CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-16

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. If one zero of the quadratic polynomial x^2+3x+k is 2, then the value of k is [1]

(a) 10 (b) -10

(c) 5 (d) -5

Ans : (b) -10

$$\begin{aligned}(2)^2+3\times 2+k &= 0\\ 10+k &= 0\\ k &= -10\end{aligned}$$

2. Which of the following equations has two distinct real roots? [1]

(a) $2x^2-3\sqrt{2x}+\frac{9}{4}=0$ (b) $x^2+x-5=0$

(c) $x^2+3x+2\sqrt{2}=0$ (d) $5x^2-3x+1=0$

Ans : (b) $x^2+x-5=0$

Out of the four quadratic equations given, only for the quadratic equation given in option (b).

i.e. $x^2+x-5 = 0$

$$\begin{aligned}D &= (1)^2-4\times 1\times(-5)\\ &= 1+20 = 21>0\end{aligned}$$

roots are real and distinct

3. If 2nd term of an A.P. is 13 and the 5th term is 25, then its 7th term is [1]

(a) 30 (b) 33

(c) 37 (d) 38

Ans : (b) 33

Given, $a+d = 13$

and $a+4d = 25$

Solving these equations, we get

$$a = 9,\ d=4$$

So, $$\begin{aligned}\text{7th term} &= a+6d\\ &= 9+6\times 4\\ &= 33\end{aligned}$$

4. Which of the following can be the probability of an event? [1]

(a) -0.04 (b) 1.004

(c) $\frac{18}{23}$ (d) $\frac{8}{7}$

Ans : (c) $\frac{18}{23}$

$$0 \le P(E) \le 1$$

So, $\frac{18}{23}$ can be the probability of an event

5. A bag contains 3 red balls, 5 white balls and 7 black balls. The probability that a ball drawn from the bag at random will be neither red nor black is [1]

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$

(c) $\frac{7}{15}$ (d) $\frac{8}{15}$

Ans : (b) $\frac{1}{3}$

Total number of outcomes $= 3+5+7 = 15$

Favourable number of outcomes $= 5$

$$\text{Required probability} = \frac{5}{15} = \frac{1}{3}$$

6. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is [1]

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) $\frac{1}{3}$

Ans : (a) $\frac{1}{5}$

Total number of outcomes $= 40$
Favourable outcomes are 5, 10, 15, ..., 40
These are 8 in numbers

$$\text{Required probability} = \frac{8}{40} = \frac{1}{5}$$

7. The volume of the largest right circular cone that can be carved out from a cube of edge 4.2 cm is [1]

(a) $9.7\ \text{cm}^3$ (b) $77.6\ \text{cm}^3$

(c) $58.2\ \text{cm}^3$ (d) $19.4\ \text{cm}^3$

Ans : (d) $19.4\ \text{cm}^3$

The diameter of the base of largest cone= 4.2 cm

$$\text{radius} = 2.1 \text{ cm}$$

Also, the height of the largest cone= 4.2 cm

So, $$\text{volume} = \frac{1}{3}\pi \times (2.1)^2 \times 4.2$$

$$= \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4.2$$

$$= 19.404 \text{ cm}^3 = 19.4 \text{ cm}^3$$

8. The area of a sector of an angle p (in degrees) of a circle with radius R is [1]

(a) $\frac{p}{360^\circ} \times 2\pi R$ (b) $\frac{p}{180^\circ} \times \pi R^2$

(c) $\frac{p}{720^\circ} \times 2\pi R$ (d) $\frac{p}{720^\circ} \times 2\pi R^2$

Ans : (d) $\frac{p}{720^\circ} \times 2\pi R^2$

$$\text{Area of sector} = \frac{p}{360^\circ} \times \pi R^2 = \frac{p}{720^\circ} \times 2\pi R^2$$

9. It is given that $\triangle ABC \sim \triangle DFE, \angle A = 30^\circ, \angle C = 50^\circ, AB = 5\text{cm}, AC = 8\text{cm}$, and $DF = 7.5\text{cm}$. Then which of the following is true? [1]

(a) $DE = 12\text{cm}, \angle F = 50^\circ$

(b) $DE = 12\text{cm}, \angle F = 100^\circ$

(c) $EF = 12\text{cm}, \angle D = 100^\circ$

(d) $EF = 12\text{cm}, \angle D = 30^\circ$

Ans : (b) $DE = 12\text{cm}, \angle F = 100^\circ$

Here, $$\angle A = 30^\circ, \angle C = 50^\circ$$

$$\angle B = 180^\circ - (30^\circ + 50^\circ) = 100^\circ$$

Given, $$\triangle ABC \sim \triangle DEF$$

$$\frac{AB}{DF} = \frac{AC}{DE} = \frac{BC}{FC}$$

and $$\angle A = \angle D, \angle B = \angle F$$

$$\angle C = \angle E$$

$$\frac{5}{7.5} = \frac{8}{DE}$$

and $$\angle D = 30^\circ, \angle F = 100^\circ, \angle E = 50^\circ$$

$$DE = \frac{8 \times 7.5}{.5}$$

$$DE = 12 \text{ cm}$$

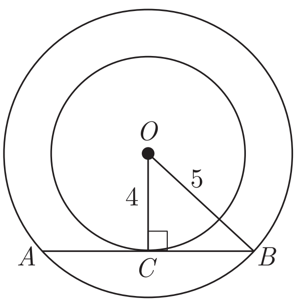
and $$\angle F = 100^\circ$$

10. If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other is [1]

(a) 3 cm (b) 6 cm

(c) 9 cm (d) 1 cm

Ans : (b) 6 cm



$$BC = \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

$$\text{Length of chord } AB = 2BC = 6 \text{ cm}$$

(Q.11-Q.15) Fill in the blanks.

11. A coin and a dice are thrown together, then the number of possible outcomes is .... [1]

Ans : 12

Possible outcomes are H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

The number of possible outcomes = 12

12. The length of a minute hand of a wall clock is 7 cm, then the area swept by it in 30 minutes is .... [1]

Ans : 77 cm^2

Angle turned by minute hand in 30 minutes = 180°

$$\text{So, area swept by it} = \frac{180^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7^2 = 77 \text{ cm}^2$$

13. If a real number α is a zero of a polynomial $f(x)$, then $x - \alpha$ is a .... of $f(x)$. [1]

Ans : Factor

14. If one root of a quadratic equation $ax^2 + bx + c = 0$ with rational coefficients is $2 + \sqrt{3}$, then the other root is .... [1]

Ans : $2 - \sqrt{3}$ (irrational roots occur in pairs)

or

If a and b are the roots of the quadratic equation $ax^2 - bx + c = 0$, then $\alpha + \beta =$ ....

Ans : $\frac{b}{a}$

$$\alpha + \beta = -\frac{(-b)}{a} = \frac{b}{a}$$

15. If $\triangle ABC \sim \triangle DEF$ and $\angle A = 47^\circ, \angle E = 83^\circ$, then $\angle C =$ .... [1]

Ans : 50°

Given, $$\triangle ABC \sim \triangle DEF$$

$$\angle D = \angle A = 47^\circ$$

$$\angle B = \angle E = 83^\circ$$

So, $$\angle C = 180^\circ - (47F + 83^\circ) = 50^\circ$$

$$\angle C = 50^\circ$$

(Q.16-Q.20) Answer the following.

16. If $\frac{p}{q}$ is a rational number $(q \neq 0)$, what is the condition on q so that the decimal representation of $\frac{p}{q}$ is terminating? [1]

Ans :

q is of the form $2^m \times 5^n$, where m, n are non-negative integers, where p, q are coprime.

17. Find the value of k so that the following system has no solution [1]

$$3x - y - 5 = 0,$$

$$6x - 2y - k = 0$$

Ans :

Given, $3x - y - 5 = 0$

$6x - 2y - k = 0$

For no solution, we have

$$\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-k}$$

$$k \neq 10$$

or

Find the 10th term of the $AP\ \sqrt{2}, \sqrt{8}, \sqrt{18}, ...$

Ans :

Given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}, ...$

i.e. $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, ...$

$$a = \sqrt{2},\ d = \sqrt{2}$$

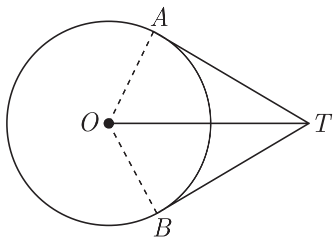
$$T_{10} = a + (10-1)\,d$$

$$= \sqrt{2} + 9\sqrt{2}$$

$$= 10\sqrt{2} = \sqrt{200}$$

Hence, 10th term of given AP is $\sqrt{200}$.

18. In the given figure, if $\angle ATO = 40^\circ$, find $\angle AOB$. [1]



Ans :

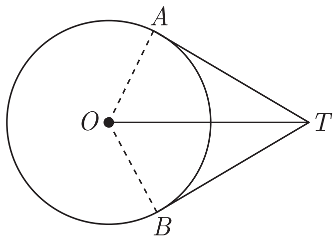
Given, $\angle ATO = 40^\circ$

and $\angle OAT = 90^\circ$

(tangent is perpendicular to the radius)

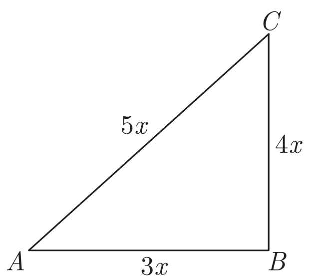
$$\angle AOT = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$$

$$\angle AOB = 2\angle AOT = 2 \times 50^\circ = 100^\circ$$



19. If $\cos A = \frac{3}{5}$, find $9\cot^2 A - 1$ [1]

Ans :



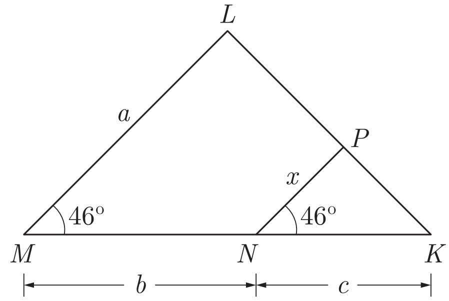
$$\cos A = \frac{3}{5}$$

$$\cot A = \frac{3x}{4x} = \frac{3}{4}$$

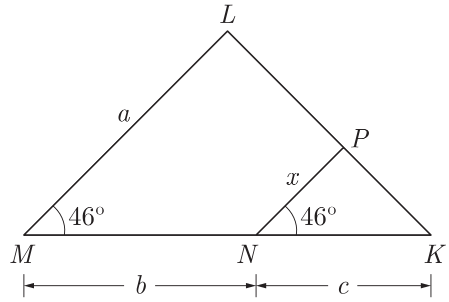
$$9\cot^2 A - 1 = 9\left(\frac{3}{4}\right)^2 - 1$$

$$= \frac{81}{16} - 1 = \frac{65}{16}$$

20. In the given figure, $\angle M = \angle N = 46^\circ$. Express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively. [1]



Ans :



In ΔKPN and ΔKLM,

$\angle N = \angle M = 46^\circ$ (given)

$\angle K = \angle K$ (common)

$$\Delta KPN \sim \Delta KLM$$

$$\frac{PN}{LM} = \frac{KN}{KM}$$

$$\frac{PN}{LM} = \frac{KN}{KN + NM}$$

$$\frac{x}{a} = \frac{c}{c+b}$$

$$x = \frac{ac}{b+c}$$

Section B

Question 21 to 26 carry 2 marks each.

21. For any natural number n check whether 6^n end with digit 0. [2]

Ans :

6^n can end with digit 0 only if 6^n is divisible by 2 and 5 both.

But prime factors of $6^n = 2^n \times 3^n$, so 6^n is not divisible by 5.

By Fundamental Theorem of Arithmetic, there is no natural number n for which 6^n ends with digit zero.

Hence, 6^n does not end with digit zero.

or

Find the LCM and HCF of 26 and 91 using prime factorisation method.

Ans :

Prime factorisation 26 and 91 are

$$26 = 2 \times 13$$
$$91 = 7 \times 13$$
$$\text{LCM } (26, 91) = 2 \times 7 \times 13 = 182$$
$$\text{HCF } (26, 91) = 13$$

22. Simplify: $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$ [2]

Ans :

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$$
$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$$
$$(a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$
$$= \sin^2\theta + \cos^2\theta - \sin\theta\cos\theta + \sin\theta\cos\theta$$
$$= \sin^2\theta + \cos^2\theta = 1$$

23. If A, B and C are the interior angles of a triangle ABC, show that. [2]

$$\sin\frac{B+C}{2} = \cos\frac{A}{2}$$

Ans :

As A, B and C are the interior angles of ΔABC,

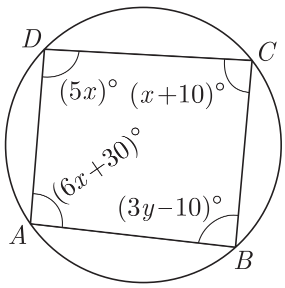
$$A + B + C = 180^\circ$$
$$B + C = 180^\circ - A$$
$$\frac{B+C}{2} = 90^\circ - \frac{A}{2} \quad \text{...(i)}$$

Now, $\sin\frac{B+C}{2} = \sin\left(90^\circ - \frac{A}{2}\right)$ (using (i))

$$\sin\frac{B+C}{2} = \cos\frac{A}{2} \quad [\sin(90^\circ - \theta) = \cos\theta]$$

24. The angles of a cyclic quadrilateral $ABCD$ are $\angle A = (6x+30)^\circ$, $\angle B = (5x)^\circ$, $\angle C = (x+10)^\circ$ and $\angle D = (3y-10)^\circ$. Find x and y. [2]

Ans :



In cyclic quadrilateral $ABCD$

$$\angle A + \angle C = 180^\circ$$
$$(6x+30)^\circ + (x+10)^\circ = 180$$
$$7x = 140$$
$$x = 20 \quad \text{...(i)}$$

and

$$\angle B + \angle D = 180^\circ$$
$$(5x)^\circ + (3y-10)^\circ = 180^\circ$$
$$5x + 3y = 190$$
$$5 \times 20 + 3y = 190$$
$$3y = 90$$
$$y = 30 \quad \text{(using (i))}$$

$$x = 20$$
$$y = 30$$

25. Find the middle term (s) of the following AP.
213, 205, 197, ..., 37 [2]

Ans :

Given AP is 213, 205, 197, ..., 37.

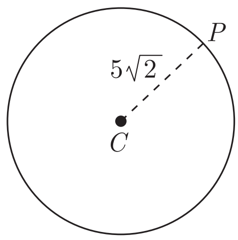
Here, $a = 213,\ d = -8,\ l = 37$

$$l = a + (n-1)d$$
$$37 = 213 + (n-1)(-8)$$
$$n - 1 = \frac{-176}{-8}$$
$$n - 1 = 22$$
$$n = 23$$
$$\text{Total number of terms} = 23$$
$$\text{Middle term} = \left(\frac{23+1}{2}\right)\text{th term}$$
$$= 12\text{th term}$$
$$= 213 + (12-1) \times (-8)$$
$$= 213 - 88 = 125$$

Hence, the middle term of given AP is 125.

26. The centre of a circle is $[2a(a-7)]$. Find the values of a if the circle passes through the point $(11, -9)$ and has diameter $10\sqrt{2}$ units. [2]

Ans :



$$\text{Radius of circle} = \frac{1}{2} \times 10\sqrt{2} = 5\sqrt{2}.$$

The centre of the circle is $C(2a, a-7)$ and it passes through the point $P(11, -9)$, so CP = radius of circle

$$\sqrt{(2a-11)^2 + (a-7+9)^2} = 5\sqrt{2}$$
$$(2a-11)^2 + (a+2)^2 = (5\sqrt{2})^2$$
$$4a^2 - 44a + 121 + a^2 + 4a + 4 = 50$$
$$5a^2 - 40a + 75 = 0$$
$$a^2 - 8a + 15 = 0$$
$$(a-3)(a-5) = 0$$
$$a = 3, 5$$

Hence, the values of a are 3, 5.

or

If the point $P(3,4)$ is equidistant from the points $A(-2,3)$ and $B(k,-1)$, find the values of k. Also, find the distance AB.

Ans :

Given, $PA = PB$

$$\sqrt{(3+2)^2 + (4-3)^2} = \sqrt{(3-k)^2 + (4+1)^2}$$
$$25 + 1 = (3-k)^2 + 25$$

$$(3-k)^2 = 1$$
$$3-k = \pm 1$$
$$k = 2, 4$$

So, B is $(2, -1)$ or $(4, -1)$.

When A is $(-2, 3)$ and B is $(2, -1)$, then

$$AB = \sqrt{(-2-)^2+(3+1)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2} \text{ units}$$

When A is $(-2, 3)$ and B is $(4, -1)$, then

$$AB = \sqrt{(-2-4)^2+(3+1)^2}$$
$$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

Section C

Question 27 to 34 carry 3 marks each.

27. If the roots of the equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal, prove that $2a=b+c$. [3]

Ans :

The given equation is $(a-b)x^2+(b-c)x+(c-a)=0$
Comparing it with

$$Ax^2+Bx+C = 0$$

we get, $A = a-b,\ B=b-c$

$$C = c-a$$

$$\text{Discriminant} = B^2-4AC$$
$$= (b-c)^2-4(a-b)(c-a)$$

For equal roots,

$$\text{Discriminant} = 0$$
$$(b-c)^2-4(a-b)(c-a) = 0$$
$$b^2-2bc+c^2-4(ca-a^2-bc+ab) = 0$$
$$4a^2+b^2+c^2-4ab-4ca+2bc = 0$$
$$(2a-b-c)^2 = 0$$
$$2a-b-c = 0$$
$$2a = b+c$$

28. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28th term of AP. [3]

Ans :

Let the first term and common difference of AP be a and d respectively.

Given, $S_7 = 63$

$$\frac{7}{2}[2a+(7-1)d] = 63$$
$$2a+6d = 18 \quad \text{...(i)}$$

and sum of next 7 terms $= 161$

i.e. $S_{14}-S_7 = 161$

$$S_{14} = 161+63$$
$$S_{14} = 224$$
$$\frac{14}{2}[2a+(14-1)d] = 224$$
$$2a+13d = 32 \quad \text{...(ii)}$$

Subtracting equation (i) from (ii), we get

$$2a+13d-2a-6d = 32-18$$
$$7d = 14$$
$$d = 2$$

Substituting $d=2$ in equation (i), we get

$$2a+6\times 2 = 18$$
$$2a = 6$$
$$a = 3$$

Now, $T_{28} = 3+(28-1)\times 2 = 3+54 = 57$

Hence, 28th term of AP is 57.

29. Show that the square of any positive integer cannot be of the form $5m+2$ or $5m+3$ for some integer m. [3]

Ans :

Let n be any positive integer. Applying Euclid's division lemma with divisor $=5$, we get

$$n = 5q,\ 5q+1,\ 5q+2,\ 5q+3 \text{ or } 5q+4,$$

where q is some whole number.

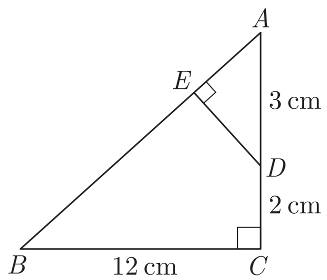
Now, $(5q)^2 = 25q^2 = 5m$

where $m = 5q^2$, which is an integer;

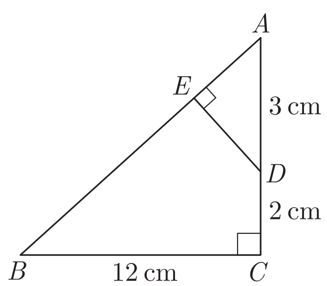
$$(5q+1)^2 = 25q^2+10q+1 = 5(5q^2+2q)+1$$
$$= 5m+1, \text{ where}$$
$$m = 5q^2+2q, \text{ which is an integer;}$$
$$(5q+2)^2 = 25q^2+20q+4 = 5(5q^2+4q)+4$$
$$= 5m+4, \text{ where}$$
$$m = 5q^2+4q, \text{ which is an integer;}$$
$$(5q+3)^2 = 25q^2+30q+9$$
$$= 5(5q^2+6q+1)+4 = 5m+4, \text{ where}$$
$$m = 5q^2+6q+1, \text{ which is an integer;}$$
$$(5q+4)^2 = 25q^2+40q+16$$
$$= 5(5q^2+8q+3)+1 = 5m+1, \text{ where}$$
$$m = 5q^2+8q+3, \text{ which is an integer;}$$

Thus, the square of any positive integer is of the form $5m, 5m+1$ or $5m+4$ for some integer m. It follows that the square of any positive integer cannot be of the form $5m+2$ or $5m+3$ for some integer m.

30. In the given figure, $\triangle ABC$ is right angled at C and $DE \perp AB$. Prove that $\triangle ABC \sim \triangle ADE$, and hence find the lengths of AE and DE. [3]



Ans :



In ΔABC and ΔADE,

$$\angle BAC = \angle EAD \quad \text{(same angle)}$$

$$\angle ACB = \angle AED \quad \text{(each } = 90^\circ\text{)}$$

$$\Delta ABC \sim \Delta ADE \quad \text{(AA similarity criterion)}$$

$$\frac{BC}{DE} = \frac{AC}{AE} = \frac{AB}{AD} \quad ...(i)$$

From figure, $AC = AD + DC = 3\text{ cm} + 2\text{ cm}$

$= 5\text{ cm}$

In ΔABC, $\angle C = 90^\circ$

By Pythagoras Theorem, we have

$$AB^2 = BC^2 + AC^2 = 12^2 + 5^2 = 169$$

$$AB = 13\text{ cm}$$

From (i), we get

$$\frac{12\text{ cm}}{DE} = \frac{5\text{ cm}}{AE} = \frac{13\text{ cm}}{3\text{ cm}}$$

$$AE = \left(5 \times \frac{3}{13}\right)\text{cm} = \frac{15}{13}\text{ cm}$$

and $$DE = \left(12 \times \frac{3}{13}\right)\text{cm} = \frac{36}{13}\text{ cm}$$

31. The average score of boys in the examination of a school is 71 and that of the girls is 73. The average score of the boys and girls in the examination is 71.8. Find the ratio of number of boys to the number of girls who appeared in the examination. [3]

Ans :

Let the number of boys who appeared in the examination be x and that of the girls be y, so the total number of students who appeared in the examination $= x + y$.

Since the average score is boys in the examination is 71 and that of the girls is 73, therefore, the sum of scores of boys $= 71x$ and the sum of scores of girls $= 73y$.

The total sum of scores of boys and girls $= 71x + 73y$

The average score of the boys and girls

$$= \frac{71x + 73y}{x + y} = 71.8 \text{ (given)}$$

$$71x + 73y = 71.8x + 71.8y$$

$$(73 - 71.8)\,y = (71.8 - 71x)$$

$$1.2y = 0.8x$$

$$\frac{12}{10}y = \frac{8}{10}x$$

$$12y = 8x$$

$$\frac{x}{y} = \frac{12}{8} = \frac{3}{2}$$

$$x : y = 3 : 2$$

Hence, the ratio of number of boys to that of girls is $3:2$.

or

Given below is the distribution of weekly pocket money received by students of a class. Calculate the pocket money that is received by most of the students.

Pocket money (in ₹)	0-20	20-40	40-60	60-80	80-100	100-120	120-140
No. of students	2	2	3	12	18	5	2

Ans :

Pocket money (in ₹)	**Number of students**
0-20	2
20-40	2
40-60	3
60-80	12
80-100	18
100-120	5
120-140	2

Note that the frequency distribution is continuous and all classes are of equal size.

The maximum class frequency is 18. So, the model class is 80-100.

Here, $l = 80,\ h = 20,\ f_1 = 18$

$f_0 = 12,\ f_2 = 5$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 80 + \frac{18 - 12}{36 - 12 - 5} \times 20$$

$$= 80 + \frac{6}{19} \times 20 = 80 + 6.32$$

$$= 86.32 \text{ (approx)}$$

Hence, the pocket money received by most of the students is ₹ 86.32.

32. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \frac{p^2 - 1}{p^2 + 1}$ [3]

Ans :

Given,

$$\sec\theta + \tan\theta = p$$

$$\text{RHS} = \frac{p^2 - 1}{p^2 + 1} = \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1}$$

$$= \frac{\sec^2\theta - 1 + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + 1 + \tan^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$\left(\frac{\sec^2\theta - 1 = \tan^2\theta}{1 + \tan^2\theta = \sec^2\theta}\right)$$

$$= \frac{2\tan^2\theta + 2\sec\theta \cdot \tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)}$$

$$= \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta} \times \frac{\cos\theta}{1}$$

$$= \sin\theta = \text{LHS}$$

or

Evaluate the following

$$\sec 41^\circ \sin 49^\circ + \cos 49^\circ \operatorname{cosec} 41^\circ - \frac{2}{\sqrt{3}}\tan 20^\circ \cdot \tan 60^\circ$$

$$\tan 70^\circ - 3(\cos^2 45^\circ - \sin^2 90^\circ)$$

Ans :

$$\sec 41^\circ \sin 49^\circ + \cos 49^\circ \operatorname{cosec} 41^\circ - \frac{2}{\sqrt{3}}$$

$$(\tan 20^\circ \tan 60^\circ \tan 70^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$$

$= \sec 41^\circ \sin(90^\circ - 41^\circ) + \cos(90^\circ - 41^\circ)\operatorname{cosec} 41^\circ$

$-\frac{2}{\sqrt{3}}(\tan 20^\circ \cdot \sqrt{3} \cdot \tan(90^\circ - 20^\circ)) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - 1^2\right)$

$= \sec 41^\circ \cos 41^\circ + \sin 41^\circ \operatorname{cosec} 41^\circ - 2(\tan 20^\circ \cot 20^\circ)$

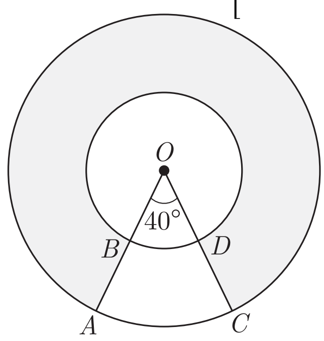
$-3\left(\frac{1}{2} - 1\right)$

$= 1 + 1 - 2 \times 1 - 3\left(-\frac{1}{2}\right)$

$(\sec A \cos A = 1, \sin A \operatorname{cosec} A = 1, \tan A \cot A = 1)$

$= 1 + 1 - 2 + \frac{3}{2} = \frac{3}{2}$

33. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm, where $\angle AOC = 40^\circ$ $\left[\text{Use } \pi = \frac{22}{7}\right]$ [3]



Ans :

Let R and r be the radii of the outer and inner concentric circles then $R = 14$ cm and $r = 7$ cm.
The central angle made by the major sector of the outer and inner circles $\theta = 360^\circ - 40^\circ = 320^\circ$.
Area of shaded region = area of major sector $OAC -$ area of major sector OBD

$$= \frac{\theta}{360}\pi R^2 - = \frac{\theta}{360}\pi r^2 = \frac{\theta\pi}{360}(R^2 - r^2)$$

$$= \frac{320}{360} \times \frac{22}{7}(14^2 - 7^2)\text{ cm}^2$$

$$= \frac{8}{9} \times \frac{22}{7} \times 147\text{ cm}^2$$

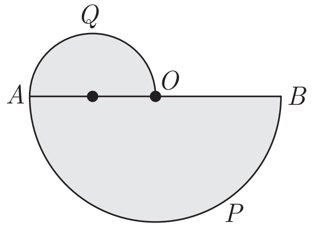
$$= \left(\frac{8}{3} \times 22 \times 7\right)\text{cm}^2 = \frac{1232}{3}\text{ cm}^2$$

$$= 410.66\text{ cm}^2$$

Hence, area of the shaded region $= 410.66\text{ cm}^2$.

or

In the given figure, APB and AQO are semicircles and $OA = OB$. If the perimeter of the figure is 40 cm, find the area of the shaded region.



Ans :

Let, $OA = OB = r$ cm
AO is the diameter of small semicircle
Radius of small semicircle,

$$R = \frac{r}{2}$$

Given, perimeter of shaded region= 40 cm
Perimeter of small semicircle + perimeter of big semicircle + OB = 40 cm

$$\pi R + \pi r + r = 40$$

$$\pi\left(\frac{r}{2}\right) + \pi r + r = 40$$

$$\frac{3}{2}\pi r + r = 40$$

$$\frac{3}{2} \times \frac{22}{7} \times r + r = 40$$

$$\frac{33}{7}r + r = 40$$

$$\frac{40r}{7} = 40$$

$$r = 7\text{ cm}$$

Area of shaded region

= area of small semicircle

+ area of big semicircle

$$= \frac{1}{2}\pi R^2 + \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{r}{2}\right)^2 + \frac{1}{2}\pi r^2$$

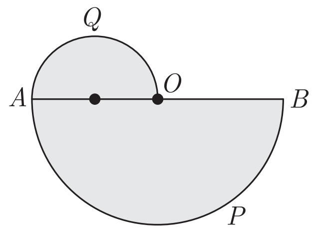
$$= \frac{\pi}{2}\left(\frac{r^2}{4} + r^2\right) = \frac{\pi}{2} \times \frac{5r^2}{4}$$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{5}{4} \times (7)^2 = \frac{11 \times 5 \times 7}{4}$$

$$= \frac{385}{4}$$

$$= 96\frac{1}{4}\text{ cm}^2 = 96.25\text{ cm}^2$$

Hence, area of the shaded region is 96.25 cm^2.

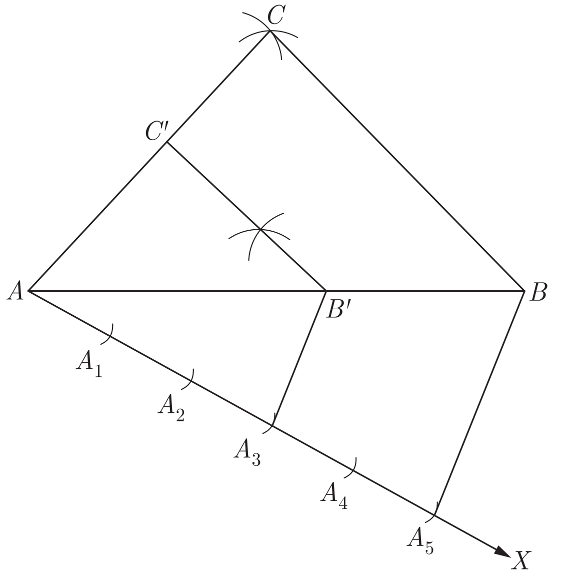


34. Construct a triangle with sides 5 cm, 5.5 cm and 6.5 cm. Now construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.[3]

Ans :

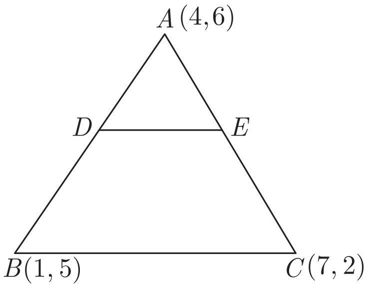
Steps of construction:

(i) Draw $AB = 6.5$ cm.
(ii) With A as centre and radius 5.5 cm draw an arc.
(iii) With B as as centre and radius 5.5 cm draw an arc to meet the previous arc at C.
(iv) Join AC and BC, then ABC is a triangle with $AB = 6.5$ cm, $AC = 5.5$ cm and $BC = 5$ cm.
(v) Draw any ray AX making an acute angle with AB on the side opposite to the vertex C.
(vi) Locate 5 points A_1, A_2, A_3, A_4 and A_5 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
(vii) Join A_5B. Through A_3 draw a line parallel to A_5B to intersect AB at B'.
(viii) Through B' draw a line parallel to BC to intersect the AC at C'. Then $AB'C'$ is the required triangle.

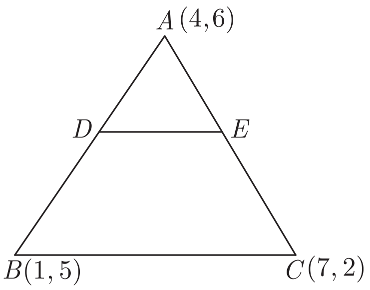


Section D

35. In the given figure, the vertices of $\Delta\, ABC$ are $A(4, 6)$, $B(1, 5)$ and $C(7, 2)$. A line segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of $\Delta\, ADE$ and compare it with area of $\Delta\, ABC$. [4]



Ans :



Given the vertices of $\Delta\, ABC$ are $A(4, 6)$ $B(1, 5)$ and $C(7, 2)$.

Also, $\frac{AD}{AB} = \frac{1}{3}$

$$\frac{AD}{AD + DB} = \frac{1}{3}$$

$$\frac{AD + DB}{AD} = \frac{3}{1}$$

$$1 + \frac{DB}{AD} = 3$$

$$\frac{DB}{AD} = 2$$

$$AD : DB = 1 : 2$$

i.e. D divides the join of A and B internally in the ratio $1 : 2$

Similarly, $\frac{AE}{AC} = \frac{1}{3}$

$AE : EC = 1 : 2$ i.e. E divides the join of A and C internally in the ratio $1 : 2$.

Coordinates of D are $\left(\frac{1 \times 1 + 2 \times 4}{1 + 2}, \frac{1 \times 5 + 2 \times 6}{1 + 2}\right)$

i.e. $\left(3, \frac{17}{3}\right)$

and coordinates of E are $\left(\frac{1 \times 7 + 2 \times 4}{1 + 2}, \frac{1 \times 2 + 2 \times 6}{1 + 2}\right)$

i.e. $\left(5, \frac{14}{3}\right)$

Area of $\Delta\, ADE$

$$= \frac{1}{2}\left|4\left(\frac{17}{3} - \frac{14}{3}\right) + 3\left(\frac{14}{3} - 6\right) + 5\left(6 - \frac{17}{3}\right)\right|$$

$$= \frac{1}{2}\left|4(1) + 3\left(\frac{-4}{-3}\right) + 5\left(\frac{1}{3}\right)\right| = \left|4 - 4 + \frac{5}{3}\right|$$

$$= \frac{1}{2} \times \frac{5}{3} = \frac{5}{6} \text{ sq. units}$$

Area of $\Delta\, ABC$

$$= \frac{1}{2}\left|4(5 - 2) + 1(2 - 6) + 7(6 - 5)\right|$$

$$= \frac{1}{2}\left|4(3) + 1(-4) + 7(1)\right| = \frac{1}{2}\left|12 - 4 + 7\right|$$

$$= \frac{15}{2} \text{ sq. units}$$

Now, $\frac{\text{ar}(\Delta\, ADE)}{\text{ar}(\Delta\, ABC)} = \frac{\frac{5}{6}}{\frac{15}{2}} = \frac{1}{9}$

$$\text{ar}(\Delta\, ADE) : \text{ar}(\Delta\, ABC) = 1 : 9$$

or

Find the point on the x-axis which is equidistant from the points $(5, 4)$ and $(-2, 3)$. Also find the area of the triangle formed by these points.

Ans :

Let the point on the x-axis be $P(x, 0)$ and the given points are $A(5, 4), B(-2, 3)$

As the point P is equidistant from A and B, we have

$$PA = PB$$

$$(PA)^2 = (PB)^2$$

$$(5 - x)^2 + (4 - 0)^2 = (-2 - x)^2 + (3 - 0)^2$$

$$25 - 10x + x^2 + 16 = 4 + 4x + x^2 + 9$$

$$14x = 28$$

$$x = 2$$

Hence, the point on x-axis is (2, 0).

Now, area of $\Delta\, PAB$

$$= \frac{1}{2}\left|2(4 - 3) + 5(3 - 0) + (-2)(0 - 4)\right|$$

$$= \frac{1}{2}\left|2 + 15 + 8\right| = \frac{25}{2} \text{ sq. units}$$

36. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k. Also find the zeroes of the polynomial $p(x)$. [4]

Ans :

Given α and β are the zeroes of the polynomial,

$$p(x) = 2x^2 + 5x + k$$

$$\text{Sum of the zeroes} = \alpha + \beta = -\frac{5}{2} \quad \text{...(i)}$$

$$\text{and} \quad \text{product of the zeroes} = \alpha\beta = \frac{k}{2} \quad \text{...(ii)}$$

Also given $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$$(\alpha+\beta)^2 - 2\alpha\beta + \alpha\beta = \frac{21}{4}$$

$$(\alpha+\beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\left(-\frac{5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = \frac{25}{4} - \frac{21}{4}$$

$$\frac{k}{2} = 1$$

$$k = 2$$

Hence, the given polynomial

$$p(x) = 2x^2 + 5x + 2$$

$$p(x) = 2x^2 + 4x + x + 2$$

$$= 2x(x+2) + 1(x+2)$$

$$= (x+2)(2x+1)$$

Zeroes of the polynomial are $-2, -\frac{1}{2}$.

or

If the remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$

Ans :

Dividing the given polynomial $x^3 + 2x^2 + kx + 3$ by $x - 3$, we get

$$\begin{array}{r|l} & x^2+5x+(k+15) \\ \hline x-3 & x^3+2x^2+kx+3 \\ & x^3-3x^2 \\ & \underline{-\quad +} \\ & 5x^2+kx+3 \\ & 5x^2-15x \\ & \underline{-\quad +} \\ & (k+15)x+3 \\ & (k+15)x-3(k+15) \\ & \underline{-\quad +} \\ & 3k+48 \end{array}$$

$$\text{Remainder} = 3k + 48 = 21 \text{ (given)}$$

$$3k = -27$$

$$k = -9$$

$$\text{Quotient} = x^2 + 5x + (k+15)$$

$$= x^2 + 5x + (-9+15)$$

$$= x^2 + 5x + 6$$

Now, $x^3 + 2x^2 + kx + 3 = (x-3)(x^2+5x+6) + 21$

$$x^3 + 2x^2 + kx - 18 = (x-3)(x^2+5x+6)$$

$$= (x-3)(x+2)(x+3)$$

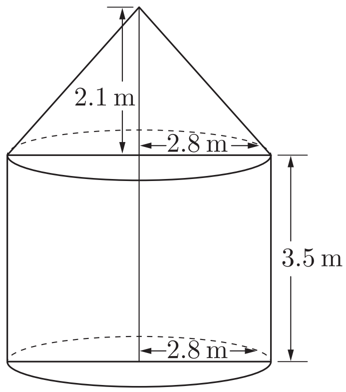
If follows that the value of the polynomial $x^3 + 2x^2 + kx - 18$ will be zero when $x - 3 = 0$ or $x + 2 = 0$ or $x + 3 = 0$ i.e. when $x = 3, -2, -3$.

Hence, the zeroes of the polynomial $x^3 + 2x^2 + kx - 18$ are $3, -2, -3$.

37. Due to heavy floods in a State, thousands were rendered homeless. 50 schools collectively offered to the State Government to provide place and canvas for 1500 tests to be fixed by the Government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹120 per sq. m, find the amount shared by each school to set up the tents. $\left[\text{Use } \pi = \frac{22}{7}\right]$ [4]

Ans :

Let r and h be the radius and height of the cylindrical part and l be the slant height of conical part.



Given, $r = 2.8\text{ m}$, $l = 3.5\text{ m}$

$$\text{height of cone} = 2.1\text{ m}$$

Slant height of cone,

$$l = \sqrt{(2.8)^2 + (2.1)^2}$$

$$= \sqrt{7.84 + 4.41} = \sqrt{12.25}$$

$$= 3.5\text{ m}$$

Area of canvas required for one tent

$$= \text{curved surface area of cylinder} + \text{curved surface area of cone}$$

$$= 2\pi rh + \pi rl = \pi r(2h + l)$$

$$= \frac{22}{7} \times 2.8 \times (2 \times 3.5 + 3.5)$$

$$= 22 \times 0.4 \times 10.5 = 92.4\text{ m}^2$$

Area of canvas required for 1500 tents

$$= (92.4 \times 1500)\text{ m}^2$$

$$= 138600\text{ m}^2$$

Cost of 1500 tents at the rate of ₹ 120 per m^2

$$= 138600 \times ₹\ 120$$

$$= ₹\ 16632000$$

$$\text{Share of each school} = \frac{\text{Total cost}}{\text{Number of school}}$$

$$= \frac{16632000}{50}$$

$$= ₹\ 332640$$

38. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. [4]

Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Ans :

Construct the cumulative frequency distribution table as under:

Class intervals	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$

It is given, $n = 100$

so $76+x+y = 100$

$x+y = 24$ ...(i)

As the median is 525, which lies in the class $500-600$, so the median class is $500-600$.

$l = 500,\ f = 20,$

$c.f. = 36+x$

and $h = 100$

$$\text{Median} = l + \frac{\frac{n}{2} - c.f.}{f} \times h = 525 \text{ (given)}$$

$$500 + \frac{50-(36+x)}{20} \times 100 = 525$$

$5(14-x) = 25$

$14-x = 5$

$x = 9$

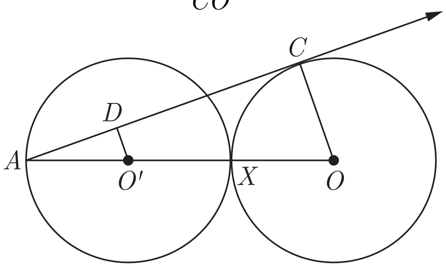
From (i), $9+y = 24$

$y = 15$

Hence, $x = 9$

and $y = 15$

39. In the adjoining figure, two equal circles with centres O and O', touch each other at X. OO' produced meets the circle with O' at A. AC is tangent to the circle with centre O, at the point C. $O'D$ is perpendicular to AC. Find the value of $\frac{DO'}{CO}$. [4]



Ans :

As AC is tangent to the circle with centre O at the point C and OC is radius,

$OC \perp AC$

$\angle ACO = 90^\circ$

Given, $O'D \perp AC$

so $\angle ADO' = 90^\circ$

$AO' = r$

and $AO = AO' + O'X + XO$

$= r+r+r = 3r$

In $\Delta ADO'$ and ΔACO,

$\angle ADO' = \angle ACO$ (each $= 90^\circ$)

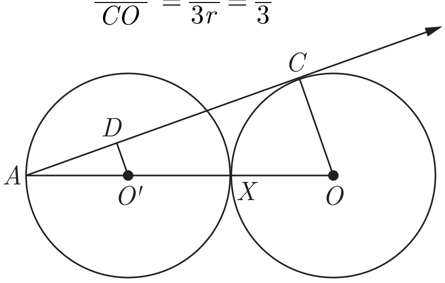
$\angle DAO' = \angle CAO$ (same angle)

$\Delta ADO' \sim \Delta ACO$

(AA criterion of similarity)

$$\frac{DO'}{CO} = \frac{AO'}{AO}$$

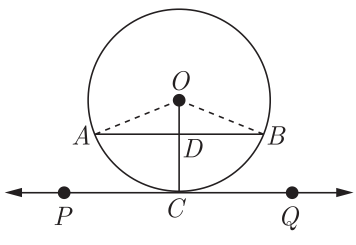
$$\frac{DO'}{CO} = \frac{r}{3r} = \frac{1}{3}$$



or

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Ans :



Let AB be an arc of a circle with centre O and C be mid-point of arc AB.

Let PQ be tangent to the arc AB of the circle at the

point C and OC meet the chord AB at D.
As PQ is tangent to the arc at C and OC is radius through C, $OC \perp PQ$

$$\angle OCP = 90^\circ \quad \text{...(i)}$$

In ΔOAD and ΔOBD,

$$OA = OB \quad \text{(each = radius)}$$

$$OD = OD \quad \text{(common)}$$

$$\angle AOD = \angle BOD$$

(equal arcs subtend equal angles at the centre of circle)

$$\Delta OAD \cong \Delta OBD$$

(SAS criterion of congruency)

$$\angle ODA = \angle ODB$$

But,

$$\angle ODA + \angle ODB = 180^\circ$$

$$\angle ODA + \angle ODA = 180^\circ \quad (AB \text{ is a straight line})$$

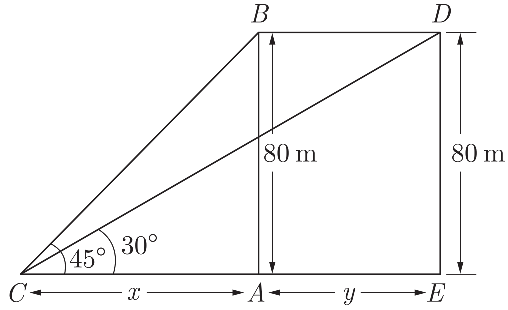
$$\angle ODA = 90^\circ$$

$$\angle ODA = \angle OCP \quad \text{(using (i))}$$

But these are corresponding angles, therefore, $AB \parallel PQ$.
Hence, the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

40. A bird is sitting on the top of a 80 m high tree. From a point on the gound, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$) [4]

Ans :



Let AB be the tree and B be the position of bird.
From a point C on the ground the angle of elevation of bird is 45°. Bird flies away horizontally and reach the point D in 2 seconds. The angle of elevation from point to the bird is now 30°.

Let, $AC = x$ m

and $AE = y$ m

In ΔABC, $\angle A = 90^\circ$

$$\tan 45^\circ = \frac{AB}{AC}$$

$$1 = \frac{80}{x}$$

$$x = 80 \text{ m} \quad \text{...(i)}$$

In ΔCDE, $\angle E = 90^\circ$

$$\tan 30^\circ = \frac{DE}{CE}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{x+y}$$

$$x + y = 80\sqrt{3}$$

$$y = 80\sqrt{3} - 80 \quad \text{(using (i))}$$

$$y = 80(\sqrt{3} - 1)$$

$$= 80(1.732 - 1)$$

$$y = 80 \times 0.732$$

$$y = 58.56 \text{ m}$$

Hence, the speed of flying of the bird

$$= \frac{\text{Distance}}{\text{Time}} = \frac{y \text{ m}}{2 \text{ s}}$$

$$= \frac{58.56}{2} \text{ m/s} = 29.28 \text{ m/s}$$

Hence, the speed of flying of the bird

$$= 29.28 \text{ m/s}$$