE = 3$ cm

$AE = AC - CE = 8 - 3 = 5$ cm

$AF = AE;\ AF = 5$ cm

Also, $FB = BD = 4$ cm

$AB = AF + FB = 5 + 4 = 9$ cm

13. The probability of an event that cannot happen is.......... [1]

Ans : 0

14. The length of a rope by which a cow must be tethered in order that it may be able to graze an area of 616 m^2 is.......... [1]

Ans : 14 m

Let the length of rope by l,

then $\pi l^2 = 616$

$$\frac{22}{7} \times l^2 = 616$$

$$l^2 = \frac{616 \times 7}{22} = 196$$

$$l = 14\text{ m}$$

15. The distance of a point (2,-3) from the $x-axis$ is......... [1]

Ans : 3 unit

Distance of (2,-3) from $x-axis$ is 3 units

(Q.16-Q.20) Answer the following

16. Explain why 13233343563715 is a composite number? [1]

Ans :

The given number ends in 5, so it is a multiple of 5, hence it is a composite number.

or

If HCF $(a,8) = 4$ and LCM $(a,8) = 24$, then find the value of a.

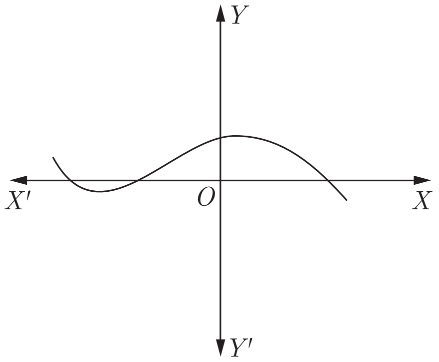
Ans :

$$\text{HCF} \times \text{LCM} = \text{product of numbers}$$

$$4 \times 24 = a \times 8$$

$$a = \frac{4 \times 24}{8} \Rightarrow a = 12.$$

17. Write number of zeroes of the polynomial $y = f(x)$ whose graph is given alongside. [1]



Ans :

Since, the graph of the polynomial

$y = f(x)$ intersects the $x-$ axis at three points. So, the number of zeroes of the given polynominal is 3.

18. If $(x+a)$ is a factor of $2x^2+2ax+5x+10=0$, then find the value of a. [1]

Ans :

Given $(x+a)$ is a factor of

$$2x^2+2ax+5x+10 = 0$$

On putting $x=-a$ in the given quadratic equation

$$2(-a)^2+2a(-a)+5(-a)+10 = 0$$
$$2a^2-2a^2-5a+10 = 0$$
$$-5a+10 = 0$$
$$5a = 10$$
$$a = 2.$$

19. If $\tan\theta+\cot\theta=5$, find the value of $\tan^2\theta+\cot^2\theta$. [1]

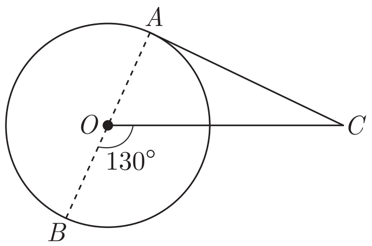
Ans :

Given $\tan\theta+\cot\theta = 5$

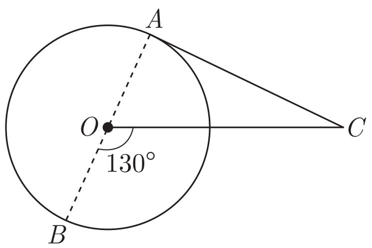
Squaring both the sides, we get

$$\tan^2\theta+\cot^2\theta+2.\tan\theta\frac{1}{\tan\theta} = 25$$
$$\tan^2\theta+\cot^2\theta = 25-2$$
$$\tan^2\theta+\cot^2\theta = 23.$$

20. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC=130°$, then find $\angle ACO$ [1]



Ans :



Given, $\angle BOC = 130°$

Also, given AC is a tangent and OA is radius

$$\angle OAC = 90°$$

In ΔABC, $\angle BOC = \angle ACO+\angle OAC$

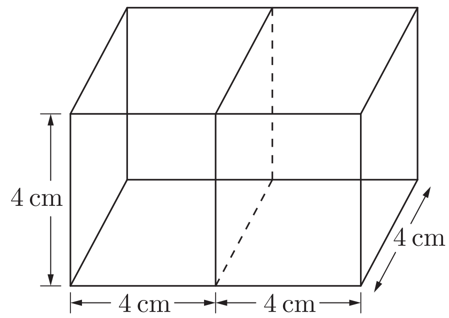
$$130° = \angle ACO+90°$$
$$\angle ACO = 130°-90°$$
$$\angle ACO = 40°.$$

Section B

21. Two cubes, each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid. [2]

Ans :

When two cubes are joined end to end, the resulting solid (cuboid) is shown in the given figure.



Length of cuboid, $l = 8\text{ cm}$,

breadth of cuboid, $b = 4\text{ cm}$ and

height of cuboid, $h = 4\text{ cm}$.

$$\text{Surface area of the cuboid} = 2(lb+bh+hl)$$
$$= 2(8\times4+4\times4+4\times8)\text{ cm}^2$$
$$= 160\text{ cm}^2$$

22. If the product of zeroes of the polynomial ax^2-6x-6 is 4, find the value of a. [2]

Ans :

Given polynomial is ax^2-6x-6.

$$\text{Product of zeroes} = \frac{\text{Constanterm}}{\text{Coefficient of } x^2}$$
$$4 = \frac{-6}{a}$$
$$a = \frac{-6}{4} \Rightarrow a = \frac{-3}{2}.$$

23. The first and last term of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference. [2]

Ans :

Given, in an AP

$$a = 5,\ l = 45,\ \text{sum} = 400.$$

Let the common difference be d and number of terms be n.

$$S_n = \frac{n}{2}(a+l)$$
$$400 = \frac{n}{2}(5+45)$$
$$n = \frac{400\times2}{50}$$
$$n = 16$$

Now, $l = a+(n-1)d$

$40 = 15d$

$d = \frac{40}{15} = \frac{8}{3}.$

Hence, the common difference is $\frac{8}{3}$.

or

Find the sum of all 13 terms of an AP whose middle term is 42.

Ans :

Let the AP be a, $a+d, a+2d, ...$

As there are 13 terms in AP so its middle term is 7th term.

Given, middle term $= 42$

$a+6d = 42$

Now, sum of 13 terms $= \frac{13}{12}[2a+(13-1)d]$

$= \frac{13}{2}(2a+12d)$

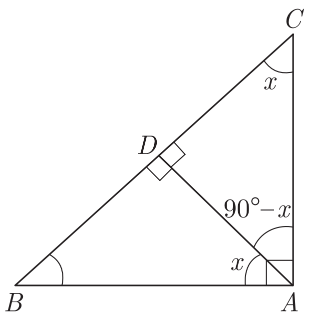
$= 13 \times (a+6d)$

$= 13 \times 42$ [using(i)]

$= 546$

24. In $\Delta ABC, \angle BAC = 90^\circ$ and $AD \perp BC$. Prove that $AD^2 = BD \times DC$. [2]

Ans :



In right ΔABC, $\angle BAC = 90^\circ$ and $AD \perp BC$,

so $\angle ADB = \angle ADC = 90^\circ$

Let $\angle ACD$ be x,

then $\angle DAC = 90^\circ - x$

$\angle BAC = 90^\circ$

$\angle BAD + \angle DAC = 90^\circ$

$\angle BAD = 90^\circ - (90^\circ - x)$

$\angle BAD = x$

In ΔADB and ΔCDA,

$\angle ADB = \angle CDA$ (each $= 90^\circ$)

$\angle DCA = \angle DAB$ (proved above)

So $\Delta ADB \sim \Delta CDA$ (by AA similarity criterion)

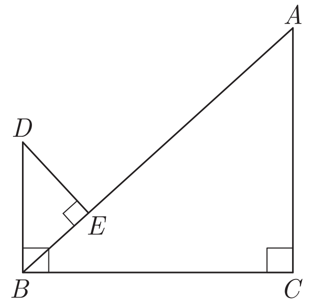
$\frac{BD}{AD} = \frac{AD}{DC}$

$AD^2 = BD \times DC.$

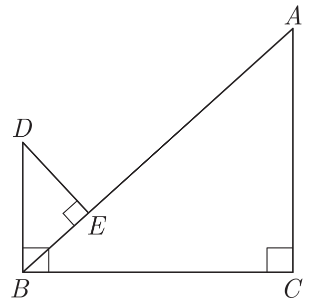
or

In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.

Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Ans :



Given, $DB \perp BC$, $AC \perp BC$

$\angle DBC = \angle ACB = 90^\circ$

$AC \parallel DB$

$\left(\begin{array}{l}\text{sum of interior} \angle s \text{ on the same side of} \\ BC \text{ is } 180^\circ\end{array}\right)$

$\angle ABD = \angle BAC$ (alt$\angle s$)

Now, in ΔBED and ΔACB,

$\angle EBD = \angle ABD = \angle BAC$

$\angle C = \angle E$ (each $= 90^\circ$)

$\Delta BED \sim \Delta ACB$ (AA similarity)]

$\frac{BE}{AC} = \frac{DE}{BC}$

$\frac{BE}{DE} = \frac{AC}{BC}$, as required.

25. If $\cos(A+B) = 0$ and $\sin(A-B) = \frac{1}{2}$, then find the values of A and B, where A and B are acute angles. [2]

Ans :

Given $\cos(A+B) = 0$

and $\sin(A-B) = \frac{1}{2}$

$\cos(A+B) = \cos 90^\circ$

and $\sin(A-B)$ $\sin = 30^\circ$

$A+B = 90^\circ$ ...(i)

and $A-B = 30^\circ$ ...(ii)

Adding equation (i) and (ii), we get

$2A = 120^\circ \Rightarrow A = 60^\circ$

Putting $A = 60^\circ$ in equation (i), we get

$60^\circ + B = 90^\circ \Rightarrow B = 30^\circ$

Hence, $A = 60^\circ;\ B = 30^\circ$.

26. If $x = p\sec\theta + q\tan\theta$ and $y = p\tan\theta + q\sec\theta$, then prove that $x^2 - y^2 = p^2 - q^2$. [2]

Ans :

Given $x = p\sec\theta + q\tan\theta$ ...(i)

and $y = p\tan\theta + q\sec\theta$ ...(ii)

Squaring and subtracting equation (i) and (ii), we get

$$\begin{aligned} x^2 - y^2 &= (p\sec\theta + q\tan\theta)^2 - (p\tan\theta + q\sec\theta)^2 \\ &= p^2\sec^2\theta + q^2\tan^2\theta + 2pq\sec\theta\tan\theta \\ &\quad -(p^2\tan^2\theta + q^2\sec^2\theta + 2pq\sec\theta\tan\theta) \\ &= p^2\sec^2\theta + q^2\tan^2\theta + 2pq\sec\theta\tan\theta \\ &\quad -(p^2\tan^2\theta - q^2\sec^2\theta - 2pq\sec\theta\tan\theta) \\ &= p^2(\sec^2\theta - \tan^2\theta) - q^2(\sec^2\theta - \tan^2\theta) \\ &= p^2(1) - q^2(1) \qquad (\sec^2\theta - \tan^2\theta = 1) \\ &= p^2 - q^2 \end{aligned}$$

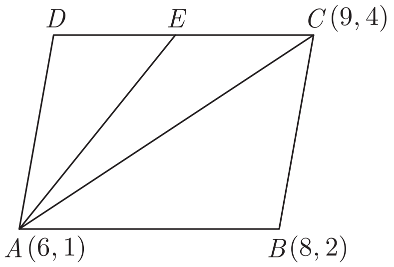
Hence,

$$x^2 - y^2 = p^2 - q^2.$$

Section C

27. $A(6,1)$, $B(8,2)$ and $C(9,4)$ are the three vertices of a parallelogram ABCD. If E is the mid-point of DC, find the area of ΔADE. [3]

Ans :



$$\begin{aligned} \text{Area of } \Delta ABC &= \frac{1}{2}|6(2-4)+8(4-1)+9(1-2)| \\ &= \frac{1}{2}|-12+24-9| = \frac{1}{2}|3| \\ &= \frac{3}{2} \text{ sq. units.} \end{aligned}$$

We know that a diagonal of a parallelogram divides it into two triangles of equal area, therefore,

$$\text{area of } \Delta ACD = \frac{3}{2} \text{sq. units}$$

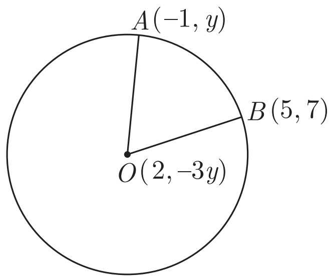
As E is mid-point of DC, AE is a median of ΔACD. Also, we know that a median of a triangle divides it into two triangles of equal area, therefore,

$$\begin{aligned} \text{area of } \Delta ADE &= \frac{1}{2}(\text{area of } \Delta ACD) \\ &= \left(\frac{1}{2} \times \frac{3}{2}\right) = \frac{3}{4} \text{ sq. units.} \end{aligned}$$

or

Points $A(-1,y)$ and $B(5,7)$ lie on a cicrle with centre $O(2,-3y)$. Find the value of y. Hence, find the radius of the circle.

Ans :



As points $A(-1,y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$,

$$\begin{aligned} OA &= OB \qquad \text{(each being radius)} \\ OA^2 &= OB^2 \\ (-1-2)^2 + (y-(3y))^2 &= (5-2)^2 + (7-(-3y))^2 \\ 9 + (4y)^2 &= 9 + (7+3y)^2 \\ 16y^2 &= 49 + 42y + 9y^2 \\ 7y^2 - 42y - 49 &= 0 \\ y^2 - 6y - 7 &= 0 \\ (y-7)(y+1) &= 0 \quad y = 7, -1. \end{aligned}$$

$$\begin{aligned} \text{Radius of circle} &= OA \\ &= \sqrt{(-1-2)^2 + (y-(-3y))^2} \\ &= \sqrt{9+16y^2} \end{aligned}$$

When, $y = 7$,

$$\begin{aligned} \text{radius} &= \sqrt{9+16\times 7^2} = \sqrt{9+784} \\ &= \sqrt{793} \text{ units;} \end{aligned}$$

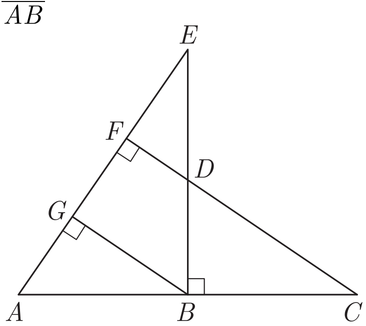
When, $y = -1$,

$$\begin{aligned} \text{radius} &= \sqrt{9+16\times(-1)^2} = \sqrt{9+16} \\ &= 5 \text{ units.} \end{aligned}$$

28. In the given figure, $EB \perp AC$, $BG \perp AE$ and $CF \perp AE$. Prove that: [3]

(i) $\Delta ABG \sim \Delta DCB$

(ii) $\dfrac{BC}{BD} = \dfrac{BE}{AB}$



Ans :

(i)Given $BG \perp AE$ and $CF \perp AE$

$$\begin{aligned} BG &\parallel CF \\ \angle ABG &= \angle BCD \end{aligned}$$

In ΔABG and ΔDCB,

$$\begin{aligned} \angle ABG &= \angle BCD \qquad \text{(proved above)} \\ \angle AGB\ &\angle CBD \end{aligned}$$

$$\left(\begin{aligned} &\text{each} = 90^\circ \text{ because } BG \perp AE \\ &\qquad\qquad \text{and } EB \perp AC \end{aligned}\right)$$

$$\Delta ABG \sim \Delta DCB$$

(AA similarity criterion)

$$\angle GAB = \angle BDC \text{ i.e. } \angle EAB = \angle BDC$$

(ii) In $\wedge ABE$ and ΔDBC,

$$\begin{aligned} \angle EAB &= \angle BDC \qquad \text{(proved above)} \\ \angle ABE &= \angle DBC \end{aligned}$$

$$\left(\begin{aligned} &\text{each} = 90^\circ \text{ because } BG \perp AE \\ &\qquad\qquad \text{and } EB \perp AC \end{aligned}\right)$$

$$\Delta ABE \sim \Delta DBC$$

(AA similarity criterion)

$$\frac{AB}{BD} = \frac{BE}{BC}$$

$$\frac{BC}{BD} = \frac{BE}{AB}, \text{ as required}$$

29. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$. [3]

Ans :

Let a be the first term and d be the common difference of the AP, then

$$3(S_8 - S_4) = 3\left[\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)\right]$$

$$= 3[4(2a+7d) - 2(2a+3d)]$$

$$= 6(4a+14d-2a-3d)$$

$$= \frac{12}{2}(2a+11d) = S_{12}.$$

Hence, $S_{12} = 3(S_8 - S_4)$.

or

The sum of the first five terms and the sum of first seven terms of the same AP is 167. If the sum of first ten terms of this AP is 235, find the sum of its first twenty terms.

Ans :

Let a be the first term and d be the common difference of the A.P.

According to given,

$$S_5 + S_7 = 167$$

$$\frac{5}{2}(2a+4d) + \frac{7}{2}(2a+6d) = 167$$

$$5(a+2d) + 7(a+3d) = 167 \quad \text{...(i)}$$

$$12a + 31d = 167$$

and $S_{10} = 235$

$$\frac{10}{2}(2a+9d) = 235 \quad \text{...(ii)}$$

$$2a + 9d = 47$$

Multiplying equation (ii) by 6, we get ...(iii)

$$12a + 54d = 282$$

Substituting $d = 5$ in equation (ii), we get

$$2a + 9 \times 5 = 47$$

$$2a = 2$$

$$a = 1.$$

Sum of first 20 terms

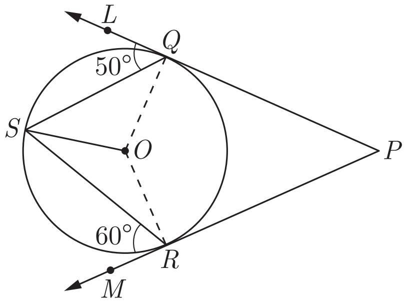
$$S_{20} = \frac{20}{2}(2a+19d)$$

$$= 10(2 \times 1 + 19 \times 5)$$

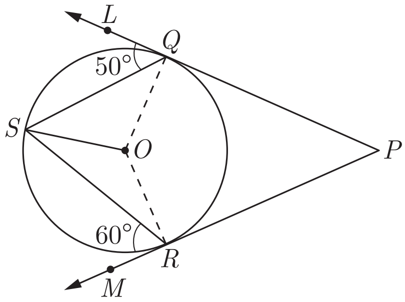
$$= 10 \times 97$$

$$= 970.$$

30. In the given figure, PQL and PRM are tangents to the circle with centre O at the points Q and R, respectively and S is a point on the circle such that $\angle SQL = 50^\circ$ and $\angle SRM = 60^\circ$. Find $\angle QSR$. [3]



Ans :



Since PQ is tangent to the circle and OQ is radius,

$$OQ \perp PQ.$$

Similarly, $OR \perp PR$.

$$\angle OQS = \angle OQL - \angle SQL = 90^\circ - 50^\circ$$

$$= 40^\circ.$$

But $\angle OSQ = \angle OQS$ (in $\Delta OSQ, OS = OQ$ both are radius of circle)

$$\angle OSQ = 40^\circ.$$

$$\angle ORS = \angle ORM - \angle SRM = 90^\circ - 60^\circ = 30^\circ.$$

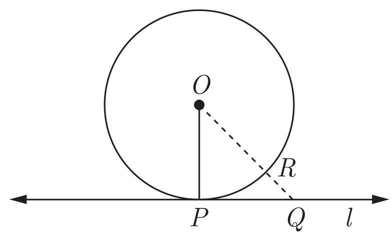
But $\angle OSR = \angle ORS$ (in $\Delta OSR, OS = OR$)

$$\angle OSR = 30^\circ$$

$$\angle QSR = \angle OSQ + \angle OSR = 40^\circ + 30^\circ = 70^\circ$$

31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. [3]

Ans :



Given: A circle with centre O and a tangent l at a point P of the circle.

To prove: $OT \perp l$.

Construction: Take a point of Q, other than P, on l and join OQ.

Proof: Since Q is a point on the tangent l, other than the point of contact P, so Q lies outside the circle.

(because every point on the tangent to a circle, other than the point P, lies outside the circle).

Let OQ intersect the circle at the point R.

Now, $OQ = OR + RQ$ (from figure)

$$OQ > OR$$

$OQ > OP \quad (OP = OR = \text{radius of the circle})$

$OP < OQ.$

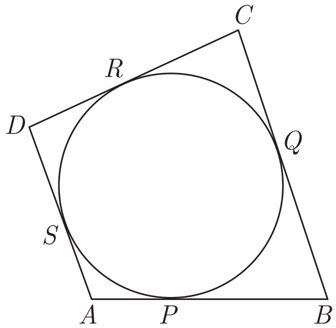
Thus, OP is shorter than any other line segment joining O to any point of l, other than the point P i.e. OP is the shortest distance between point O and the line l.

But the shortest distance between a point and a line is the perpendicular distance from the point to the line.

Hence, $OP \perp l$.

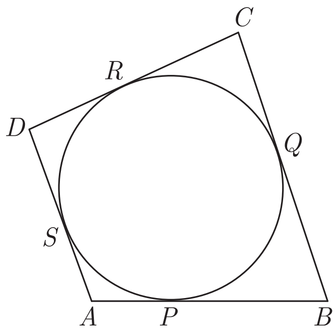
or

In the given figure, a quadrilateral ABCD circumscribes a circle. Prove that $AB + CD = AD + BC$



Ans :

The circle touches the sides of the quad. ABCD at the points P,Q,R and S as shown in the figure.



Since the lengths of tangents drawn from an external point to a circle are equal,

$AP = AS,\ BP = BQ,\ CR = CQ$

and $DR = DS$.

On adding these results, we get

$(AP + BP) + (CR + DR)$

$= AS + BQ + CQ + DS$

$AB + CD = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC.$

32. Prove that:

$$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$$ [3]

Ans :

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

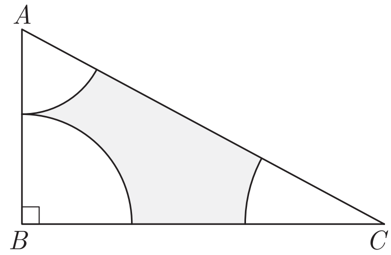
$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2 - \cos^2\theta}$$

$$= \frac{1 - 2\sin\theta\cos\theta + 1 + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{2}{\sin^2\theta - 1 + \sin^2\theta} = \frac{2}{2\sin^2\theta - 1} = \text{RHS}$$

33. In the given figure, ABC is a triangle right angled at B with $AB = 14$ cm and $BC = 24$ cm. With vertices A, B and C as centers, arcs are drawn, each of radius 7 cm. Find the area of the shaded region. [3]



Ans :

Let, $\angle BAC = \theta_1$,

$\angle ACB = \theta_2$ and $\angle CBA = \theta_3$.

$\angle BAC + \angle ACB + \angle CBA$ (sum of all interior $\angle s$ of a Δ)

$\theta_1 + \theta_2 + \theta_3 = 180°$

Now, area of shaded region

$= ar(\Delta ABC) - [ar(\text{sector at A}) + ar(\text{sector at B}) + ar(\text{sector at C})]$

$$= \frac{1}{2}\,base \times height - \left[\frac{\theta_1}{360°} \times \pi r^2 + \frac{\theta_2}{360°} \times \pi r^2 + \frac{\theta_3}{360°} \times \pi r^2\right]$$

$$= \frac{1}{2} \times 24 \times 14 - \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3)$$

$$= 12 \times 14 - \frac{\pi}{360°} \times 7^2 \times 180° \quad \text{(given } r = 7\text{)} \quad \text{(using } (i)\text{)}$$

$$= 168 - \frac{1}{2} \times \frac{22}{7} \times 7^2 = 168 - 77 = 91 \text{ cm}^2$$

Hence, the area of shaded region

$= 91 \text{ cm}^2$

34. Find the mean of the following data: [3]

Class	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	15	37	74	99	120

Ans :

Class	Cumulative frequency	Class interval	Frequency f_i	Mid value x_i	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
Less than 20	55	0-20	15	10	-2	-30
Less than 40	37	20-40	22	30	-1	-22

Class	Cumulative frequency	Class interval	Frequency f_i	Mid value x_i	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
Less than 60	74	40-60	37	50	0	0
Less than 80	99	60-80	25	70	1	25
Less than 100	120	80-100	21	90	2	42
			$\Sigma f_i = 120$			$\Sigma f_i u_i = 15$

$$\text{Mean } = a + h \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 50 + 20 \times \frac{15}{120}$$

$$= 50 + 2.5 = 52.5$$

Hence, mean $= 52.5$

Section D

35. Prove that one and only one out of $n, n+2$ or $n+4$ is divisible by 3, where n is any positive integer. [4]

Ans :

Given, n is any positive integer.
Applying Euclid's division lemma with divisor $= 3$, we get
$n = 3q$, $3q+1$ or $3q+2$, where q is some whole number.
Three cases arise.
Case I. When $n = 3q$
n is divisible by 3;
$n+2 = 3q+2$, which leaves remainder 2 when divided by 3
$n+2$ is not divisible by 3;
$n+4 = 3q+4 = 3(q+1)+1$, which leaves remainder 1 when divided by 3
$n+4$ is not divisible by 3.
Thus, in this case, n is divisible by 3 but both $n+2$ and $n+4$ are not divisible by 3.
Case II. When $n = 3q+1$
n leaves remainder 1 when divided by 3
n is not divisible by 3;
$n+2 = (3q+1)+2 = 3(q+1)$, which is divisible by 3;
$n+4 = (3q+1)+4 = 3q+5 = 3(q+1)+2$, which leaves remainder 2 when divided by 3
$n+4$ is not divisible by 3.
Thus, in this case, $n+2$ is divisible by 3 but both n and $n+4$ are not divisible by 3.
Case III When $n = 3q+2$
n leaves remainder 2 when divided by 3
n is not divisible by 3;
$n+2 = (3q+2)+2 = 3q+4 = 3(q+1)+1$, which leaves remainder 1 when divided by 3
$n+2$ is not divisible by 3;
$n+4 = (3q+2)+4 = 3q+6 = 3(q+2)$, which is divisible by 3.
Thus, in all the three cases, we find that one and only one out of n, $n+2$ and $n+4$ is divisible by 3.

36. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and $x-$axis. [4]

Ans :

The given equations can be written as

$$y = x + 1 \quad ...(i)$$

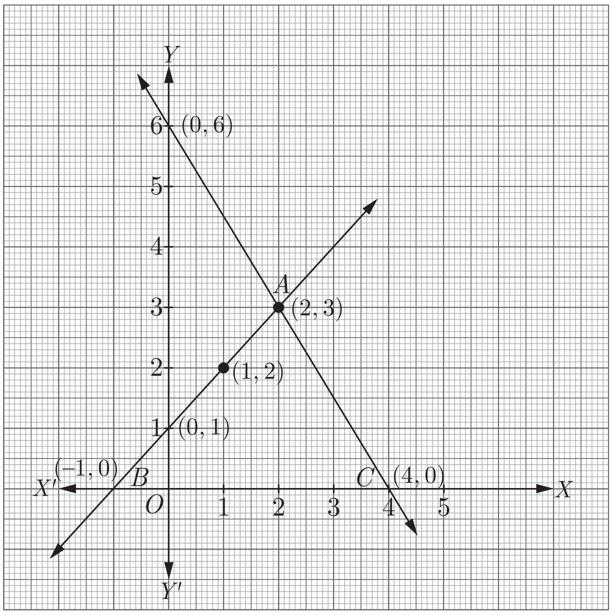
and $$y = \frac{12 - 3x}{2} \quad ...(ii)$$

Table of values of (i)

x	0	1
y	1	2

Table of values of (ii)

x	0	2
y	6	3



Select the coordinate axes on a graph paper and take1 cm $= 1$ unit on both axes. To draw the line representing equation (i) plot the points (0,1) and (1,2) and draw the line passing through these points. To draw the line representing equation (ii) plot the points (0,6) and (2,3) and draw the line passing through these points.
The lines represented by equations (i) and (ii) intersect at $A(2,3)$ and these lines cut the $x-axis$ at $B(-1,0)$ and $C(4,0)$.
Hence, the vertices of triangle formed by the given line and $x-axis$ are (2,3), (-1,0) and (4,0).

37. .Two water pipes together can fill a tank $9\frac{3}{8}$ hours. The pipe of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each pipe can fill the tank separately. [4]

Ans :

The tank is filled by the two pipes together in $9\frac{3}{8}$ hours *i.e.* in $\frac{75}{8}$ hours, the part of tank filled in one hour $= \frac{8}{75}$.

Let the time taken by the pipe of larger diameter to fill the tank separately be x hours, then the time taken by the pipe of smaller diameter to fill the tank separately $= (x+10)$ hours.

The part of the tank filled by the pipe of larger diameter in one hour

$$= \frac{1}{x} \text{ and}$$

the part of the tank filled by the pipe of smaller diameter in one hour

$$= \frac{1}{x+10}.$$

According to problem,

$$\frac{1}{x} + \frac{1}{x+10} = \frac{8}{75} \Rightarrow \frac{(x+10)+x}{x(x+10)} = \frac{8}{75}$$

$$8x(x+10) = 75(2x+10)$$

$$8x^2 + 80x = 150x + 750$$

$$8x^2 - 70x - 750 = 0$$

$$4x^2 - 35x - 375 = 0$$

$$(x-15)(4x+25) = 0$$

$$x = 15 \text{ or } x = -\frac{25}{4}$$ but x being the time taken cannot be negative

$$x = 15.$$

Hence, the time taken to fill the tank by the two pipes separately is 15 hours and $(x+10)$ hours *i.e.* 25 hours.

or

Solve the following equation:

$$\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}, x \neq \frac{4}{3}.$$

Ans :

Given $$\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2},\ x \neq \frac{4}{3}$$

Let, $\frac{3x-4}{7} = y$, then equation (i) becomes

$$y + \frac{1}{y} = \frac{5}{2}$$

$$\frac{y^2+1}{y} = \frac{5}{2}$$

$$2y^2 + 2 = 5y$$

$$2y^2 - 5y + 2 = 0$$

$$2y^2 - 4y - y + 2 = 0$$

$$2y(y-2) - 1(y-2) = 0$$

$$(y-2)(2y-1) = 0$$

$$y = 2 \text{ or } \frac{1}{2}$$

When

$$y = 2,\ \frac{3x-4}{7} = 2$$

$$3x - 4 = 14$$

$$3x = 18\ x = 6$$

When

$$y = \frac{1}{2},\ \frac{3x-4}{7} = \frac{1}{2}$$

$$6x - 8 = 7$$

$$6x = 15$$

$$x = \frac{15}{6}$$

$$x = \frac{5}{2}$$

Hence, $x = 6,\ \frac{5}{2}$.

38. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. [4]

Ans :

Radius of the base of metal cylinder $= 4.2$ cm

$= $ radius of hemisphere.

Height of cylinder $= 10$ cm

Volume of metal in the rest of cylinder after scooping out hemispheres from each end

$$= \text{volume of cylinder} - 2(\text{volume of hemisphere})$$

$$= \left[\pi \times (4.2)^2 \times 10 - 2 \times \frac{2}{3}\pi \times (4.2)^3\right] \text{cm}^3$$

$$= \left[\pi \times (4.2)^2 \left(10 - \frac{4}{3} \times 4.2\right)\right] \text{cm}^3$$

$$= \left[\pi \times (4.2)^2 \times (4.4)\right] \text{cm}^3$$

Radius of wire

$$= \frac{1.4}{2} \text{ cm} = 0.7 \text{ cm}$$

Let l cm be the length of the cylindrical wire, then volume of wire

$$= \left[\pi \times (0.7)^2 \times l\right] \text{cm}^3$$

Volume of metal in wire

$$= \text{volume of metal in the rest of cylinder}$$

$$\pi \times (0.7)^2 \times l = \pi \times (4.2)^2 \times 4.4$$

$$l = (6^2 \times 4.4) \text{cm}$$

$$= (36 \times 4.4) \text{ cm} = 158.4 \text{ cm}$$

or

A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

Ans :

Radius of the well $= \frac{4}{2}$m $= 2$ m.

Volume of earth dug out $= (\pi \times 2^2 \times 14)\text{m}^3$

$= 56\pi\text{m}^3$

Height of embankment $= 40$ cm

$= \frac{40}{100}$ m $= \frac{2}{5}$ m

Inner radius of embankment $= 2$ m.

Let the width of the embankment be x metres $(x > 0)$, then

outer radius of embankment $= (x+2)$ m.

Volume of embankment $= \left[\pi(x+2)^2 - 2^2 \times \frac{2}{5}\right]\text{m}^3$

Since the earth dug out is to be converted into embankment,

$$\pi\left[(x+2)^2-2^2\right]\times\frac{2}{5} = 56\pi$$

$$x^2+4x = 140$$

$$x^2+4x-140 = 0$$

$$(x-10)(x+14) = 0$$

$$x = 10 \text{ or } x = -14 \text{ but } x>0$$

$$x = 10.$$

Hence, the width of embankment

$$= 10\,\text{m}.$$

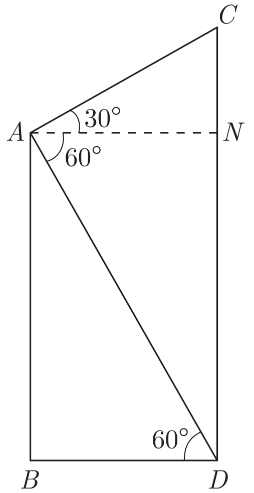
39. The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60 m high, are 30° and 60° respectively. Find the difference between the height of the building and tower and the distance between them. $(use\sqrt{3} = 1.732)$. [4]

Ans :

Let AB be the building and CD be the tower.

$AB = 60\,\text{m}$ (given).

AN is horizontal, so ABDN is a rectangle.



The angles of elevation and depression are shown in the figure. Note that

$\angle CAN = 30°$ and $\angle ADB = 60°$.

From right angled $\Delta\,ABD$,

$$\tan 60° = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{60}{BD}\,\text{m}$$

$$BD = \frac{60}{\sqrt{3}}\,\text{m} = 20\sqrt{3}\,\text{m}$$

$$= (20\times 1.732)\text{m} = 34.64\,\text{m}$$

From right angled $\Delta\,CAN$,

$$\tan 30° = \frac{CN}{AN} = \frac{1}{\sqrt{3}} = \frac{CN}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{CN}{20\sqrt{3}\,\text{m}}$$

$$CN = 20\,\text{m}$$

The difference between the heights of building and tower

$$\text{CN} = 20\,\text{m}.$$

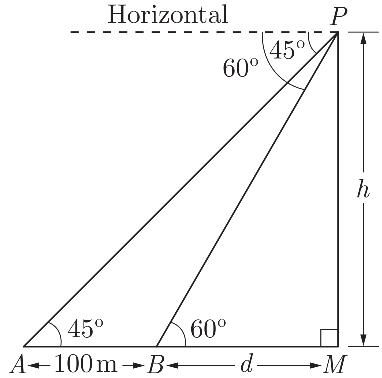
and distance between them $= BD = 34.64\,\text{m}$

or

From a balloon vertically above a straight road, the angles of depression of two cars at an instant are found to be 45° and 60°.If the cars are 100 m apart, find the height of the balloon.

Ans :

Let the height of the ballon at P be h metres. Let A and B be the two cars.



Thus $AB = 100$ m

Angles of depression are shown in the given figure.

Note that $\angle PAM = 45°$ and $\angle PBM = 60°$.

Let $BM = d$ metres.

From right angled $\Delta\,MBP$,

$$\tan 60° = \frac{h}{d}$$

$$\sqrt{3} = \frac{h}{d}$$

$$d = \frac{h}{\sqrt{3}} \quad \text{...(i)}$$

From right angled $\Delta\,MAP$,

$$\tan 45° = \frac{MP}{AM} \quad 1 = \frac{h}{100+d}$$

$$100+d = h$$

$$100+\frac{h}{\sqrt{3}} = h \quad \text{...(using (i))}$$

$$100\sqrt{3} = \sqrt{3}\,h - h$$

$$h = \frac{100\sqrt{3}}{\sqrt{3}-1} = \frac{100\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 500(3+\sqrt{3}).$$

Hence, the height of the balloon is 500 $(3+\sqrt{3})$ m.

40. Find the median of the following data: [4]

Marks	**0 or above**	**10 or above**	**20 or above**	**30 or above**	**40 or above**	**50 or above**	**60 or above**	**70 or above**	**80 or above**	**90 or above**	**100 or above**
Number of students	80	77	72	65	55	43	28	16	10	8	0

Ans :

Marks	No. of students	Classes	Frequency	Cumulative frequency
0 or above	80	0-10	3	3
10 or above	77	10-20	5	8
20 or above	72	20-30	7	15
30 or above	65	30-40	10	25
40 or above	55	40-50	12	37
50 or above	43	50-60	15	52
60 or above	28	60-70	12	64
70 or above	16	70-80	6	70
80 or above	10	80-90	2	72
90 or above	8	90-100	8	80
100 or above	0	100 or above	0	-

Note that the cumulative frequency of the class 50-60 is 52, which is greater than $\frac{n}{2}$ i.e. 40 and is nearest to it, so the median class is 50-60.

Here, $l = 50,\ c.f. = 37,\ f = 15$ and $h = 10$.

$$\text{Median} = l + \frac{\frac{n}{2} - c.f.}{f} \times h$$

$$= 50 + \frac{40 - 37}{15} \times 10$$

$$= 50 + \frac{3}{15} \times 10$$

$$= 50 + 2 = 52$$

Hence, the median of the given data is 52.