CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-17

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

() All questions are compulsory.
(i) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The least number that is divisible by all natural numbers from 1 to 10 (both inclusive) is [1]
(a) 10 (b) 100
(c) 504 (d) 2520

Ans : (d) 2520

The least number which is divisible by all natural numbers from 1 to 10 (both inclusive) is the LCM of numbers 1 to 10.
The LCM is 2520.

2. The decimal expansion of the rational number $\frac{33}{2^2\times5}$ will terminate after [1]
(a) one decimal place (b) two decimal places
(c) three decimal places (d) four decimal places

Ans : (b) two decimal places

$$\frac{33}{2^2\times5}=\frac{33\times5}{2^2\times5^2}=\frac{165}{(10)^2}$$

Hence, the decimal expansion will terminate after two decimal places.

3. If the equation $3x^2-kx+2k=0$ has equal roots, then the value(s) of k is (are) [1]
(a) 6 (b) 0 only
(c) 24 only (d) 0 or 24

Ans : (d) 0 or 24

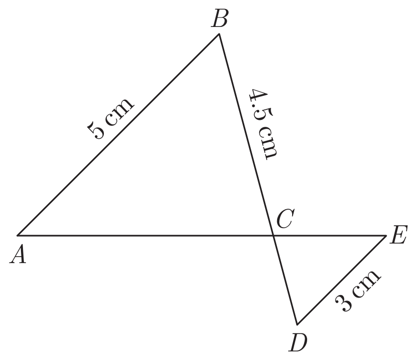
$$b^2-4ac=0$$
$$(-k)^2-4\times3\times2k=0$$
$$k^2-24k=0$$
$$k(k-24)=0$$
$$k=0 \text{ or } 24$$

4. If a ladder 10 m long reaches a window 8 m above the ground, then the distance of the foot of the ladder from the base of the wall is [1]
(a) 18 m (b) 8 m
(c) 6 m (d) 4 m

Ans : (c) 6 m

$$\text{Distance of ladder from the wall}=\sqrt{10^2-8^2}=\sqrt{36}=6\text{ m}$$

5. In given figure, $AB\parallel DE$. The length of CD is [1]



(a) 2.5 cm (b) 2.7 cm
(c) $\frac{10}{3}$ cm (d) 3.5 cm

Ans : (b) 2.7 cm

$\because$ $AB\parallel DE$,

So, $\angle ABC=\angle EDC$ (alternate $\angle s$)
$\angle BAC=\angle DEC$ (alternate $\angle s$)
$\Delta ABC\sim\Delta EDC$

$\therefore$ $$\frac{BC}{CD}=\frac{AB}{DE}$$
$$\frac{4.5}{CD}=\frac{5}{3}$$
$$CD=2.7\text{ cm}$$

6. If $\sin\theta=\frac{a}{b}$, the $\cos\theta$ is equal to [1]

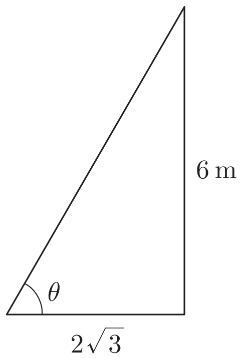
(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$
(c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

Ans : (c) $\frac{\sqrt{b^2-a^2}}{b}$

$$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\frac{a}{b}\right)^2}=\frac{\sqrt{b^2-a^2}}{b}$$

7. If a pole 6 m high casts shadow $2\sqrt{3}$ m long on the ground, then the sun's elevation is [1]
(a) $60°$ (b) $45°$
(c) $30°$ (d) $90°$

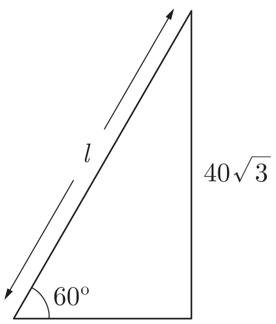
Ans : (a) $60°$



$$\tan\theta = \frac{6}{2\sqrt{3}} = \sqrt{3}$$
$$\theta = 60°$$

8. If a kite is flying at a height of $40\sqrt{3}$ metres from the level ground, attached to a string inclined at $60°$ to the horizontal, then the length of the string is [1]
(a) 80 m (b) $60\sqrt{3}$
(c) $80\sqrt{3}$ m (d) 120 m

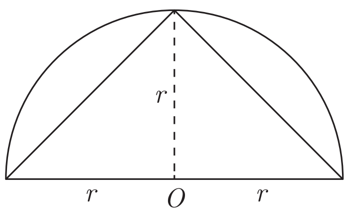
Ans : (a) 80 m



$$\sin 60° = \frac{40\sqrt{3}}{l}$$
$$\frac{\sqrt{3}}{2} = \frac{40\sqrt{3}}{l}$$
$$l = 80 \text{ m}$$

9. Area of the largest triangle that can be inscribed in a semicircle of radius r units is [1]
(a) r^2 sq. units (b) $\frac{1}{2}r^2$ sq. units
(c) $2r^2$ sq. units (d) $\sqrt{2}\,r^2$ sq. units

Ans : (a) r^2 sq. units



$$\text{Area of largest triangle} = \frac{1}{2} \times 2r \times r$$
$$= r^2 \text{ sq. units}$$

10. A mason constructs a wall of dimensions 300 cm × 270 cm × 350 cm with the bricks of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that $\frac{1}{8}$ space is covered by mortar. Then the number of bricks used to construct the wall is [1]
(a) 11100 (b) 11200
(c) 11000 (d) 11300

Ans : (b) 11200

It is given that $\frac{1}{8}$ space of wall is covered by mortar, so $\frac{7}{8}$ space of the wall should be constructed with bricks.

$$\therefore \text{ Number of bricks} = \frac{300 \times 270 \times 350 \times \frac{7}{8}}{22.5 \times 11.25 \times 8.75}$$
$$= 11200$$

(Q.11-Q.15) Fill in the blanks.

11. All decimal numbers (terminating, non-terminating repeating or non-terminating non-repeating) are .... numbers. [1]

Ans : real

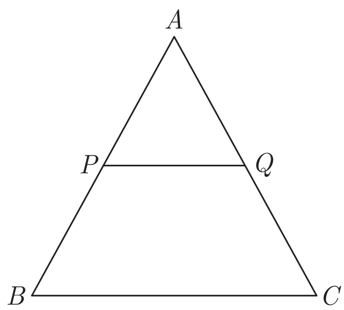
12. In two similar triangles, if the ratio of their corresponding medians is 3 : 5, then the ratio of their corresponding sides is .... [1]

Ans :

In two similar triangles, ratio of areas of Δs
= Square of ratio of corresponding sides
Also, ratio of areas of Δs
= Square of ratio of corresponding medians
∴ Ratio of corresponding sides
= Ratio of corresponding medians
$= 3:5$

or

In the given figure, P and Q are mid-points of sides AB and AC respectively. If $PQ = 2.3$ cm, then the length of BC is ....



Ans :

Line joining the mid-points of two sides of a triangle is parallel to third side and half of it.

$$PQ = \frac{1}{2}BC$$
$$BC = 2PQ$$
$$BC = 2 \times 2.3 \text{ cm}$$
$$= 4.6 \text{ cm}$$

13. The lengths of tangents drawn from an external point to a circle are .... [1]

Ans : equal

14. If the difference between circumference and radius of a circle is 37 cm, then the circumference of that circle is .... [1]

Ans :

$$2\pi r - r = 37$$
$$\left(2 \times \frac{22}{7} - 1\right)r = 37$$
$$\frac{33}{7}r = 37$$
$$r = 7 \text{ cm}$$

So, $\text{Circumference} = 2 \times \frac{22}{7} \times 7$

$$= 44 \text{ cm}$$

15. The value of $\sin^2 30^\circ \cdot \tan 60^\circ + \cos^2 30^\circ \cdot \tan 60^\circ$ is .... [1]

Ans :

$$\sin^2 30^\circ \cdot \tan 60^\circ + \cos^2 30^\circ \cdot \tan 60^\circ$$
$$= (\sin^2 30^\circ + \cos^2 30^\circ)\tan 60^\circ$$
$$= 1 \times \sqrt{3} = \sqrt{3}$$

(Q.16-Q.20) Answer the following

16. If the sum of zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k. [1]

Ans :

$$3 = \frac{-(-k)}{3}$$
$$k = 9$$

or

If 1 is a root of both the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then find the value of ab.

Ans :

Given 1 is a root of both the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$

$$a \cdot 1^2 + a \cdot 1 + 3 = 0$$
$$a + a + 3 = 0$$
$$2a + 3 = 0$$
$$a = -\frac{3}{2}$$

and
$$1^2 + 1 + b = 0$$
$$1 + 1 + b = 0$$
$$2 + b = 0$$
$$b = -2$$

$\therefore$
$$ab = \left(-\frac{3}{2}\right) \times (-2)$$
$$= 3$$

Hence, the value of ab is 3.

17. Consider the following distribution, find the frequency of class $30 - 40$. [1]

Marks obtained	No. of students
0 or more	63
10 or more	58
20 or more	55
30 or more	51
40 or more	48
50 or more	42

Ans :

Marks obtained	No. of students	Class Interval	Frequency
0 or more	63	$0 - 10$	5
10 or more	58	$10 - 20$	3
20 or more	55	$20 - 30$	4
30 or more	51	$\boxed{30 - 40}$	$\boxed{3}$
40 or more	48	$40 - 50$	6
50 or more	42	$50 - 60$	42

Hence, the frequency of class $30 - 40$ is 3.

18. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number. [1]

Ans :

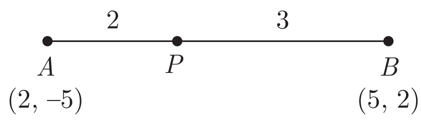
Total number of cards = 48

Perfect square number cards from 3 to 50 are 4, 9, 16, 25, 36 and 49. These are 6 in number.

$\therefore$ $P(\text{Perfect Square Number}) = \frac{6}{48} = \frac{1}{8}$

19. In which quadrant, the point P that divides the line segment joining the points $A(2, -5)$ and $B(5, 2)$ in the ratio 2 : 3 lies? [1]

Ans :



The coordinates of P are

$$\left(\frac{2 \times 5 + 3 \times 2}{2 + 3}, \frac{2 \times 2 + 3 \times (-5)}{2 + 3}\right) \text{ i.e. } \left(\frac{16}{5}, -\frac{11}{5}\right).$$

The point P lies in IV quadrant.

20. If $\sec 2A = \operatorname{cosec}(A - 27^\circ)$, where $2A$ is an acute angle, find the measure of $\angle A$. [1]

Ans :

$$\sec 2A = \operatorname{cosec}(A - 27^\circ)$$
$$\operatorname{cosec}(90^\circ - 2A) = \operatorname{cosec}(A - 27^\circ)$$
$$90^\circ - 2A = A - 27^\circ$$
$$3A = 117^\circ$$

Hence, $\angle A = 39^\circ$

Section B

21. Find whether the following pair of linear equations is consistent or inconsistent: [2]
$2x - 3y = 8;\ 4x - 6y = 9$

Ans :

The given pair of linear equations can be written as:

$$2x - 3y - 8 = 0$$

and $$4x - 6y - 9 = 0$$

Here, $a_1 = 2$, $b_1 = -3$, $c_1 = -8$

and $a_2 = 4$, $b_2 = -6$

and $c_2 = -9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2},$$

$$\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The given pair of linear equations is inconsistent.

22. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,-5)$ and $R(-3,6)$, find the coordinates of P. [2]

Ans :

Given the coordinates of $Q(2,-5)$ and $R(-3,6)$.
Let the coordinates of P be $(2y, y)$.
According to given,

$$PQ = PR$$

$$(PQ)^2 = (PR)^2$$

$$(2-2y)^2 + (-5-y)^2 = (-3-2y)^2 + (6-y)^2$$

$$4 - 8y + 4y^2 + 25 + 10y + y^2 = 9 + 12y + 4y^2 + 36 - 12y + y^2$$

$$2y + 29 = 45$$

$$2y = 16$$

$$y = 8$$

Hence, coordinates of P are (16, 8).

or

If the point $(m, 3)$ lies on the line segment joining the points $\left(-\frac{2}{5}, 6\right)$ and (2, 8), find the value of m.

Ans :

Points $(m, 3)$, $\left(-\frac{2}{5}, 6\right)$ and (2, 8) are collinear.

The area of triangle formed by these points is zero.

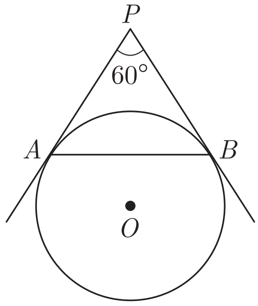
$$\frac{1}{2}\left|m(6-8) + \left(-\frac{2}{5}\right)(8-3) + 2(3-6)\right| = 0$$

$$\left|-2m - 2 - 6\right| = 0$$

$$2m + 8 = 0$$

$$m = -4$$

23. In the given figure, AP and BP are tangents to a circle with centre O, such that $AP = 5\,\text{cm}$ and $\angle APB = 60^\circ$, find the length of chord AB [2]



Ans :

Given, $\angle APB = 60^\circ$

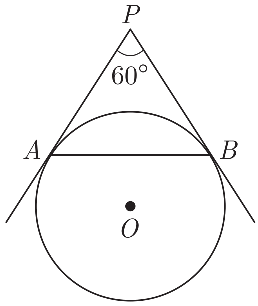
and $AP = 5$ cm

In ΔAPB, $PA = PB$

(tangents drawn from an external point are equal)

$\therefore \quad \angle PAB = \angle PBA$

(angles opposite to equal sides are equal)



Now, $\angle PAB + \angle PBA + \angle APB = 180^\circ$

(angle sum property of a triangle)

$$2\angle PAB = 120^\circ$$

$$\angle PAB = 60^\circ$$

ΔAPB is an equilateral triangle.

So, $AB = AP = 5$ cm

24. How many terms of the AP 18, 16, 14, ... be taken so that their sum is zero? [2]

Ans :

Given AP is 18, 16, 14, ...

Here, $a = 18$, $d = -2$

If the sum of n terms of the given AP is zero,

Then, $S_n = 0$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{n}{2}[2 \times 18 + (n-1)(-2)] = 0$$

$$\frac{n}{2}[36 - 2n + 2] = 0$$

$$n(19 - n) = 0$$

$$n = 0$$

or $$n = 19$$

But n cannot be zero.
Hence, the required number of terms is 19.

or

The sum of 5[th] and 7[th] terms of an AP is 52 and the 10[th] term is 46. Find the AP.

Ans :

Let the first term and common difference of AP be a and d respectively,

Then, $a_5 + a_7 = 52$

$(a+4d)(a+6d) = 52$

$2a + 10d = 52$

$a + 5d = 26$ ...(i)

Also, $a_{10} = 46$

$a + 9d = 46$ ...(ii)

Solving equation (i) and (ii), we get,

$a = 1,$

$d = 5$

Hence, the AP is 1, 6, 11, 16, ...

25. Show that the mode of the sequences obtained by combining the two sequences S_1 and S_2 taken separately: [2]

S_1 : 3, 5, 8, 8, 9, 12, 13, 9, 9

S_2 : 7, 4, 7, 8, 7, 8, 13

Ans :

Given, S_1 : 3, 5, 8, 8, 9, 12, 13, 9, 9

S_2 : 7, 4, 7, 8, 7, 8, 13

In, S_1 : Number 9 occurs maximum number of times i.e. 3 times

Hence, the mode of sequences,

$S_1 = 9$

In, S_2 : Number 7 occurs maximum number of times i.e. 3 times

Hence, the mode of sequences,

$S_2 = 7$

After combining sequences S_1 and S_2, we have,

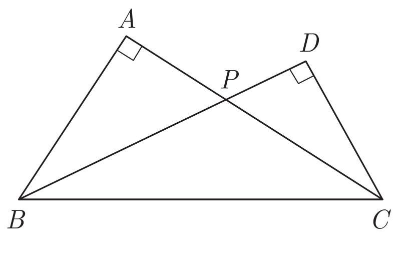
3, 4, 5, 7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 12, 13, 13

Here number 8 occurs maximum number of times i.e. 4 times.

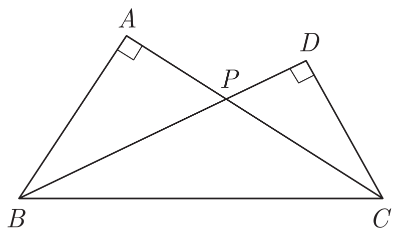
Hence,The mode of combined sequence = 8

Hence, mode of S_1 and S_2 taken combined is different from that of S_1 and S_2 taken separately.

26. In the adjoining figure, ABC and DBC are two right triangles. Prove that $AP \times PC = BP \times PD$. [2]



Ans :



In ΔAPB and ΔDPC,

$\angle BAP = \angle CDP$ (each 90°)

$\angle APB = \angle DPC$ (vert. opp. $\angle s$)

$\therefore$ $\Delta APB \sim \Delta DPC$ (by AA similarity criterion)

So, $\frac{BP}{PC} = \frac{AP}{PD}$

$AP \times PC = BP \times PD$

Section C

27. Sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number. [3]

Ans :

Let the two digit number be $10x + y$,
Then, according to given,

$x + y = 8$

and $|10x + y - (10y + x)| = 18$

$|9x - 9y| = 18$

$|x - y| = 2$

$x - y = \pm 2$

When, $x + y = 8$

and $x - y = 2$,

Then, on adding these equations, we get,

$2x = 10$

$x = 5$

Putting $x = 5$ in $x + y = 8$,

We get, $y = 5$

The original number is 53 or 35.

28. Divide 56 in four parts in AP such that the ratio of the product of their extremes (1[st] and 4[th]) to the product of means (2[nd] and 3[rd]) is 5 : 6. [3]

Ans :

Let the four numbers in AP be,

$a - 3d,$

$a - d,$

$a + d$

and $a + 3d$

Then, according to given,

$a - 3d + a - d + a + d + a + 3d = 56$

$4a = 56$

$a = 14$

and $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$

$$\frac{a^2-9d^2}{a^2-d^2} = \frac{5}{6}$$

$$6a^2-54d^2 = 5a^2-5d^2$$

$$a^2 = 49d^2$$

$$d^2 = \frac{a^2}{49}$$

$$d^2 = \frac{14^2}{7^2}$$

$$d^2 = 4$$

$$d = 2, -2$$

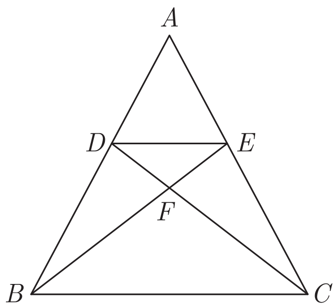
When $d=2$, the numbers are $14-3\times 2$, $14-2$, $14+2$, $14+3\times 2$ i.e. 8, 12, 16, 20;

When $d=-2$, the numbers are $14-3\times(-2)$, $14-(-2)$, $14+(-2)$, $14+3\times(-20)$ i.e. 20, 16, 12, 8

Hence, the numbers are 8, 12, 16, 20.

29. D and E are points on the sides AB and AC respectively of ΔABC such that DE is parallel to BC, and $AD:DB = 4:5$. CD and BE intersect each other at F. Find the ratio of the areas of ΔDEF and ΔCBF. [3]

Ans :



Given, $AD:DB = 4:5$

$$\frac{AD}{DB} = \frac{4}{5}$$

$$5AD = 4DB$$

$$5AD = 4(AB-AD)$$

$$9AD = 4AB$$

$$\frac{AD}{AB} = \frac{4}{9} \quad ...(i)$$

As $DE \parallel BC$,

$$\angle ADE = \angle ABC$$

and $\angle AED = \angle ACB$ (corres. $\angle s$)

$\therefore$ $\Delta ADE \sim \Delta ABC$ (AA similarity)

$$\frac{DE}{BC} = \frac{AD}{AB}$$

$$\frac{DE}{BC} = \frac{4}{9} \quad ...(ii) \text{ (using (i))}$$

In ΔDEF and ΔCBF,

$$\angle DEF = \angle CBF$$

(alt. $\angle s$, as $DE \parallel BC$)

$\angle DFE = \angle CFB$ (vert. opp. $\angle$)

$\therefore$ $\Delta DEF \sim \Delta CBF$

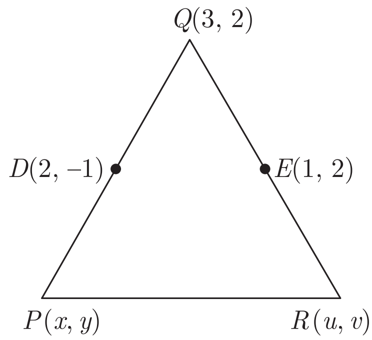
$\therefore$ $\frac{ar(\Delta DEF)}{ar(\Delta CBF)} = \left(\frac{DE}{BC}\right)^2$

$$= \left(\frac{4}{9}\right)^2 \quad \text{[using (ii)]}$$

$$ar(\Delta DEF):ar(\Delta CBF) = 16:81$$

30. Find the area of ΔPQR with $Q(3,2)$ and the mid-points of the sides through Q being $(2,-1)$ and $(1,2)$. [3]

Ans :



Let D and E are the mid-points of the sides through Q.

Let the coordinates of P and R be (x,y) and (u,v) respectively.
D is the mid-point of QP.

Coordinates of D are $\left(\frac{3+x}{2}, \frac{2+y}{2}\right)$ which is given $(2,-1)$.

$\therefore$ $\frac{3+x}{2} = 2$

and $\frac{2+y}{2} = -1$

$$x = 1$$

and $y = -4$

So, coordinates of P are $(1,-4)$.
Again,

E is the mid-point of QR.

Coordinates of E are $\left(\frac{3+u}{2}, \frac{2+v}{2}\right)$ which is given $(1, 2)$.

$$\frac{3+u}{2} = 1$$

and $\frac{2+v}{2} = 2$

$$u = -1$$

and $v = 2$

So, coordinates of R are $(-1,2)$.

$P(1,-4)$, $Q(3,2)$, $R(-1,2)$

Area of $\Delta PQR = \frac{1}{2}|1(2-2)+3(2+4)$

$$+(-1)(-4-2)|$$

$$= \frac{1}{2}|1\times 0+3\times 6-1(-6)|$$

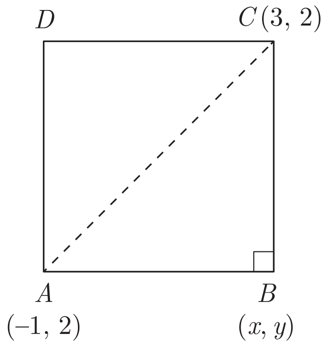
$$= \frac{1}{2}|18+6| = 12 \text{ sq. units}$$

Hence,The area of $\Delta PQR = 12$ sq. units

or

The two opposite vertices of a square are $(-1,2)$ and (3, 2). Find the coordinates of the other two vertices.

Ans :



Let $A(-1,2)$ and $C(3,2)$ be two opposite vertices of the square $ABCD$ and vertex B be (x,y).

Since $ABCD$ is a square,

$$AB = BC$$
$$AB^2 = BC^2$$
$$(x+1)^2+(y-2)^2 = (x-3)^2+(y-2)^2$$
$$x^2+2x+1 = x^2-6x+9$$
$$8x = 8$$
$$x = 1$$

Also as $ABCD$ is a square,

$$\angle B = 90^\circ$$
$$AB^2+BC^2 = AC^2$$
$$(x+1)^2+(y-2)^2+(x-3)^2+(y-2)^2 = (3+1)^2+(2-2)^2$$
$$2x^2-4x+10+2y^2-8y+8 = 16$$
$$2\times 1^2-4\times 1+2y^2-8y+2 = 0$$
$$2y^2-8y = 0$$
$$y(y-4) = 0$$
$$y = 0, 4$$

Hence, the other two vertices are (1, 2) and (1, 4).

31. Prove the following identity: [3]

$$\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta} = \tan\theta$$

Ans :

$$\text{LHS} = \frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$
$$= \frac{\sin\theta[1-2(1-\cos^2\theta)]}{\cos\theta(2\cos^2\theta-1)}$$
$$= \frac{\sin\theta(2\cos^2\theta-1)}{\cos\theta(2\cos^2\theta-1)}$$
$$= \tan\theta$$
$$= \text{RHS}$$

32. Three alarm clocks ring at intervals of 4, 12 and 20 minutes respectively. If they start ringing together, after how much time will they next ring together? [3]

Ans :

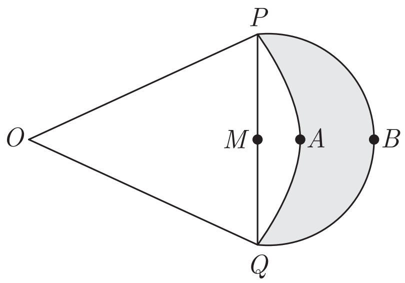
To find out the time when the clocks will next ring together, we have to find the LCM of 4, 12 and 20.

Prime factorisation 4, 12 and 20

$$4 = 2\times 2$$
$$12 = 2\times 2\times 3$$
$$20 = 2\times 2\times 5$$
$$\therefore \quad \text{LCM of 4, 12 and 20} = 2\times 2\times 3\times 5$$
$$= 60$$

Hence, the alarm clocks will ring together again after 60 minutes i.e. one hour.

33. The given figure shows two arcs PAQ and PBQ. Arc PAQ is a part of a circle with centre O and radius OP while arc PBQ is a semicircle drawn on PQ as diameter. If $OP = PQ = 10$ cm, show that the area of the shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)\text{cm}^2$. [3]



Ans :

$$OQ = OP \quad \text{(radii of same circle)}$$

Given, $$OP = PQ = 10 \text{ cm}$$
$$OP = OQ = PQ = 10 \text{ cm}$$

Thus, OPQ is an equilateral triangle of side 10 cm.

Also, $\angle POQ = 60^\circ$ (angle of an equilateral triangle)

Area of shaded region

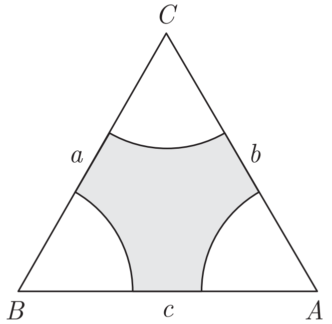
= Area of a semicircle with PQ as diameter i.e. radius 5 cm + Area of equilateral $\Delta\, OPQ$ of side 10 cm − Area of sector of a circle of radius 10 cm and central angle 60°

$$= \left(\frac{1}{2}\pi\times 5^2+\frac{\sqrt{3}}{4}(10)^2-\frac{60}{360}\times\pi\times 10^2\right)\text{cm}^2$$
$$= \left(\frac{25\pi}{2}+25\sqrt{3}-\frac{50\pi}{3}\right)\text{cm}^2$$
$$= \left(25\sqrt{3}-\frac{25\pi}{6}\right)\text{cm}^2$$
$$= 25\left(\sqrt{3}-\frac{\pi}{6}\right)\text{cm}^2$$

or

Sides of a triangular field are 15 m, 16 m, 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.

Ans :



Let ABC be the triangular field with,

$$a = 15 \text{ m},$$
$$b = 16 \text{ m}$$
and $$c = 17 \text{ m}$$
$\therefore$ $$s = \frac{15+16+17}{2} \text{ m}$$
$$= 24 \text{ m}$$
$\therefore$ $$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{24 \times 9 \times 8 \times 7} \text{ m}^2$$
$$= 24\sqrt{21} \text{ m}^2$$

The region of the field left ungrazed by the three animals is shown shaded.

Sum of areas of three sectors of radius 7 m (each) and having central angles as $\angle A$, $\angle B$ and $\angle C$

= Area of a sector of radius 7 m and having central angle as sum of $\angle A$, $\angle B$ and $\angle C$

= Area of a sector of radius 7 m and having central angle as $\angle A + \angle B + \angle C$ i.e. an angle of 180°

$$= \left(\frac{180}{360} \times \frac{22}{7} \times 7^2\right) \text{m}^2$$
$$= 77 \text{ m}^2$$

$\therefore$ Required area = Area of triangular field − Area of the field grazed by three animals

$$= (24\sqrt{21} - 77) \text{ m}^2$$

34. Two different dice are tossed together. Find the probability that the product of the two numbers on the top of dice is (i) 6 (ii) a perfect square number. [3]

Ans :

When two different dice are tossed together, the total number of outcomes is 36 and all the outcomes are equally likely.

(i) The outcomes favourable to the event 'the product of two numbers on the top of two dice is 6' are (1, 6), (6, 1), (2, 3) and (3, 2). These are 4 in number.

$$P \text{ (Product of two numbers is 6)} = \frac{4}{36} = \frac{1}{9}$$

(ii) The outcomes favourable to the event 'the product of two numbers on the top of dice is a perfect square number' are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 4), (4, 1). These are 8 in number.

P (Product of two numbers is a perfect square number.)

$$= \frac{8}{36} = \frac{2}{9}$$

or

A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a trader, will only accept the shirts which are good, but Kewal, another trader, will only reject the shirts have major defects. One shirt is drawn at random from the carton. What is the probability that:
(i) it is acceptable to Ramesh?
(ii) it is acceptable to Kewal?

Ans :

One shirt is drawn at random from a carton containing 100 shirts means all shirts are equally likely to be drawn. So, the sample space of the experiment has 100 equally likely outcomes.

(i) Since Rames accepts only good shirts and there are 88 good shirts in the carton.
Therefore, the number of favourable (i.e. acceptable) outcomes to Ramesh = 88

$$P \text{ (Acceptable to Ramesh)} = \frac{88}{100}$$
$$= 0.88$$

(ii) As Kewal rejects the shirts which have major defects and the number of such shirts is 4,
So, The number of shirts acceptable to Kewal

$$= 100 - 4$$
$$= 96$$

Therefore, the number of favourable (acceptable) outcomes to Kewal = 96

$$P \text{ (Acceptable to Kewal)} = \frac{96}{100}$$
$$= 0.96$$

Section D

35. If α and β are the zeroes of the polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$. [4]

Ans :

Given α and β are the zeroes of the polynomial,

$$p(x) = 3x^2 + 2x + 1$$

So, $$\text{Sum of zeroes} = \alpha + \beta = \frac{-2}{3} \quad \text{...(i)}$$

and $$\text{Product of zeroes} = \alpha\beta = \frac{1}{3} \quad \text{...(ii)}$$

Now, $$\frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta} = \frac{(1-\alpha)(1+\beta)+(1+\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$
$$= \frac{1-\alpha+\beta-\alpha\beta+1-\alpha\beta-\beta+\alpha}{1+\alpha+\beta+\alpha\beta}$$
$$= \frac{2-2\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$
$$= \frac{2-2\times\frac{1}{3}}{1+\left(-\frac{2}{3}\right)+\frac{1}{3}} \quad \text{[using (i) and (ii)]}$$

$$\frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1+\beta}=\frac{\frac{4}{3}}{\frac{2}{3}}=2 \quad \text{...(iii)}$$

and $$\frac{1-\alpha}{1+\alpha}\times\frac{1-\beta}{1+\beta}=\frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$=\frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$=\frac{1-\left(-\frac{2}{3}\right)+\frac{1}{3}}{1+\left(-\frac{2}{3}\right)+\frac{1}{3}}$$

$$\frac{1-\alpha}{1+\alpha}\times\frac{1-\beta}{1+\beta}=\frac{\frac{6}{3}}{\frac{2}{3}}=3 \quad \text{...(iv)}$$

The polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$ is given by,

x^2 − (sum of zeroes)x + Product of zeroes

$$=x^2-\left(\frac{1-\alpha}{1+\alpha}\times\frac{1-\beta}{1+\beta}\right)x+\frac{1-\alpha}{1+\alpha}\times\frac{1-\beta}{1+\beta}$$

$$=x^2-2x+3 \quad \text{[using equation (iii) and (iv)]}$$

36. If $\sec\theta-\tan\theta=x$, show that $\sec\theta+\tan\theta=\frac{1}{x}$ and hence, find the values of $\cos\theta$ and $\sin\theta$. [4]

Ans :

Given, $$\sec\theta-\tan\theta=x \quad \text{...(i)}$$

$\because$ $$\sec^2\theta-\tan^2\theta=1$$

$$(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)=1$$

$$x\cdot(\sec\theta+\tan\theta)=1$$

$$\sec\theta+\tan\theta=\frac{1}{x} \quad \text{...(ii)}$$

Adding equation (i) and (ii), we get,

$$\sec\theta-\tan\theta+\sec\theta+\tan\theta=x+\frac{1}{x}$$

$$2\sec\theta=\frac{x^2+1}{x}$$

$$\sec\theta=\frac{x^2+1}{2x}$$

$$\frac{1}{\cos\theta}=\frac{x^2+1}{2x}$$

$$\cos\theta=\frac{2x}{x^2+1} \quad \text{....(iii)}$$

Subtracting equation (i) from (ii), we get,

$$\sec\theta+\tan\theta-(\sec\theta-\tan\theta)=\frac{1}{x}-x$$

$$\sec\theta+\tan\theta-\sec\theta+\tan\theta=\frac{1-x^2}{x}$$

$$2\tan\theta=\frac{1-x^2}{x}$$

$$\tan\theta=\frac{1-x^2}{2x}$$

$$\frac{\sin\theta}{\cos\theta}=\frac{1-x^2}{2x}$$

$$\sin\theta=\frac{1-x^2}{2x}\times\cos\theta$$

From equation (iii) we get,

$$\sin\theta=\frac{1-x^2}{2x}\times\frac{2x}{1+x^2}$$

$$\sin\theta=\frac{1-x^2}{1+x^2}$$

or

If $\tan(A+B)=\sqrt{3}$, $\tan(A-B)=\frac{1}{\sqrt{3}}$, $0^\circ<A+B<90^\circ$, $A>B$, find A and B. Also calculate $\tan A\sin(A+B)+\cos A\tan(A-B)$.

Ans :

Given, $$\tan(A+B)=\sqrt{3}$$

$$\tan(A+B)=\tan 60^\circ$$

$$A+B=60^\circ \quad \text{...(i)}$$

Also, $$\tan(A-B)=\frac{1}{\sqrt{3}}$$

$$\tan(A-B)=\tan 30^\circ$$

$$A-B=30^\circ \quad \text{...(ii)}$$

Solving equation (i) and (ii), we get,

$$A=45^\circ$$

and $$B=15^\circ$$

$\tan A\sin(A+B)+\cos A\tan(A-B)$

$$=\tan 45^\circ\sin 60^\circ+\cos 45^\circ\tan 30^\circ$$

$$=1+\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{3}}$$

$$=\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{6}}$$

$$=\frac{\sqrt{3}}{2}+\frac{\sqrt{6}}{6}$$

$$=\frac{3\sqrt{3}+\sqrt{6}}{6}$$

37. Sushant has a vessel of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which $\frac{2}{5}th$ of the water in the vessel flows out. Find how many balls were put in the vessel. [4]

Ans :

Volume of water in the cone = Volume of cone of height 11 cm and radius 2.5 cm

$$=\left(\frac{1}{3}\pi\times(2.5)^2\times 11\right)\text{cm}^2$$

$$=\frac{2.5\times 2.5\times 11}{3}\pi\text{ cm}^3$$

Volume of one metallic ball = Volume of sphere of radius 0.25 cm

$$=\left(\frac{4}{3}\pi\times(0.25)^3\right)\text{cm}^3$$

$$=\frac{4\times 0.25\times 0.25\times 0.25}{3}\pi\text{ cm}^3$$

Since $\frac{2}{5}th$ of the water flows out when metallic balls are put into the vessel, therefore, $\frac{2}{5}th$ of the volume of water is occupied by the metallic balls.

Number of metallic balls put into vessel

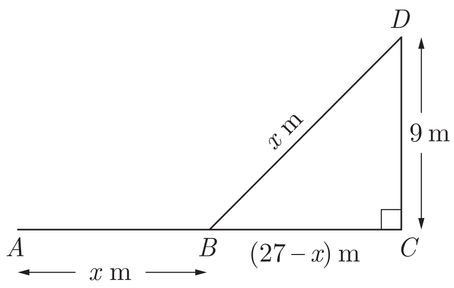
$$= \frac{\frac{2}{5} \text{ of volume of water}}{\text{Volume of one metallic ball}}$$

$$\frac{\frac{2}{5} \times \frac{2.5 \times 2.5 \times 11}{3}\pi}{\frac{4 \times 0.25 \times 0.25 \times 0.25}{3}\pi} = 440$$

Hence, the number of metallic balls put into the vessel $= 440$

38. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of a pillar, a snake is coming to its hole at the base of a pillar, seeing the snake, the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught? [4]

Ans :



Let CD be the pillar and initially the snake be at the point A and the peacock is sitting at the top of pillar i.e. at the point D.

Then, $CD = 9$ m

and $AC = 27$ m

Let the snake be caught at the point B. As the snake and peacock have equal speeds, they cover equal distance in same time.

Let, $AB = x$ metres

Then, $BD = AB = x$ metres

and $BC = AC - AB$

$= (27 - x)$ metres

In ΔBCD,

$\angle C = 90°$

By Pythagoras theorem, we get,

$$BD^2 = BC^2 + CD^2$$

$$x^2 = (27 - x)^2 + 9^2$$

$$x^2 = 729 - 54x + x^2 + 81$$

$$54x = 810$$

$$x = 15$$

$\therefore$ $BC = (27 - x)$ metres

$= (27 - 15)$ metres

$= 12$ metres

Hence, the snake is caught at a distance of 12 metres from its hole.

or

Find the value(s) of p for which the quadratic equation $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots.

Also find these roots.

Ans :

The given equation is:

$$(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0 \quad ...(i)$$

Comparing it with, $ax^2 + bx + c = 0$

We get, $a = 2p + 1$,

$b = -(7p + 2)$,

$c = 7p - 3$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (-(7p + 2))^2 - 4(2p + 1)(7p - 3)$$

$$= 49p^2 + 28p + 4 - 4(14p^2 - 6p + 7p - 3)$$

$$= 49p^2 + 28p + 4 - 56p^2 4p + 12$$

$$= -7p^2 + 24p + 16$$

For equal roots, Discriminant $= 0$

$$-7p^2 + 24p + 16 = 0$$

$$7p^2 - 24p - 16 = 0$$

$$7p^2 - 28p + 4p - 16 = 0$$

$$7p(p - 4) + 4(p - 4) = 0$$

$$(p - 4)(7p + 4) = 0$$

$$p - 4 = 0$$

or $7p + 4 = 0$

$7p = 4$

or $p = -\frac{4}{7}$

When, $p = 4$

The given equation becomes,

$$9x^2 - 30x + 25 = 0$$

$$(3x - 5)^2 = 0$$

$$x = \frac{5}{3}, \frac{5}{3}$$

When, $p = -\frac{4}{7}$

The given equation becomes,

$$\left(-\frac{8}{7} + 1\right)x^2 - (-4 + 2)x + (-4 - 3) = 0$$

$$-\frac{1}{7}x^2 + 2x - 7 = 0$$

$$x^2 - 14x + 49 = 0$$

$$(x - 7)^2 = 0$$

$$x = 7, 7$$

Hence, the values of p are $4, -\frac{4}{7}$.

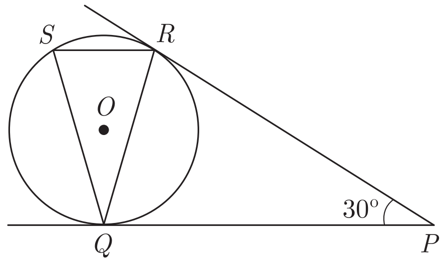
When, $p = 4$,

Roots are $\frac{5}{3}, \frac{5}{3}$;

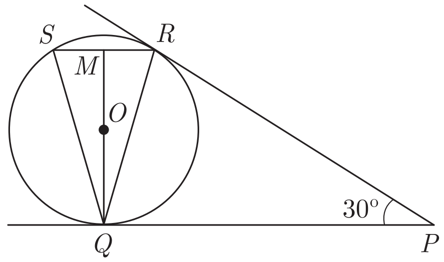
When, $p = -\frac{4}{7}$,

Roots are 7, 7.

39. In the given figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$. [4]



Ans :



Join QO and produce it to meet SR at M.

In ΔPRQ,

$$PR = PQ \quad \text{(lengths of tangents)}$$

$$\angle PRQ = \angle PQR \quad ...(i)$$

$$\angle PQR + \angle PRQ + \angle RPQ = 180^\circ$$

$$\angle PQR + \angle PQR + 30^\circ = 180^\circ \quad \text{[using (i)]}$$

$$2\angle PQR = 150^\circ$$

$$\angle PQR = 75^\circ$$

As $RS \parallel PQ$ and RQ is transversal,

$$\angle SRQ = \angle PQR \quad (\text{alt. } \angle s)$$

$$\angle SRQ = 75^\circ \quad ...(ii)$$

Since PQ is tangent to the circle with centre O at the point Q and OQ is radius,

$$OQ \perp PQ$$

Also, $RS \parallel PQ$ (given),

So, $QM \perp RS$

i.e. $OM \perp RS$

$$MR = MS$$

(Perpendicular from centre to a chord bisects it.)

In ΔQRM and ΔQSM,

$$MR = MS \quad \text{(proved above)}$$

$$\angle QMR = \angle QMS$$

(each $= 90^\circ$, as $QM \perp RS$)

$$QM = QM \quad \text{(common)}$$

$\therefore$ $\Delta QRM \cong \Delta QSM$

$\therefore$ $\angle MSQ = \angle MRQ$ (c.p.c.t.)

$$\angle RSQ = \angle SRQ$$

In ΔQRS,

$$\angle RQS + \angle SRQ + \angle RSQ = 180^\circ$$

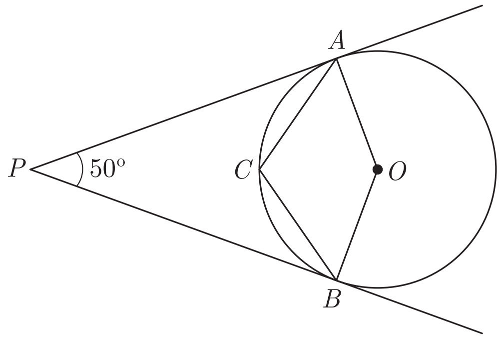
$$\angle RSQ + \angle SRQ + \angle SRQ = 180^\circ$$

$$\angle RQS + 2 \times 75^\circ = 180^\circ$$

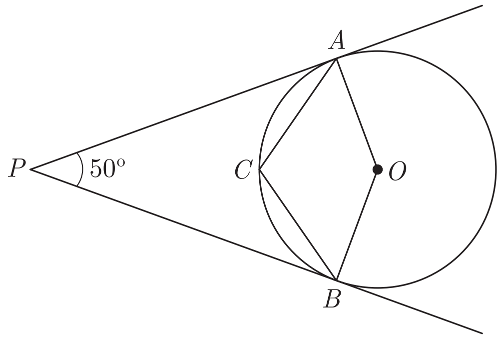
$$\angle RQS = 30^\circ$$

or

In the given figure, O is the centre of the circle. Determine $\angle ACB$, if PA and PB are tangents and $\angle APB = 50^\circ$.



Ans :



As PA is tangent to the circle at A and OA is radius,

$$OA \perp AP$$

i.e. $\angle OAP = 90^\circ$

In quadrilateral $OAPB$,

$$\angle AOB + \angle OAP + \angle APB + \angle OBP = 360^\circ$$

$$\angle AOB + 90^\circ + 50^\circ + 90^\circ = 360^\circ$$

$$\angle AOB = 130^\circ$$

$\therefore$ Reflex $\angle AOB = 360^\circ - 130^\circ = 230^\circ$

$\therefore$ $\angle ACB = \frac{1}{2}$ of reflex $\angle AOB$

(Angle at the centre = Double the angle at the remaining part of circle)

$$\angle ACB = \frac{1}{2} \times 230^\circ = 115^\circ$$

40. The following table gives the daily income of 50 workers of a factory. Draw both types (less than type and greater than type) gives. Hence, obtain the median income. [4]

Daily income (in ₹)	No. of workers
100 – 120	12
120 – 140	14
140 – 160	8
160 – 180	6
180 – 200	10

Ans :

For more than ogive:

Daily income (in ₹)	No. of workers (frequency)	Daily income more than	Cumulative frequency
100 – 120	12	More than 100	50

Daily income (in ₹)	No. of workers (frequency)	Daily income more than	Cumulative frequency
120 – 140	14	More than 120	38
140 – 160	8	More than 140	24
160 – 180	6	More than 160	16
180 – 200	10	More than 180	10
		More than 200	0

Take 1 cm along x-axis = ₹ 20 and 1 cm long y-axis = 10 workers.

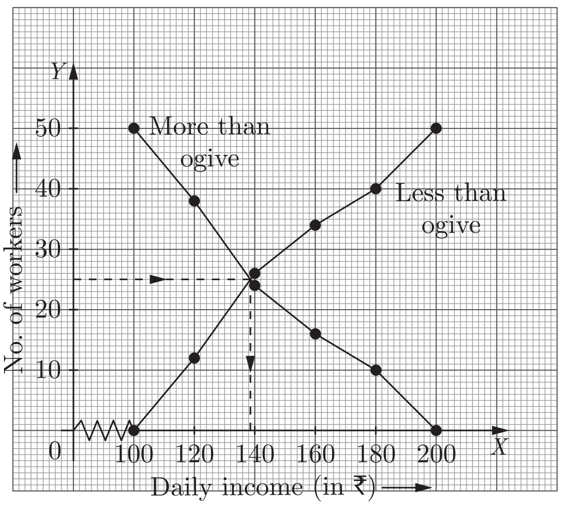
Plot the points (100, 50), (120, 38), (140, 24), (160, 16), (180, 10) and (200, 0).

Join these points by a free-hand drawing. More than type ogive is drawn on the graph sheet.

For less than type ogive:

Daily income (in ₹)	No. of workers (frequency)	Daily income more than	Cumulative frequency
100 – 120	12	Less than 100	0
120 – 140	14	Less than 120	12
140 – 160	8	Less than 140	26
160 – 180	6	Less than 160	34
180 – 200	10	Less than 180	40
		Less than 200	50

Choose the same scale (as above). Plot the points (100, 0), (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50). Join these points by a freehand drawing. Less than type ogive is drawn on the same graph sheet (with same axes).



The abscissa of point of intersection of more than ogive and less than ogive represents 138. Hence, the median daily = ₹ 138.