CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-18

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. It is given that $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR} = \frac{1}{3}$, then $\frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to [1]

(a) 9 (b) 3
(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

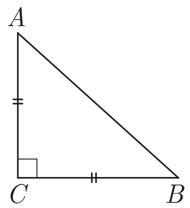
Ans : (a) 9

$$\frac{ar(\Delta PRQ)}{ar(\Delta BCA)} = \left(\frac{QR}{BC}\right)^2 = \left(\frac{3}{1}\right)^2 = 9$$

2. If ABC is an isosceles right triangle, right angled at C, then [1]

(a) $AC^2 = 2AB^2$ (b) $AB^2 = 2BC^2$
(c) $AB^2 + BC^2 = AC^2$ (d) $2AB^2 = BC^2$

Ans : (b) $AB^2 = 2BC^2$



Now,
$$AB^2 = AC^2 + BC^2$$
$$AB^2 = BC^2 + BC^2 \qquad (AC = BC)$$
$$AB^2 = 2BC^2$$

3. The diameters of the two circular ends of a bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. The capacity of the bucket is [1]

(a) 32.7 litres (b) 33.7 litres
(c) 34.7 litres (d) 31.7 litres

Ans : (a) 32.7 litres

The capacity of bucket

$$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 35\left[(22)^2 + (12)^2 + 22 \times 12\right]$$
$$= \frac{110}{3}(484 + 144 + 264)\,\text{cm}^3$$
$$= \frac{110}{3} \times 892\,\text{cm}^3$$
$$= 32706.66\,\text{cm}^3 = 32.7 \text{ litres}$$

4. If a solid circular cylinder of iron whose diameter is 15 cm and height 10 cm is melted and recasted into a sphere, then the radius of the sphere is [1]

(a) 15 cm (b) 10 cm
(c) 7.5 cm (d) 5 cm

Ans : (c) 7.5 cm

$$\pi \times \left(\frac{15}{2}\right)^2 \times 10 = \frac{4}{3}\pi r^3$$
$$r^3 = \frac{15}{2} \times \frac{15}{2} \times 10 \times \frac{3}{4}$$
$$r^3 = \left(\frac{15}{2}\right)^3$$
$$r = 7.5\,\text{cm}$$

5. The pair of linear equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have [1]

(a) unique solution
(b) exactly two solutions
(c) infinitely many solutions
(d) no solutions

Ans : (d) no solutions

$$\frac{1}{-3} = \frac{2}{-6} \neq \frac{5}{1}$$

no solution

6. The zeroes of the quadratic polynomial $x^2 + 99x + 127 = 0$ are [1]

(a) both positive
(b) both negative
(c) one positive and one negative
(d) none of these

Ans : (b) both negative

Let the zeroes be α and β are of same sign

$$\alpha + \beta = -99 \text{ (–ve)}$$
and
$$\alpha\beta = 127 \text{ (+ve)}$$

α and β are of same sign
$\alpha + \beta$ is –ve
α, β both are –ve

7. In an AP, if $a = -5$, $l = 21$ and $S = 200$, then n is equal to [1]
(a) 50 (b) 40
(c) 32 (d) 25

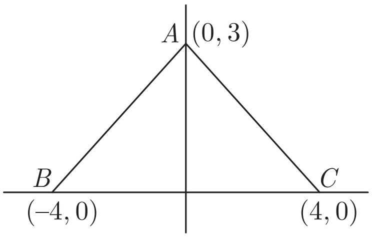
Ans : (d) 25

$$S = \frac{n}{2}(a+l)$$
$$200 = \frac{n}{2}(-5+21)$$
$$n = \frac{400}{16} = 25$$

8. The points (-4, 0), (0, 3) are the vertices of a [1]
(a) right triangle (b) isosceles triangle
(c) equilateral triangle (d) scalene triangle

Ans : (b) isosceles triangle

Let $A(0,3)$, $B(-4,0)$ and $C(4,0)$.



$$AB = AC$$
so, triangle is isosceles

9. The end points of a diameter of a circle are (-2, 3) and (4, -5), then the coordinates of its centre are [1]
(a) (2, –2) (b) (1, –1)
(c) (–1, 1) (d) (–2, 2)

Ans : (b) (1, –1)

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-2+4}{2}, \frac{3-5}{2}\right) = (1, -1)$$

10. If θ and $\theta + 36°$ are acute angles and $\sin(\theta + 36°) = \cos\theta$, then the value of θ is [1]
(a) 27° (b) 18°
(c) 54° (d) 36°

Ans : (a) 27°

$$\sin(\theta + 36°) = \cos\theta = \sin(90° - \theta)$$
$$\theta + 36° = 90° - \theta$$
$$2\theta = 54°$$
$$\theta = 27°$$

(Q.11-Q.15) Fill in the blanks.

11. The base radii of a cone and a cylinder are equal. If their volumes area also equal, then the ratio of height of cone to height of cylinder is .... [1]

Ans :

Let the radius and height of cone be r and h and radius and height of cylinder be R and H.

Given $$V_{\text{Cone}} = V_{\text{Cylinder}}$$
$$\frac{1}{3}\pi r^2 h = \pi R^2 H$$
$$\frac{h}{H} = 3\frac{R^2}{r^2}$$
$$\frac{h}{H} = \frac{3}{1} \qquad (r = R, \text{ given})$$

Hence, the ratio is 3 : 1

12. The (directed) distance of a point from the y-axis is called its .... [1

Ans : x-coordinate or abscissa

13. The graph of a linear equation in two variables is always a .... [1]

Ans : straight line

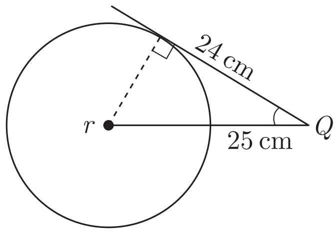
14. There is no tangent to a circle passing through a point lying .... the circle. [1]

Ans : inside

or

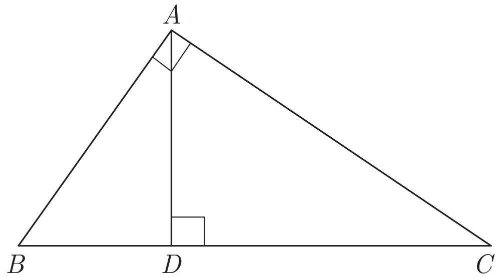
From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is ....

Ans :



$$r = \sqrt{(25)^2 - (24)^2}$$
$$= \sqrt{625 - 576} = \sqrt{49}$$
radius $= 7$ cm

15. In the adjoining figure, $\angle BAC = 90°$ and $AD \perp BC$, then $AD^2 = BD \times$ .... [1]



Ans :

$$\angle BAC = 90° \text{ and } AD \perp BC,$$
$$\Delta DBA \sim \Delta DAC \qquad \text{(AA similarity)}$$
$$\frac{AD}{DC} = \frac{BD}{AD}$$
$$AD^2 = BD \times DC$$

(Q.16-Q.20) Answer the following

16. a and b are two positive integers such that the least prime factor of a is 3 and least prime factor of b is 5. Then calculate the least prime factor of $(a+b)$. [1]

Ans :

As 3 is the least prime factor of a and 5 is the least prime factor of b, therefore, a and b both are odd integers and sum of two odd integers is an even integer i.e. $a+b$ is an even integer.
Hence, least prime factor of $(a+b)$ is 2.

17. Find the value of k, so that system of linear equations $2x+3y=4$ and $(k+2)x+6y=3k+2$ has an infinite number of solutions. [1]

Ans :

Given $2x+3y = 4$

and $(k+2)x+6y = 3k+2$

For infinite number of solutions, we have

$$\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

$$\frac{2}{k+2} = \frac{3}{6}$$

$$3k+6 = 12$$

$$k = 2$$

Also $\frac{3}{6} = \frac{4}{3k+2}$

$$9k+6 = 24$$

$$k = 2$$

Hence, $k = 2$

18. Find the common difference of the following AP: [1]
$\frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, ...$

Ans :

Given AP is $\frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, ...$

i.e. $\frac{1}{2b}, \frac{1}{2b}-3, \frac{1}{2b}-6, ...$

Common difference $= \frac{1}{2b} - 3 - \frac{1}{2b} = -3.$

19. If ΔABC is right angled at B, what is the value of $\sin(A+C)$? [1]

Ans :

In ΔABC,

$$A+B+C = 180°$$

$$A+C+90° = 180° \qquad (\angle B = 90°, \text{given})$$

$$A+C = 90°$$

$$\sin(A+C) = \sin 90°$$

$$\sin(A+C) = 1$$

or

If $\sqrt{3}\sin\theta = \cos\theta$, find the value of $\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta+2}$.

Ans :

Given $\sqrt{3}\sin\theta = \cos\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

$$\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta+2} = \frac{\cos\theta(3\cos\theta+2)}{3\cos\theta+2}$$

$$\cos\theta = \cos 30° = \frac{\sqrt{3}}{2}$$

20. Form the following distribution, find the median class: [1]

Class interval	1400-1550	1550-1700	1700-1850	1850-2000
Frequency	8	15	21	8

Ans :

Class interval	Frequency	Cumulative frequency
1400-1550	8	8
1550-1700	15	23
1700-1850	**21**	**44**
1850-2000	8	52
	$n = 52$	

Here, $n = 52$

$$\frac{n}{2} = 26$$

Median class is 1700-1850

Section B

21. If the zeroes of the polynomial x^2+px+q are double in value to the zeroes of $2x^2-5x-3$, find the values of p and q. [2]

Ans :

$$2x^2-5x-3 = 2x^2-6x+x-3$$

$$= (2x+1)(x-3)$$

Zeroes are $2x+1 = 0$

or $x-3 = 0$

$$x = -\frac{1}{2} \text{ or } 3$$

Hence, zeroes of the polynomial x^2+px+q are $2\times\left(-\frac{1}{2}\right)$ and 2×3 i.e. -1 and 6

Sum of zerores $= -1+6 = 5$

Product of zeroes $= -1\times 6 = -6$

Quadratic polynomial is x^2- (sum of zeroes)$x+$ product of zeroes i.e. x^2-5x-6

Comparing it with x^2+px+q, we have

$$p = -5$$

$$q = -6.$$

22. Find the prime factorisation of the denominator of rational number expressed as $6.\overline{12}$ in simplest form. [2]

Ans :

Let $x = 6.\overline{12} = 6.12121212$ ...(i)

Multiplying both sides of (i) by 100, we get

$100x = 612.121212$ ...(ii)

Subtracting equation (i) from (ii), we get

$$99x = 606$$
$$x = \frac{606}{99}$$
$$x = \frac{202}{33}, \text{ which is in simplest form}$$
$$\text{Its denominator} = 33$$

Prime factorisation of denominator

$$= 3 \times 11.$$

or

Prove that the sum of a rational number and an irrational number is always an irrational number.

Ans :

Let a be a rational number and b be an irrational number.

Let us assume that $a+b$ is rational, say r.

Then $a + b = r$

$$b = r - a$$

As r and a are both rational numbers, so $r - a$ is a rational number

b is a rational number.

But this contradicts that b is irrational

Hence, our assumption is wrong. Therefore, $a + b$ is an irrational number

i.e. the sum of a rational and an irrational number is always an irrational number.

23. Find the value, of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. [2]

Ans :

Let one root of the given quadratic equation

$$px^2 - 14x + 8 = 0$$

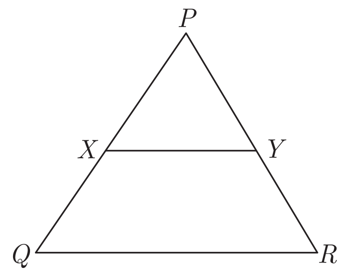
be α then the other root is 6α.

$$\alpha + 6\alpha = \text{Sum of roots}$$
$$= \frac{-\text{coeff. of } x}{\text{coeff. of } x^2} = \frac{14}{p}$$
$$7\alpha = \frac{14}{p}$$
$$\alpha = \frac{2}{p} \quad \text{...(i)}$$

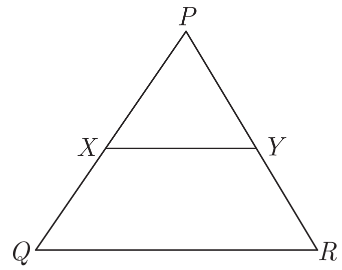
and

$$\alpha . 6\alpha = \text{Product of roots}$$
$$= \frac{\text{Constant term}}{\text{Coeff. of } x^2} = \frac{14}{p}$$
$$6\alpha^2 = \frac{8}{p}$$
$$6\left(\frac{2}{p}\right)^2 = \frac{8}{p} \quad \text{(using (i))}$$
$$\frac{24}{p^2} = \frac{8}{p}$$
$$8p^2 - 24p = 0$$
$$8p(p-3) = 0$$
$$p = 0 \text{ or } 3 \quad (p \neq 0)$$
$$p = 3$$

24. In the given figure, $XY || QR$. If $\frac{PQ}{XQ} = \frac{7}{3}$ and $PR = 6.3$ cm, find YR. [2]



Ans :



Given $\frac{PQ}{XQ} = \frac{7}{3}$

$$\frac{XQ}{PQ} = \frac{3}{7} \quad \text{...(i)}$$

As $XY \parallel QR$,

by corollary to $B.P.T.$., we have

$$\frac{YR}{PR} = \frac{XQ}{PQ}$$
$$\frac{YR}{6.3} = \frac{3}{7} \quad \text{[using (i)]}$$
$$7YR = 3 \times 6.3$$
$$YR = \frac{3 \times 6.3}{7}$$
$$YR = 2.7 \text{ cm}$$

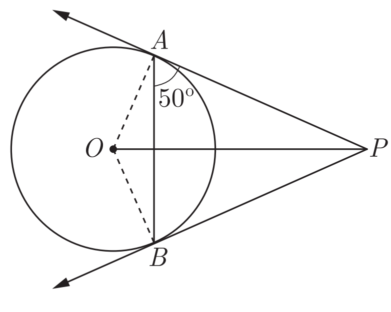
25. From an external point P tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$. [2]

Ans :

As AP is tangent to the circle at A and OA is radius,

so $OA \perp AP$

$$\angle OAP = 90°$$
$$\angle OAB = \angle OAP - \angle PAB$$
$$= 90° - 50° = 40°$$



In ΔOAB, $OA = OB$ (each being radius)

$$\angle OBA = \angle OAB$$
$$\angle OBA = 40°$$

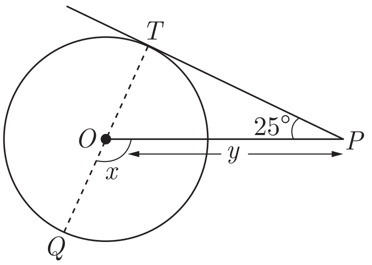
In ΔOAB,

$$\angle AOB + \angle OAB + \angle OBA = 180°$$

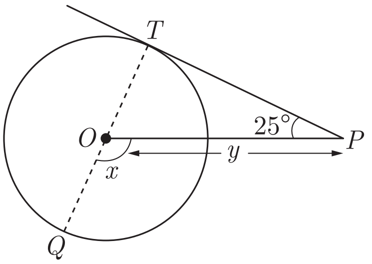
$$\angle AOB + 40^\circ + 40^\circ = 180^\circ$$
$$\angle AOB = 100^\circ$$

or

In the given figure, PT is a tangent at T to the circle with centre O and radius 3 cm. If $\angle TPO = 25^\circ$ and $PT = 4$ cm, then find the value of x and y.



Ans :



PT is tangent at T

$$\angle OTP = 90^\circ$$

In ΔOTP, $\angle QOP$ is an exterior angle.

So, $\angle QOP = \angle OTP + \angle OPT$

$$x = 90^\circ + 25^\circ$$
$$x = 115^\circ$$

Again, OT = radius = 3 cm (given)

and PT = 4 cm (given)

In ΔOTP, by Pythagoras theorem,

$$OP^2 = OT^2 + PT^2$$
$$= 3^2 + 4^2 = 9 + 16 = 25$$
$$OP = 5 \text{ cm}$$
$$y = 5 \text{ cm}$$
$$x = 115^\circ$$

and $y = 5$ cm

26. Without using the trigonometric tables, evaluate the following: [2]

$$\frac{11 \sin 70^\circ}{7 \cos 20^\circ} - \frac{4}{7} \frac{\cos 53^\circ . \operatorname{cosec} 37^\circ}{\tan 15^\circ . \tan 35 . \tan 55^\circ . \tan 75^\circ}$$

Ans :

$$\frac{11}{7} \frac{\sin 70^\circ}{\cos 20^\circ} - \frac{4}{7} \frac{\cos 53^\circ . \operatorname{cosec} 37^\circ}{\tan 15^\circ . \tan 35^\circ . \tan 55^\circ . \tan 75^\circ}$$

$$= \frac{11}{7} \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} - \frac{4}{7} \frac{\cos 53^\circ . \operatorname{cosec}(90^\circ - 53^\circ)}{\tan 15^\circ . \tan 35^\circ . \tan(90^\circ - 35^\circ) . \tan(90^\circ - 15^\circ)}$$

$$= \frac{11}{7} \frac{\cos 20^\circ}{\cos 20^\circ} - \frac{4}{7} \frac{\cos 53^\circ . \sec 53^\circ}{\tan 15^\circ . \tan 35^\circ . \cot 35^\circ . \cot 15^\circ}$$

$$= \frac{11}{7} - \frac{4}{7} \frac{\cos 53^\circ . \frac{1}{\cos 53^\circ}}{\tan 15^\circ . \tan 35^\circ . \frac{1}{\tan 35^\circ} . \frac{1}{\tan 15^\circ}}$$

$$= \frac{11}{7} - \frac{4}{7} = \frac{7}{7} = 1.$$

Section C

27. Find the greatest number of six digits exactly divisible by 18, 24 and 36. [2]

Ans :

Prime factorisation of 18, 24 and 36

$$18 = 2 \times 3 \times 3$$
$$24 = 2 \times 2 \times 2 \times 3$$

and $36 = 2 \times 2 \times 3 \times 3$

LCM of 18, 24 and 36

$$= 2 \times 2 \times 2 \times 3 \times 3 = 72$$

The greatest number of 6 digits is 999999.

Dividing 999999 by 72, we get

```
      13888
  72)999999
    -72
     279
    -216
      639
    - 576
       639
     - 576
        639
      - 576
         63
```

Here, remainder = 63

The required number $= 999999 - 63 = 999936$.

Hence, the greatest 6 digits number exactly divisible by 18, 24 and 36 is 999936.

28. If $(x + a)$ is a factor of two polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that $\alpha = \frac{n - q}{m - p}$. [3]

Ans :

Let $f(x) = x^2 + px + q$

and $g(x) = x^2 + mx + n$.

Given $x + \alpha$ is a factor of two polynomials $f(x)$ and $g(x)$

$-\alpha$ is a zero of both polynomials $f(x)$ and $g(x)$

$$f(-\alpha) = 0$$

and $g(-\alpha) = 0$

$$(-\alpha)^2 + p(-\alpha) + q = 0$$

and $(-\alpha)^2 + m(-\alpha) + n = 0$

$$\alpha^2 - p\alpha + q = 0 \quad \text{...(i)}$$

and $\alpha^2 - m\alpha + n = 0$ ...(ii)

Subtracting equation (ii) from (i), we get

$$-p\alpha + m\alpha + q - n = 0$$
$$(m - p)\alpha = n - q$$
$$\alpha = \frac{n - q}{m - p}.$$

29. Solve the following equation for x: [3]

$$\frac{x + 1}{x - 1} + \frac{x - 2}{x + 2} = 4 - \frac{2x + 3}{x - 2}, x \neq 1, -2, 2.$$

Ans :

Given $\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}$

$$\frac{(x+1)(x+2)+(x-1)(x-2)}{(x-1)(x+2)} = \frac{4(x-2)-(2x+3)}{x-2}$$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$

$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$

$$(2x^2+4)(x-2) = (x^2+x-2)(2x-11)$$

$$2x^3-4x^2+4x-8 = 2x^3+2x^2-4x-11x^2-11x+22$$

$$-4x^2+4x-8 = -9x^2-15x+22$$

$$5x^2+19x-30 = 0$$

$$5x^2+25x-6x-30 = 0$$

$$5x(x+5)-6(x+5) = 0$$

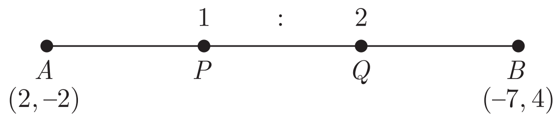
$$(x+5)(5x-6) = 0$$

$$x = -5, \frac{6}{5}$$

Hence, the roots of the given equation are $-5, \frac{6}{5}$.

30. Let P and Q be the points of trisection of the line segment joining the points $A(2,-2)$ and $B(-7,4)$ such that P is nearer to A. Find the coordinates of P and Q. [3]

Ans :



As P and Q are points of trisection of the line segment joining the points $A(2,-2)$ and $B(-7,4)$, and P is nearer to A, so

$$AP = PQ = QB$$

$$AP:PB = 1:2$$

and $AQ:QB = 2:1$

$$P\left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right) i.e. (-1,0)$$

and $Q\left(\frac{2(-7)+2(2)}{1+2}. \frac{2(4)+1(-2)}{2+1}\right) i.e. (-4,2)$

or

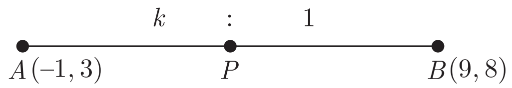
Point P divides the line segment joining the points $A(-1,3)$ and $B(9,8)$ such that $\frac{AP}{PB}=\frac{k}{1}$. If P lies on the line $x-y+2=0$, find the value of k.

Ans :

Given, $\frac{AP}{PB} = \frac{k}{1}$

$$AP:PB = k:1.$$

P divides the line segment joining the points $A(-1,3)$ and $B(9,8)$ in the ratio of $k:1$.



$$P\left(\frac{9k-1}{k+1}, \frac{8k+3}{k+1}\right).$$

As point $P\left(\frac{9k-1}{k+1}, \frac{8k+3}{k+1}\right)$ lies on the line

$x-y+2 = 0$, we have

$$\frac{9k-1}{k+1}-\frac{8k+3}{k+1}+2 = 0$$

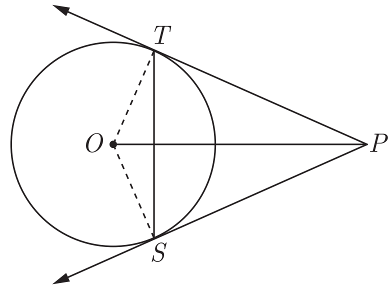
$$(9k-1)-(8k+3)+2(k+1) = 0$$

$$9k-1-8k-3+2k+2 = 0$$

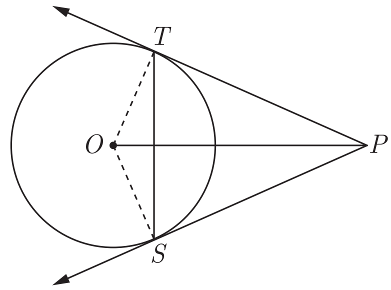
$$3k-2 = 0$$

$$k = \frac{3}{2}.$$

31. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If $OP=2r$, show that $\angle OTS = \angle OST = 30°$. [3]



Ans :



Radius of circle $= OT = OS = r$.

As PT is tangent to the circle at T and OT is radius,

$$OT \perp PT$$

$$\angle OTP = 90°.$$

Let $\angle POT = \theta$.

In right angled triangle OTP,

$$\cos\theta = \frac{OT}{OP}$$

$$\cos\theta = \frac{r}{2r} \quad (OP=2r, \text{given})$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60°$$

i.e. $\angle POT = 60°$.

We know that OP is the bisector of $\angle TOS$,

$$\angle TOS = 2\angle POT$$

$$= 2 \times 60° = 120°.$$

In $\Delta OTS, OT = OS$ (each = radius)

$$\angle OTS = \angle OST$$

(angles opp. equal sides are equal)

$$\angle TOS + \angle OTS + \angle OST = 180°$$

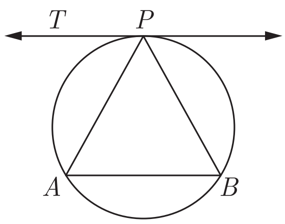
(sum of angles in a Δ)

$$120^\circ + \angle OTS + \angle OTS = 180^\circ$$
$$2\angle OTS = 60^\circ$$
$$\angle OTS = 30^\circ.$$

Hence, $\angle OTS = \angle OST = 30^\circ$.

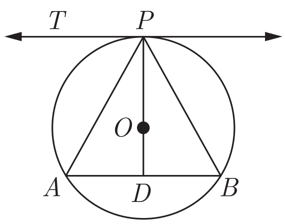
or

A tangent PT is drawn parallel to a chord AB of a circle. as shown in the adjoining figure. Prove that PAB is an isosceles triangle.



Ans :

Let O be the centre of the circle. Join PO and product to meet AB at D.



Since PT is tangent to the circle at P and OP is radius,

$OP \perp PT$

i.e. $PD \perp PT$.

Given $PT \parallel AB$,

also $PD \perp PT$

$PD \perp AB$

i.e. $OD \perp AB$.

Now, AB is a chord of a circle with centre O and

$OD \perp AB$. Therefore,

OD bisects the chord AB *i.e.*

$AD = BD$.

In ΔADP and ΔBDP,

$\angle ADP = \angle BDP$ (each $= 90^\circ$, $PD \perp AB$)

$AD = BD$ (proved above)

$PD = PD$ (common)

$\Delta ADP \cong \Delta BDP$ (SAS criterion of congruency)

$AP = BP$ (c,p.c.t.)

Hence, PAB is an isosceles triangles.

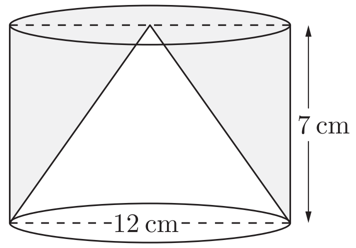
32. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. [3]

Ans :

Given,

Radius of cylinder $= r = \frac{12}{2}$ cm

$= 6$ cm $=$ radius of cone.



Height of cylinder $= h = 7$ cm $=$ height of cone.

Slant height of cone $= \sqrt{r^2 + h^2} = \sqrt{6^2 + 7^2}$ cm

$\sqrt{85}$ cm.

Required surface area

$=$ CSA of cylinder $+$ CSA of cone $+$ area of one (upper) base of cylinder

$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$

$= \left(\frac{22}{7} \times 6 \times 2 \times 7 + \sqrt{85} + 6\right)$ cm^2

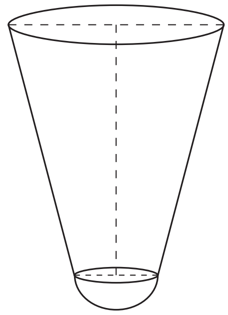
$= \frac{132}{7}(20 + 9.22)$ cm^2 $(\sqrt{85} = 9.22)$

$= \left(\frac{132}{7} \times 29.22\right)$ cm^2

$= 551.01$ cm^2 (approx.)

or

A shuttle cock used for playing Badminton has the shape of a frustum of a cone mounted on a hemisphere (as shown in the adjoining figure). The diameters of the ends of the frustum are 7 cm and 2 cm, the height of the entire shuttle cock is 7 cm. Find the external surface area of the shuttle cock.



Ans :

Upper base radius of frustum $= R$

$= \frac{1}{2}$ of 7 cm $= \frac{7}{2}$ cm,

lower base radius of frustum $= r$

$= \frac{1}{2}$ of 2 cm $= 1$ cm.

Radius of hemisphere $= 1$ cm.

Total height of shuttle cock $= 7$ cm,

height of frustum $= (7 - 1)$ cm $= 6$ cm

Slant height of frustum $= \sqrt{h^2 + (R - r)^2}$

$= \sqrt{(6)^2 + \left(\frac{7}{2} - 1\right)^2}$ cm

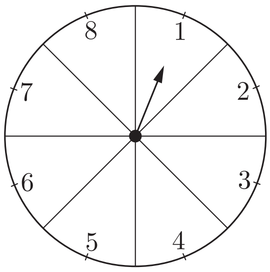
$= \sqrt{36 + \frac{25}{4}}$ cm

$= \sqrt{\frac{169}{4}}$ cm $= \frac{13}{2}$ cm.

External surface area of shuttle cock

$$= \text{CSA of frustum} + \text{CSA of hemisphere}$$
$$= \pi(r+R)l + 2\pi r^2$$
$$= \left(\pi\left(1+\frac{7}{2}\right)\times\frac{13}{2} + 2\pi\times 1^2\right)\text{cm}^2$$
$$= \pi\left(\frac{9}{2}\times\frac{13}{2}+2\right)\text{cm}^2$$
$$= \left(\frac{22}{7}\times\frac{125}{4}\right)\text{cm}^2$$
$$= \left(\frac{11\times 125}{14}\right)\text{cm}^2 = \frac{1375}{14}\,\text{cm}^2$$
$$= 98.21\,\text{cm}^2 \text{ (approx.)}$$

33. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (shown in the given figure) and these are equally likely outcomes. What is the probability that it will point at (i) an odd number? (ii) a number greater than 3? (iii) a number less than 9? [3]



Ans :

Since the arrow comes to rest at any one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and all are equally likely, so the sample space of the experiment has 8 equally likely outcomes.

(i) The outcomes favourable to the event 'arrow pointing at an odd number' are 1, 3, 5, 7; which are 4 in number.

P (arrow pointing at an odd number) $= \frac{4}{8} = \frac{1}{2}$.

(ii) The outcomes favourable to the event 'arrow pointing at a number greater then 3' are 4, 5, 6, 7, 8; which are 5 in number.
P (arrow pointing at a number greater than 3) $= \frac{5}{8}$.

(iii) The event is arrowing pointing at a number less than 9 and all the outcomes 1, 2, 3, 4, 5, 6, 7, 8 are less than 9. So, the number of favourable outcomes to the event 'arrow pointing at a number less than 9' is 8.

P (arrow pointing at a number less than 9) $= \frac{8}{8} = 1$.

34. Three digit numbers area made using the digits 4, 5, 9 (without repetition). If a number among them is selected at random, what is the probability that the number will [3]
(i) be a multiple of 5?
(ii) be multiple of 9?
(iii) end with 9?

Ans :

The three digit numbers made from the digits 4, 5, 9 (without repetition) are 459, 495, 549, 594, 945, 954. These are 6 in number.
Since any number out of these 6 numbers is selected at random, therefore, the sample space has 6 equally likely outcomes.
(i) Multiples of 5 are 495, 945.

$$P \text{ (a multiple of 9)} = \frac{2}{6} = \frac{1}{3}.$$

(ii) Note that all the six numbers are divisible by 9.

$$P \text{ (a multiple of 9)} = \frac{6}{6} = 1.$$

(iii) The numbers that end with 9 are 459, 549.

$$P \text{ (end with 9)} = \frac{2}{6} = \frac{1}{3}.$$

Section D

35. A thief runs with a uniform speed of 100m/minute. After one minute a policeman runs after the thief to catch him. He runs with a speed of 100 m/minute in the first minute and increases his speed by 10 m/ minute every succeeding minute. After how many minutes the policeman will catch the thief? [4]

Ans :

Let the total time be n minutes.
The total distance covered by the thief in n minutes

$$= (100\times n) \text{ metres} = 100\,n \text{ metres.}$$

The policeman starts after one minute, so the policeman runs for $(n-1)$ minutes. The distances (in metres) covered by the policeman in first, second, third, ..., $(n-1)$ minutes are 100, 110, 120, ..., $(n-1)$ terms.
These numbers form an AP with

$$a = 100,$$
$$d = 10 \text{ and number of terms}$$
$$= (n-1).$$

The total distance covered by the policeman

$$= \frac{n-1}{2}[2\times 100 + (\overline{n-1}-1)\times 10]$$
$$= \frac{n-1}{2}[200 + 10(n-2)]$$
$$= (n-1)(90+5n) = 90n + 5n^2 - 90 - 5n$$
$$= 5n^2 + 85n - 90.$$

As the policeman catches the thief, so the distance run by them are equal.

$$5n^2 + 85n - 90 = 100n$$
$$5n^2 - 15n - 90 = 0$$
$$n^2 - 3n - 18 = 0$$
$$(n-6)(n+3) = 0$$
$$n = 6$$

or $\quad n = -3.$

But n (time) cannot be negative, so

$$n = 6.$$

Hence, the policeman took 5 minutes to catch the thief.

or

Find the 60th term of the AP 8, 10, 12, ..., if it has a total of 60 terms and hence find the sum of its last 10 terms.

Ans :

The given AP is 8, 10, 12, ...,

so its first term $a = 8$

and common difference $d = 10 - 8 = 2$.

The AP has a total of 60 terms.

60th term of the $AP = a_{60} = a + 59d$

$= 8 + 59 \times 2 = 126$.

50th term of the $AP = a_{50} = a + 49d$

$= 8 + 49 \times 2 = 106$.

Sum of last 10 terms $= S_{60} - S_{50}$ $\quad |$use $S_n = \frac{n}{2}(a + l)$

$$= \frac{60}{2}(a + a_{60}) - \frac{50}{2}(a + a_{50})$$

$$= 30(8 + 126) - 25(8 + 106)$$

$$= 30 \times 134 - 25 \times 114$$

$$= 4020 - 2850 = 1170.$$

36. Given a rhombus $ABCD$ in which $AB = 4$ cm and $\angle ABC = 60°$. Divide it into two triangles say, ABC and ADC. Construct the triangle $AB'C'$ similar to ΔABC with scale factor $\frac{2}{3}$. Draw a line $C'D'$ parallel to CD where D' lies on AD. Is $AB'C'D'$ a rhombus? Give reasons. [4]

Ans :

Steps of construction:

(i) Construct a rhombus $ABCD$ with

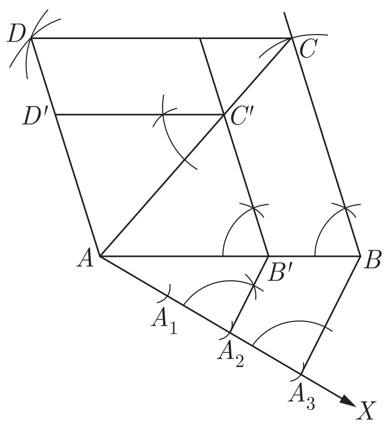
$AB = 4$ cm

and $\angle ABC = 60°$,

$BC = CD = AD = 4$ cm.

(ii) Join AC. Thus, we divide rhombus $ABCD$ into two triangles ABC and ADC.

(iii) Draw any ray AX making an acute angle with AB on the side opposite to the vertex C.



(iv) Locate 3 (the greater of 2 and 3) points on AX such that $AA_1 = A_1A_2 = A_2A_3$.

(v) Join A_3B.

(vi) Through A_2, draw a line parallel to A_3B to intersect AB at B'.

(vii) Throught B', draw a line parallel to BC to intersect AC at C'.

Then $AB'C'$ is the required triangle similar to ΔABC and with scale factor $\frac{2}{3}$.

(viii) Finally, through C' draw a line parallel to CD to intersect AD at D'. Then $AB'C'D'$ is a rhombus.

Justification:

In ΔAA_3B, $\quad A_2B' \parallel A_3B$.

By Basic Proportionality Theorem,

$$\frac{AB'}{AB} = \frac{AA_2}{AA_3}$$

But $\quad \frac{AA_2}{AA_3} = \frac{2}{3}$,

So $\quad \frac{AB'}{AB} = \frac{2}{3}$.

Since $\quad B'C' \parallel BC$, $\Delta AB'C' \sim \Delta ABC$.

$$\frac{AC'}{AC} = \frac{B'C'}{BC}$$

$$= \frac{AB'}{AB} = \frac{2}{3}.$$

Also $\quad D'C' = \parallel DC$,

So $\quad \Delta AC'D' \sim \Delta ACD$

$$\frac{C'D'}{CD} = \frac{AD'}{AD} = \frac{AC'}{AC} = \frac{2}{3}$$

Thus, $\quad AB' = B'C' = C'D' = AD' = \frac{2}{3}AB$

$AB'C'D'$ is a rhombus.

37. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then prove that $q(p^2 - 1) = 2p$. [4]

Ans :

Given $\quad \sin\theta + \cos\theta = p \quad$ ...(i)

and $\quad \sec\theta + \operatorname{cosec}\theta = q \quad$ ...(ii)

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$$

$$\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = q$$

$$\frac{p}{\sin\theta\cos\theta} = q \quad \text{(using (i))}$$

$$\sin\theta\cos\theta = \frac{p}{q} \quad \text{...(iii)}$$

On squaring equation (i), we get

$$(\sin\theta + \cos\theta)^2 = P^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = P^2$$

$$1 + 2\sin\theta\cos\theta = P^2$$

$$1 + 2\frac{p}{q} = p^2 \quad \text{(using (iii))}$$

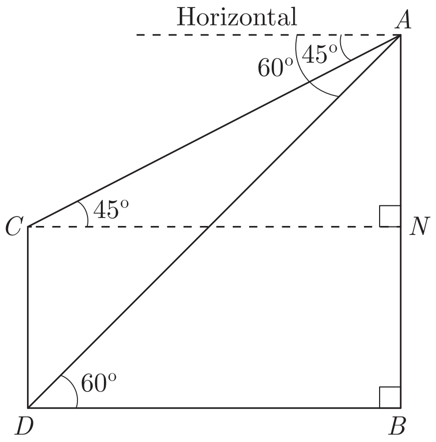
$$\frac{2p}{q} = p^2 - 1$$

$$2p = q(p^2 - 1).$$

38. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building (Use $\sqrt{3} = 1.73$) [4]

Ans :

Let CD be the building and AB be the tower. CD =50 m.



From C, draw $CN \perp AB$, then $CDBN$ is a rectangle. The angles of depression are shown is the figure. Then

$$\angle ACN = 45^\circ$$

and $$\angle ADB = 60^\circ$$

From right angled $\Delta\, CAN$,

$$\tan 45^\circ = \frac{AN}{CN}$$

$$1 = \frac{AN}{CN}$$

$$CN = AN \qquad \text{...(i)}$$

from right angled $\Delta\, ADB$,

$$\tan 60^\circ = \frac{AB}{DB}$$

$$\sqrt{3} = \frac{AN+NB}{DB} = \frac{CN+NB}{DB} \qquad \text{(using (i))}$$

$$\sqrt{3} = \frac{DB+CD}{DB} \quad \text{(As CDBN is a rectangle, } CN = DB \text{ and } NB = CD)$$

$$\sqrt{3}\,DB = DB + CD$$

$$(\sqrt{3}-1)DB = CD$$

$$DB = \frac{1}{\sqrt{3}-1} \times CD$$

$$= \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \times CD$$

$$= \frac{\sqrt{3}+1}{2} \times CD$$

$$= \left(\frac{1.73+1}{2} \times 50\right)\text{m} \qquad (CD = 50\ m)$$

$$DB = (2.73 \times 25)\,\text{m} = 68.25\,\text{m}$$

$$AB = AN + NB$$

$$= CN + CD = DB + CD$$

$$= 68.25\,\text{m} + 50\,\text{m} = 118.25\,\text{m}$$

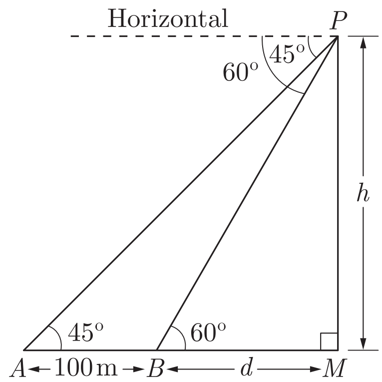
Hence, the height of tower $= 118.25\,\text{m}$

and the horizontal distance between the tower and the building $= 68.25\,\text{m}$

or

From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of tower, find the height of the tower. (Use $\sqrt{3} = 1.732$)

Ans :



Let the height of the tower MP at P be h metres. Let A and B be the two cars. Thus

$$AB = 100\,\text{m}.$$

angles of depression are shown in the given figure.

Note that $\angle PAM = 45^\circ$

and $\angle PBM = 60^\circ$.

Let $BM = d$ metres.

From right angled $\Delta\, MBP$,

$$\tan 60^\circ = \frac{h}{d}$$

$$\sqrt{3} = \frac{h}{d}$$

$$d = \frac{h}{\sqrt{3}} \qquad \text{...(i)}$$

From right angled $\Delta\, MAP$,

$$\tan 45^\circ = \frac{MP}{AM}$$

$$1 = \frac{h}{100+d}$$

$$100 + d = h$$

$$100 + \frac{h}{\sqrt{3}} = h \qquad \text{(using (i))}$$

$$100\sqrt{3} = \sqrt{3}\,h - h = (\sqrt{3}-1)h$$

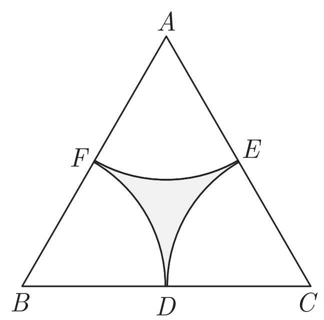
$$h = \frac{100\sqrt{3}}{\sqrt{3}-1}$$

$$\frac{100\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 50(3+\sqrt{3})$$

$$50(3+1.732) = 50 \times 4.732 = 236.6$$

Hence, the height of the tower is 236.6 metres.

39. In the given figure, arcs area drawn by taking vertices A, B and C of an equilateral triangle of side 14 cm to intersect the sides BC, CA and AB at their mid-pints D, E and F respectively. Find the area of the shaded region. $\left(\text{Take } \pi = \frac{22}{7} \text{ and } \sqrt{3} = 1.73\right)$ [4]



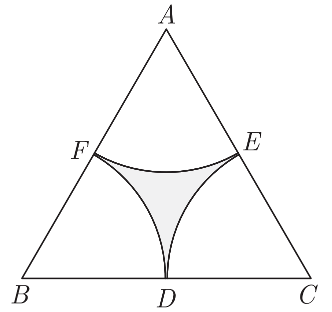
Ans :

Given, ABC is an equilateral triangle with

$$AB = BC = CA = 14\text{ cm}$$

$$\angle ABC = \angle BAC = \angle ACB = 60^\circ$$

(In equilateral Δ, all angles are 60° each)



It is given that arcs with centres A, B and C intersect the side of ΔABC at their respective mid-points D, E and F.

So, area of shaded region

$$= \text{area of } \Delta ABC - 3 \text{ (area of minor sector)}$$

$$= \frac{\sqrt{3}}{4}(\text{side})^2 - 3 \times \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \left(\frac{\sqrt{3}}{4} \times (14)^2 - 3 \times \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 7^2\right)\text{cm}^2$$

$$= \left(1.73 \times 49 - \frac{1}{2} \times 22 \times 7\right)\text{cm}^2$$

$$= (84.77 - 77)\text{ cm}^2 = 7.77\text{ cm}^2.$$

Hence, the area of shaded region is 7.77 cm^2.

40. Mode of the following frequency distribution is 65 and the sum of all the frequencies is 70. Find the missing frequencies x and y. [4]

Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-160
Frequ-ency	8	11	x	12	y	9	9	5

Ans :

Class interval	Frequency
0-20	8
20-40	11
40-60	x
60-80	**12**
80-100	y
100-120	9
120-140	9
140-160	5

Since the sum of frequencies is 70

$$8 + 11 + x + 12 + y + 9 + 9 + 5 = 70$$

$$54 + x + y = 70$$

$$x + y = 16 \qquad \text{...(i)}$$

As the mode of the given distribution is 65, therefore, the modal class is 60-80.

Here, $l = 60$,

$$h = \text{width of each class} = 20$$

$$f_1 = 12,\ f_0 = x \text{ and } f_2 = y$$

Mode $= l + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$65 = 60 + \frac{12 - x}{2(12) - x - y} \times 20$$

(given mode=65)

$$65 - 60 = \frac{12 - x}{24 - (x + y)} \times 20$$

$$5 = \frac{12 - x}{24 - 16} \times 20$$

$$5 = \frac{12 - x}{8} \times 20 \qquad \text{(using (i))}$$

$$12 - x = \frac{5 \times 8}{20}$$

$$12 - x = 2$$

$$x = 10$$

Putting $x = 10$ in equation (i), we get

$$10 + y = 16$$

$$y = 6$$

$$x = 10,$$

$$y = 6.$$

or

During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg.)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

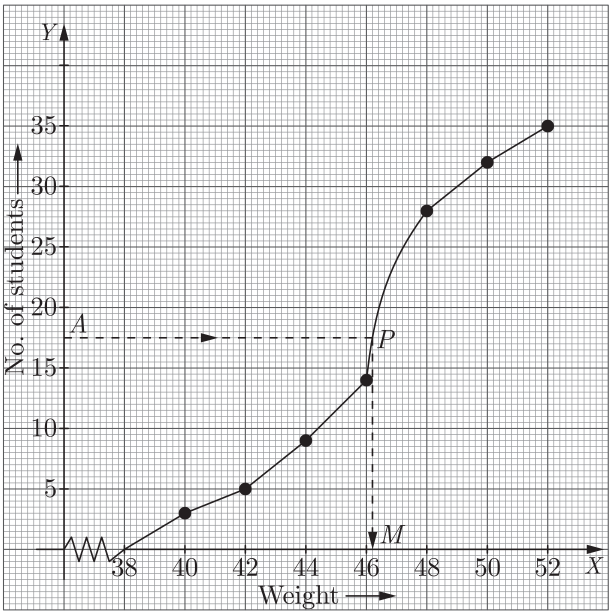
Ans :

Weight (in kg)	Classes	Frequency	Cumulative frequency
Less than 38		0	
Less than 40	38-40	3	3
Less than 42	40-42	2	5
Less than 44	42-44	4	9
Less than 46	44-46	5	14
Less than 48	46-48	14	28
Less than 50	48-50	4	32
Less than 52	50-52	3	35

Take 1 cm along x-axis = 2 kg

and 1 cm along y-axis = 5 students

Plot the point (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32), (52, 35) and (38, 0). Join these points by a freehand drawing. The required ogive (less than type) is drawn on the graph sheet given alongside.



Since the scale on x-axis starts at 38, a kink (break) is shown on the x-axis near the origin.