# CLASS X (2019-20)
# MATHEMATICS BASIC(241)
# SAMPLE PAPER-15

**Time : 3 Hours** **Maximum Marks : 80**

**General Instructions :**

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

## Section A

**Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.**

**1.** If $\alpha$ and $\frac{1}{\alpha}$ are the zeroes of the polynomial $6x^2+11x-(k-2)$, then the value of $k$ is [1]

(a) $-4$ (b) $-6$
(c) 6 (d) 4

**Ans :** (a) $-4$

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coeff. of } x^2}$$

$$\alpha\cdot\frac{1}{\alpha} = -\frac{(k-2)}{6}$$

$$k-2 = -6$$

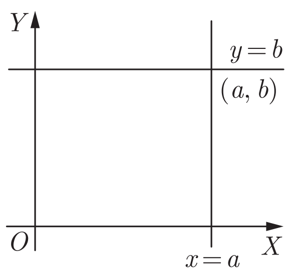
$$k = -4$$

**2.** The pair of equations $x=a$ and $y=b$ graphically represents the lines which are [1]

(a) parallel (b) intersecting at $(b,a)$
(c) coincident (d) intersecting at $(a,b)$

**Ans :** (d) intersecting at $(a,b)$

$x=a$ is a line parallel to $y$-axis and $y=b$ is a line parallel to $x$-axis.
These lines intersect at $(a,b)$




**3.** If the sum of the roots of the quadratic equation $2x^2+(2k-1)x-(k-4)=0$ is equal to the product of its roots, then the value of $k$ is [1]

(a) $-3$ (b) 3
(c) 2 (d) 0

**Ans :** (a) $-3$

Given, sum of the roots = product of the roots

$$\frac{-(2k-1)}{2} = -\frac{(k-4)}{2}$$

$$-2k+1 = -k+4$$

$$-k = 3$$

$$k = -3$$

**4.** Which term of the A.P. 21, 42, 63, 84, .......... is 210? [1]

(a) 9th (b) 10th
(c) 11th (d) 12th

**Ans :** (b) 10th

Here, $a = 21,\ d=21,\ a_n = 210$

$$a_n = a+(n-1)d$$

$$210 = 21+(n-1)\cdot 21$$

$$210 = 21[1+(n-1)]$$

$$10 = 1+n-1$$

$$n = 10$$

**5.** The perimeter of a circle is equal to that of a square, then the ratio of their areas is [1]

(a) 22 : 7 (b) 14 : 11
(c) 7 : 22 (d) 11 : 14

**Ans :** (b) 14 : 11

Let the radius of circle be $r$ and side of the square be $x$,

then, $2\pi r = 4x$

$$x = \frac{\pi r}{2}$$

Now, $$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{x^2} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2} = \frac{4}{\pi} = \frac{4}{\frac{22}{7}}$$

$$= \frac{14}{11}$$

**6.** If a bicycle wheel makes 5000 revolutions in moving $11\,\text{km}$, then the diameter of the wheel is [1]

(a) 35 cm (b) 70 cm
(c) 1.4 m (d) 70 m

**Ans :** (b) 70 cm

Distance covered by wheel in one revolution $= 2\pi r$

$$2\pi r \times 5000 = 11 \times 1000 \times 100 \text{ cm}$$
$$2 \times \frac{22}{7} r = 220$$
$$2r = 70 \text{ cm}$$

**7.** A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets she has bought? [1]
(a) 40 (b) 240
(c) 480 (d) 750

**Ans :** (c) 480

Let the girl bought $n$ tickets, then

$$\text{probability of her winning} = \frac{n}{6000}$$
$$0.08 = \frac{n}{6000}$$
$$n = 480$$

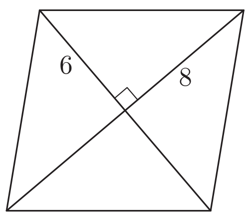
**8.** An event is very unlikely to happen. Its probability is closes to [1]
(a) 0.0001 (b) 0.001
(c) 0.01 (d) 0.1

**Ans :** (a) 0.0001

The probability will be nearest to 0 i.e., 0.0001

**9.** The lengths of the diagonals of a rhombus are 16 m and 12 m. The length of side of the rhombus is [1]
(a) 9 m (b) 10 m
(c) 8 m (d) 20 m

**Ans :** (b) 10 m




Since diagonals of a rhombus bisect each other at right angle.

$$\text{Length of side} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10 \text{ m}$$

**10.** If the area of a circle is $154 \text{ cm}^2$, then its perimeter is[1]
(a) 11 cm (b) 22 cm
(c) 44 cm (d) 55 cm

**Ans :** (c) 44 cm

$$\pi r^2 = 154$$
$$\frac{22}{7} \times r^2 = 154$$
$$r^2 = 49$$
$$r = 7 \text{ cm}$$
$$\text{Perimeter} = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$$

### (Q.11-Q.15) Fill in the blanks.

**11.** The sum of first 10 terms of AP 1, 3, 5, 7 ....., is .......... [1]

**Ans :**

As we know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2 \times 1 + (10-1) \times 2]$$
$$= 5 \times 20 = 100$$

**or**

The 9th term from the end of the AP 5, 8, 11, ....., 80 is .......... .

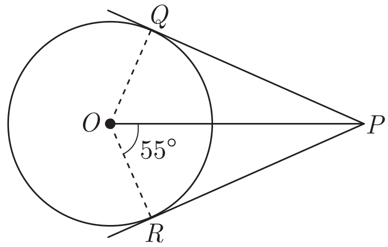
**Ans :**

5, 8, 11, ....., 80

Here, $a = 5,\ d = 3,\ l = 80$

$$\text{9th term from end} = l - (9-1)d$$
$$= 80 - 8 \times 3 = 56$$

**12.** In the given figure, $PQ$ and $PR$ are tangents from $P$ to a circle with centre $O$. If $\angle POR = 55^\circ$, then $\angle QPR =$ .......... . [1]




**Ans :**

In $\Delta OPR$, $\angle ORP = 90^\circ$

$$\angle OPR = 90^\circ - 55^\circ = 35^\circ$$

Also, $\angle QPR = 2\angle OPR = 2 \times 35^\circ = 70^\circ$

**13.** The probability of a certain or sure event is ..........[1]

**Ans :**

The probability of a certain or sure event is 1.

**14.** The area of the circular ring included between two concentric circles of radii 14 cm and 10.5 cm is .......... [1]

**Ans :**

$$\text{Area of ring} = \pi(r_1^2 - r_2^2)$$
$$= \frac{22}{7}(14^2 - 10.5^2)$$
$$= \frac{22}{7} \times (196 - 110.25)$$
$$= 269.5 \text{ cm}^2$$

**15.** If a rectangular piece of paper of dimensions $60 \text{ cm} \times 88 \text{ cm}$ is rolled to form a hollow circular cylinder of height 60 cm, then the radius of the cylinder is .......... . [1]

**Ans :**

Let the radius of cylinder be $r$, then

$$2\pi r = 88$$
$$2 \times \frac{22}{7} \times r = 88$$
$$r = 14 \text{ cm}$$

### (Q.16-Q.20) Answer the following

**16.** Which term of the sequence 114, 109, 104, ..... is the first negative term? [1]

**Ans :**

The given sequence is an AP with first term $a = 114$ and common difference $d = 5$.

$$a_n = a + (n-1)d < 0$$
$$114 + (n-1)(-5) < 0$$
$$-5n + 119 < 0$$
$$5n > 119$$
$$n > \frac{119}{5}$$
$$n > 23\frac{4}{5}$$
$$n = 24$$

Hence, 24th term is 1st negative term.

**or**

Find the value of $k$ for which the following are the consecutive terms of an AP : $k$, $2k-1$, $2k+1$.

**Ans :**

Given; $k$, $2k-1$, $2k+1$ are in AP

$$2(2k-1) = k + 2k + 1$$
$$4k - 2 = 3k + 1$$
$$4k - 3k = 2 + 1$$
$$k = 3$$

**17.** Find the distance between the points $\left(-\frac{8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$. [1]

**Ans :**

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\text{Distance} = \sqrt{\left(\frac{2}{5} + \frac{8}{5}\right)^2 + (2-2)^2}$$
$$= \sqrt{\left(\frac{10}{5}\right)^2 + 0} = \sqrt{4+0} = 2$$

Hence, the distance between $\left(-\frac{8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$ is 2 units.

**18.** Which measure of central tendency is given by the $x$ -coordinate of the point of intersection of the "more than ogive" and "less than ogive"? [1]

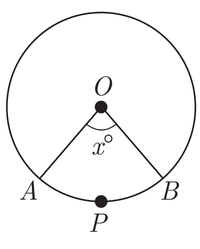
**Ans :** Median

**19.** What is the distance between two parallel tangents of a circle of radius 4 cm? [1]

**Ans :**

Distance between parallel tangents $= (4+4)$ cm $= 8$ cm

**20.** In the given figure, $O$ is the centre of a circle. The area of sector $OAPB$ is $\frac{5}{18}$ of the area of the circle. Find $x$. [1]




**Ans :**

Area of sector $OAPB = \frac{x}{360} \times \pi r^2$, where $r$ is the radius of the circle.

$$\frac{x}{360} \times \pi r^2 = \frac{5}{18} \times \pi r^2$$
$$x = \frac{5 \times 360}{18} = 100$$

Hence, $x = 100$

# Section B

**21.** Using Euclid's algorithm, find the HCF of 1656 and 4025. [2]

**Ans :**

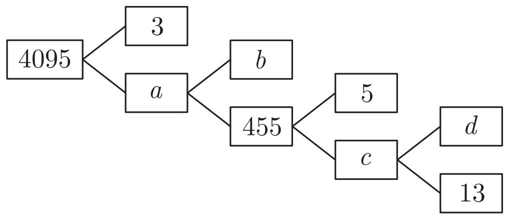
By Euclid's division algorithm, we have

$$4025 = 1656 \times 2 + 713$$
$$1656 = 713 \times 2 + 230$$
$$713 = 230 \times 3 + 23$$
$$230 = 23 \times 10 + 0$$

Thus, last non-zero remainder $= 23$
Hence, HCF of 1656 and 4025 is 23.

**or**

Write the missing numbers $a, b, c$ and $d$ in the following factor tree:




**Ans :**

Here, $3a = 4095$

$$a = \frac{4095}{3}$$
$$a = 1365$$
$$455b = a$$
$$455b = 1365$$
$$b = 3$$
$$5c = 455$$
$$c = \frac{455}{5} = 91$$
$$d = 7$$
$$a = 1365,\ b = 3,\ c = 91,\ d = 7$$

**22.** Find the two numbers whose sum is 75 and difference is 15. [2]

**Ans :**

Let the two numbers be $x$ and $y$ $(x > y)$, then according to given

$$x + y = 75$$

and $x - y = 15$

Adding equation (i) and (ii), we get

$$2x = 90$$
$$x = 45$$

Putting $x = 45$ in equation (i), we get

$$45 + y = 75$$
$$y = 30$$

Hence, two numbers are 45 and 30.

**23.** If $m$ and $n$ are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$. [2]

**Ans :**

Given, $m$ and $n$ are zeroes of the polynomial $3x^2 + 11x - 4$.

$$m + n = \frac{-11}{3}$$

and $$mn = \frac{-4}{3}$$

Now, $$\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{nm}$$
$$= \frac{(m+n)^2 - 2\,mn}{mn}$$
$$= \frac{\left(\frac{-11}{3}\right)^2 - 2 \times \left(\frac{-4}{3}\right)}{\frac{-4}{3}}$$
$$= \frac{\frac{121}{9} + \frac{8}{3}}{-\frac{4}{3}}$$
$$= \frac{\frac{121 + 24}{9}}{-\frac{4}{3}}$$
$$= -\frac{145}{9} \times \frac{3}{4} = -\frac{145}{12}$$

**24.** If the distances of $P(x, y)$ from the points $A(3, 6)$ and $B(-3, 4)$ are equal, prove that $3x + y = 5$. [2]

**Ans :**

Given, $$PA = PB$$
$$(PA)^2 = (PB)^2$$
$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$
$$x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$
$$6x + 6x - 8y + 12y = 9 + 36 - 9 - 16$$
$$12x + 4y = 20$$
$$3x + y = 5$$

**25.** If $\tan\theta = \frac{1}{\sqrt{5}}$, find the value of $\frac{\mathrm{cosec}^2\theta - \sec^2\theta}{\mathrm{cosec}^2\theta + \sec^2\theta}$. [2]

**Ans :**

Given, $$\tan\theta = \frac{1}{\sqrt{5}}$$
$$\cot\theta = \sqrt{5} \qquad \left(\cot\theta = \frac{1}{\tan\theta}\right)$$

Now, $$\sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{1}{\sqrt{5}}\right)^2$$
$$= 1 + \frac{1}{5} = \frac{6}{5}$$

and $$\mathrm{cosec}^2\theta = 1 + \cot^2\theta = 1 + (\sqrt{5})^2$$
$$= 1 + 5 = 6$$

$$\frac{\mathrm{cosec}^2\theta - \sec^2\theta}{\mathrm{cosec}^2\theta + \sec^2\theta} = \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}} = \frac{30 - 6}{30 + 6} = \frac{24}{36} = \frac{2}{3}$$

**26.** The metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed. [2]

**Ans :**

Let the edge of the cube formed be $x$ cm, then volume of the cube formed = sum of volumes of three cubes of edges 3 cm, 4 cm and 5 cm

$$x^3 = 3^3 + 4^3 + 5^3$$
$$x^3 = 27 + 64 + 125 = 216 = 6^3$$
$$x = 6$$

Hence, the edge of the cube so formed is 6 cm.

**or**

The radius and height of a cylinder are in the ratio $7:9$. If the volume of the cylinder is $1386\text{ cm}^3$. Find the total surface area of the cylinder.

**Ans :**

Let the radius and height of the cylinder be $r$ and $h$ respectively.

Given, $$\frac{r}{h} = \frac{7}{9}$$
$$h = \frac{9r}{7}$$
$$\text{Volume} = \pi r^2 h$$
$$1386 = \frac{22}{7} \times r^2 \cdot \frac{9r}{7}$$

(Volume $= 1386\text{ cm}^2$, given)

$$r^3 = \frac{1386 \times 7 \times 7}{22 \times 9}$$
$$r^3 = 343$$
$$r^3 = 7^3$$
$$r = 7\text{ cm}$$
$$h = \frac{9}{7} \times 7 = 9\text{ cm}$$

$$\text{Total surface area} = 2\pi r(h + r)$$
$$= 2 \times \frac{22}{7} \times 7(9 + 7)\text{ cm}^2$$
$$= 44 \times 16\text{ cm}^2 = 704\text{ cm}^2$$

# Section C

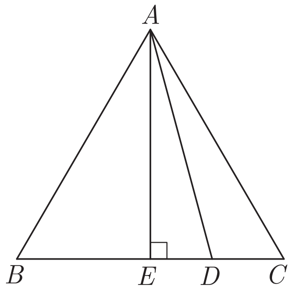
**27.** In an equilateral triangle $ABC$, a point $D$ is taken on base $BC$ such that $BD:DC = 2:1$. Prove that $9AD^2 = 7AB^2$. [3]

**Ans :**

As $\Delta\,ABC$ is equilateral,

$$BC = AB$$

Given, $$BD:DC = 2:1$$
$$BD = \frac{2}{3}BD = \frac{2}{3}AB$$




Draw $AE \perp BC$, then $E$ is mid-point of $BC$,

so $BE = \frac{1}{2}BC = \frac{1}{2}AB$

From fig., $ED = BD - BE$

$= \frac{2}{3}AB - \frac{1}{2}AB = \frac{1}{6}AB$

In $\Delta ABE$, $\angle AEB = 90°$

$AB^2 = AE^2 + BE^2$ ...(i)

In $\Delta AED$, $\angle AED = 90°$

$AD^2 = AE^2 + ED^2$ ...(ii)

Subtracting (ii) from (i), we get

$$AB^2 - AD^2 = BE^2 - ED^2$$

$$= \left(\frac{1}{2}AB\right)^2 - \left(\frac{1}{6}AB\right)^2$$

$$= \left(\frac{1}{4} - \frac{1}{36}\right)AB^2$$

$$= \frac{2}{9}AB^2$$

$$AD^2 = AB^2 - \frac{2}{9}AB^2 = \frac{7}{9}AB^2$$

$$9AD^2 = 7AB^2$$

**28.** Find the values of $a$ and $b$ so that $8x^4 + 14x^3 - 2x^2 + ax + b$ is exactly divisible by $4x^2 + 3x - 2$. [3]

**Ans :**

Dividing $8x^4 + 14x^3 - 2x^2 + ax + b$ by $4x^2 + 3x - 2$, we get

$$\begin{array}{r|l} & 2x^2+2x-1 \\ 4x^2+3x-2 & 8x^4+14x^3-2x^2+ax+b \\ & \underset{-}{8x^4}\underset{-}{+6x^3}\underset{+}{-4x^2} \\ \hline & 8x^3+2x^2+ax+b \\ & \underset{-}{8x^3}\underset{-}{+6x^2}\underset{+}{-4x} \\ \hline & -4x^2+(a+4)x+b \\ & \underset{+}{-4x^2}\underset{+}{-}\ \underset{-}{3x+2} \\ \hline & (a+7)x+(b-2) \end{array}$$

Given that $8x^4 + 14x^3 - 2x^2 + ax + b$ is exactly divisible by $4x^2 + 3x - 2$.

Remainder should be zero

$$(a+7)x + (b-2) = 0$$

$$a + 7 = 0 \text{ and } b - 2 = 0$$

$$a = -7 \text{ and } b = 2$$

**or**

If one zero of the polynomial $3x^2 - 8x - (2k+1)$ is seven times the other, find both zeroes of the polynomial and the value of $k$.

**Ans :**

The given polynomial is $3x^2 - 8x - (2k+1)$.

Let one zero of this polynomial be $\alpha$, then the other zero is $7\alpha$.

$$\text{Sum of zeroes} = \frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\alpha + 7\alpha = -\frac{-8}{3}$$

$$8\alpha = \frac{8}{3}$$

$$\alpha = \frac{1}{3}$$

so $7\alpha = 7 \times \frac{1}{3} = \frac{7}{3}$

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\frac{1}{3} \times \frac{7}{3} = \frac{-(2k+1)}{3}$$

$$\frac{7}{3} = -(2k+1)$$

$$2k + 1 = -\frac{7}{3}$$

$$2k = -\frac{7}{3} - 1$$

$$2k = -\frac{10}{3}$$

$$k = -\frac{5}{3}$$

Hence, the zeroes of the given polynomial are $\frac{1}{3}, \frac{7}{3}$ and the value of $k$ is $-\frac{5}{3}$.

**29.** Solve for $x$ and $y$: [3]

$$(a-b)x + (a+b)y = a^2 - 2ab - b^2$$

$$(a+b)(x+y) = a^2 + b^2.$$

**Ans :**

The given equations are:

$(a-b)x + (a+b)y = a^2 - 2ab - b^2$ ...(i)

$(a+b)(x+y) = a^2 + b^2$ ...(ii)

Equation (ii) can be written as

$(a+b)x + (a+b)y = a^2 + b^2$ ...(iii)

Subtracting equation (i) from equation (iii), we get

$$2bx = 2ab + 2b^2$$

$$x = a + b$$

Substituting $x = a + b$ in equation (iii), we get

$$(a+b)(a+b) + (a+b)y = a^2 + b^2$$

$$a^2 + b^2 + 2ab + (a+b)y = a^2 + b^2$$

$$(a+b)y = -2ab$$

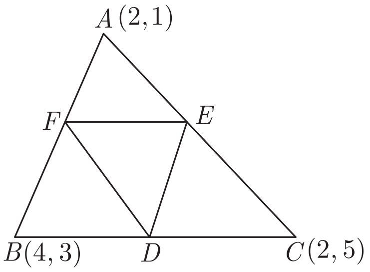
$$y = -\frac{2ab}{a+b}$$

Hence, thje solution is $x = a + b$ and $y = -\frac{2ab}{a+b}$.

**30.** Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are $A(2,1)$, $B(4,3)$ and $C(2,5)$. [3]

**Ans :**

Let $D$, $E$ and $F$ be the mid-points of the sides $BC$, $CA$ and $AB$ respectively, then




coordinates of $D$ are $\left(\frac{4+2}{2}, \frac{3+5}{2}\right)$ i.e. (3, 4)

coordinates of $E$ are $\left(\frac{2+2}{2}, \frac{5+1}{2}\right)$ i.e. (2, 3)

and coordinates of $F$ are $\left(\frac{2+4}{2}, \frac{1+3}{2}\right)$ i.e. (3, 2)

Now, ar($\Delta\,DEF$)

$$= \frac{1}{2}|3(3-2)+2(2-4)+3(4-3)|$$

$$= \frac{1}{2}|3-4+3|$$

$$= \frac{1}{2}\times 2$$

$$= 1 \text{ sq. unit}$$

Hence, the required area is 1 sq. unit.

**31.** There are 100 cards in a box on which numbers from 1 to 100 are written. A card is taken out from the box at random. Find the probability that the number on the selected card is [3]

(i) divisible by 3 and is a perfect square

(ii) a prime number greater than 80.

**Ans :**

Total number of cards in the bag = 100

By saying that a card is selected at random means all cards are equally likely to be selected.
So, the sample space of the experiment has 100 equally likely outcomes.

(i) Numbers divisible by 3 and perfect square are 9, 36, 81.
The number of outcomes favourable to the event 'divisible by 3 and is a perfect square' is 3.

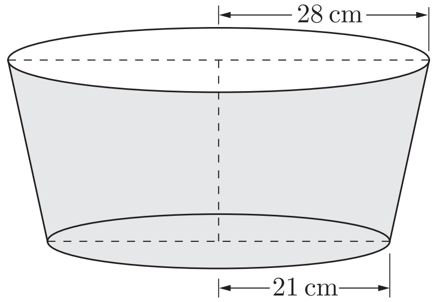
$$\text{Required probability} = \frac{3}{100}$$

(ii) Prime numbers greater than 80 but less than or equal to 100 are 83, 89, 97.
The number of outcomes favourable to the event 'prime number less than 80' is 3.

$$\text{Required probability} = \frac{3}{100}$$

**32.** A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket. [3]

**Ans :**




Let $h$ cm be height of the bucket.

Here, radius of top, $R = 28$ cm

radius of bottom, $r = 21$ cm

Volume of bucket = 28.490 litres

$= (28.490 \times 1000)\,\text{cm}^3$

$= 28490\,\text{cm}^3$

$$\frac{1}{3}\pi h(R^2+r^2+Rr) = 28490$$

$$\frac{1}{3}\times\frac{22}{7}\times h\times(28^2+21^2+28\times 21) = 28490$$

$$\frac{22}{21}\times h\times(784+441+588) = 28490$$

$$\frac{22}{21}\times 1813\times h = 28490$$

$$h = \frac{28490\times 21}{22\times 1813}$$

$$= 15$$

Hence, the height of the bucket = 15 cm.

**or**

A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. $\left(\text{Use } \pi = \frac{22}{7}\right)$

**Ans :**

Volume of water in cylinder

= Volume of cylinder

$= (\pi\times(60)^2\times 180)\,\text{cm}^3$

$= 648000\,\pi\,\text{cm}^3$

Volume of solid cone $= \left[\frac{1}{3}\times\pi\times(30)^2\times 60\right]\text{cm}^3$

$= 18000\,\pi\,\text{cm}^3$

Volume of water which overflows from the cylinder

= Volume of cone

$= 18000\,\pi\,\text{cm}^3$

Volume of water left in the cylinder

$= (648000\,\pi - 18000\,\pi)\,\text{cm}^3$

$= 630000\,\pi\,\text{cm}^3$

$= \frac{630000}{1000000}\times\frac{22}{7}\,\text{m}^3$

$= 1.98\,\text{m}^3$

Hence, the volume of water left in cylinder = $1.98\,\text{m}^3$

**33.** Prove the following: $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$. [3]

**Ans :**

$$\begin{aligned}\text{LHS} &= \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta \\ &= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta \\ &= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) \\ &\qquad + 3\sin^2\theta\cos^2\theta \\ &\qquad (a^3 + b^3 = (a+b)^3 - 3ab(a+b)) \\ &= (1)^3 - 3\sin^2\theta\cos^2\theta(1) + 3\sin^2\theta\cos^2\theta \\ &= 1 - 3\sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta \\ &= 1 = \text{RHS}\end{aligned}$$

**or**

Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

**Ans :**

$$\begin{aligned}\text{LHS} &= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \\ &= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \\ &= \frac{(\sqrt{1+\sin\theta})^2 + (\sqrt{1-\sin\theta})^2}{\sqrt{1-\sin\theta}\cdot\sqrt{1+\sin\theta}} \\ &= \frac{1+\sin\theta+1-\sin\theta}{\sqrt{1-\sin^2\theta}} \\ &= \frac{2}{\sqrt{\cos^2\theta}} \\ &= \frac{2}{\cos\theta} = 2\sec\theta = \text{RHS}\end{aligned}$$

**34.** If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}$ . [3]

**Ans :**

Given,
$$\begin{aligned}\cos\theta + \sin\theta &= \sqrt{2}\cos\theta \\ \sin\theta &= (\sqrt{2}-1)\cos\theta \\ \cos\theta &= \frac{1}{\sqrt{2}-1}\sin\theta \\ &= \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}\sin\theta \\ &= \frac{(\sqrt{2}+1)\sin\theta}{2-1} \\ \cos\theta &= \sqrt{2}\sin\theta + \sin\theta \\ \cos\theta - \sin\theta &= \sqrt{2}\sin\theta\end{aligned}$$

# Section D

**35.** If the roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$ are also real and equal. [4]

**Ans :**

Given that the roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal its discriminant $= 0$

$$(2p)^2 - 4 \times 1 \times mn = 0$$

$$p^2 = mn \qquad ...(i)$$

Now, the roots of the quadratic equation $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$ are real and equal if its discriminant $= 0$

i.e. if $(-2(m+n))^2 - 4 \times 1 \times (m^2 + n^2 + 2p^2) = 0$

i.e. if $4(m^2 + n^2 + 2mn) - 4(m^2 + n^2 + 2p^2) = 0$

i.e. if $8mn - 8p^2 = 0$ i.e. if $p^2 = mn$, which is true from equation (i).

**or**

The sums of first $n$ terms of three arithmetic progressions are $S_1, S_2$ and $S_3$ respectively. The first term of each AP is 1 and their common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

**Ans :**

$S_1$ = sum of first $n$ terms of an AP with first term 1 and common difference 1

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 1] = \frac{n(n+1)}{2}$$

$S_2$ = sum of first $n$ terms of an AP with first term 1 and common difference 2

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 2] = \frac{n}{2} \times 2n = n^2$$

$S_3$ = sum of first $n$ terms of an AP with first term 1 and common difference 3

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 3] = \frac{n(3n-1)}{2}$$

$$\begin{aligned}S_1 + S_3 &= \frac{n(n+1)}{2} + \frac{n(3n-1)}{2} \\ &= \frac{n}{2}[n + 1 + 3n - 1] \\ &= \frac{n}{2} \times 4n = 2n^2 = 2S_2\end{aligned}$$

**36.** Dudhnath has two vessels containing 720 mL of milk. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled. [4]

**Ans :**

Two vessels contain 720 mL and 405 mL of milk respectively. Since we need the minimum number of glasses of equal capacity, so the capacity of each glass should be maximum. Therefore, we have to find the HCF of 720 mL and 405 mL.

$$\begin{aligned}720 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \\ &= 2^4 \times 3^2 \times 5^1\end{aligned}$$

and $405 = 3 \times 3 \times 3 \times 3 \times 5 = 3^4 \times 5^1$

$$\text{HCF } (720, 405) = 3^2 \times 5^1 = 9 \times 5 = 45$$

Maximum capacity of each glass $= 45$ mL

Number of glasses filled from Ist vessel

$$= \frac{720}{45} = 16 \text{ and}$$

Number of glasses filled from 2nd vessel

$$= \frac{405}{45} = 9$$

Total number of glasses $= 16 + 9 = 25$

**or**

Show that the cube of any positive integer is of the form $4m, 4m+1$ or $4m+3$ for some integer $m$.

**Ans :**

Let $n$ be any positive integer. Applying Euclid's division lemma with divisor 4, we get

$$n = 4q,\ 4q+1,\ 4q+2 \text{ or } 4q+3$$

where $q$ is some whole number.

Four cases arise :

**Case I.** $n = 4q$, then

$$n^3 = (4q)^3 = 64q^3 = 4m$$

where $m = 16q^3$

which is an integer.

**Case II.** If $n = 4q+1$, then

$$\begin{aligned} n^3 &= (4q+1)^3 \\ &= 64q^3 + 3(4q)^2 \times 1 + 3(4q) \times 1^2 + 1^3 \\ &= 64q^3 + 48q^2 + 12q + 1 \\ &= 4(16q^3 + 12q^2 + 3q) + 1 \\ &= 4m + 1 \end{aligned}$$

where $m = 16q^3 + 12q^2 + 3q$

which is an integer.

**Case III.** If $n = 4q+2$, then

$$\begin{aligned} n^3 &= (4q+2)^3 \\ &= 64q^3 + 3\cdot(4q)^2 \times 2 + 3(4q) \times 2^2 + 2^3 \\ &= 64q^3 + 96q^2 + 48q + 8 \\ &= 4(16q^3 + 24q^2 + 12q + 2) = 4m \end{aligned}$$

where, $m = 16q^3 + 24q^2 + 12q + 2$

which is an integer.

**Case IV.** If $n = 4q+3$, then

$$\begin{aligned} n^3 &= (4q+3)^3 \\ &= 64q^3 + 3\cdot(4q)^2 \times 3 + 3(4q) \times 3^2 + 3^3 \\ &= 64q^3 + 144q^2 + 108q + 27 \\ &= 4(16q^3 + 36q^2 + 27q + 6) + 3 \\ &= 4m + 3 \end{aligned}$$

where, $m = 16q^3 + 36q^2 + 27q + 6$

which is an integer.

Hence, the cube of any positive integer is of the form $4m$, $4m+1$ or $4m+3$ for some integer $m$.

**37.** The following table gives production yield per hectare of wheat of 100 farms of a village. [4]

| Production in yield (in kg/ha) | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 | 75-80 |
|---|---|---|---|---|---|---|
| No. of farms | 2 | 8 | 12 | 24 | 38 | 16 |

Change the distribution to more than type, and draw its ogive. Using the ogive, find the median of the given data.

**Ans :**

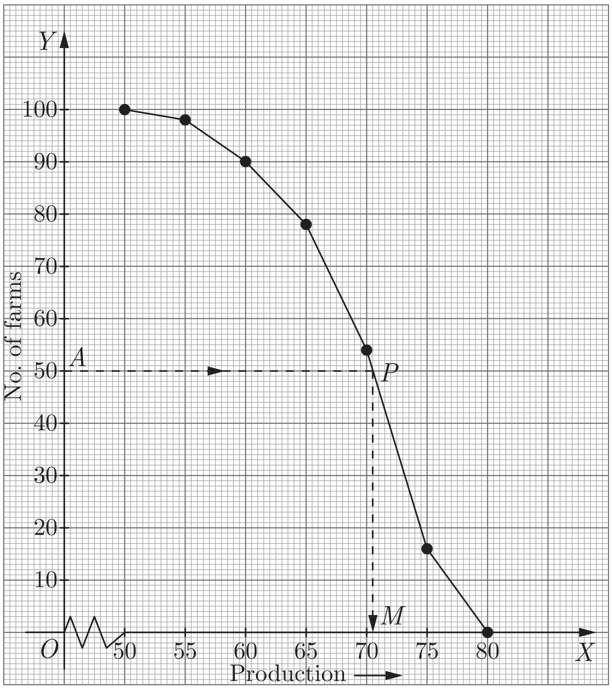
Construct the table for cumulative frequency distribution of more than type as under:

| Classes Productive yield | Frequency No. of farms | More than | Cumulative frequency |
|---|---|---|---|
| 50-55 | 2 | More than of equal to 50 | 100 |
| 50-60 | 8 | More than of equal to 55 | 98 |
| 60-65 | 12 | More than of equal to 60 | 90 |
| 65-70 | 24 | More than of equal to 65 | 78 |
| 70-75 | 38 | More than of equal to 70 | 54 |
| 75-80 | 16 | More than of equal to 75 | 16 |
| | | More than of equal to 80 | 0 |

Take 1 cm along $x$-axis $= 5$ kg of production/ha and 1 cm along $y$-axis $= 10$ farms.

Plot the points (50, 100), (55, 98), (60, 90), (65, 78), (70, 54), (75, 16) and (80, 0). Join these points by a free hand drawing. The required ogive (more than type) is drawn on the graph sheet given alongside.

Since, the scale on $x$-axis starts at 50, a kink (break) is shown on the $x$-axis near the origin.




Let $A$ be the point on $y$-axis representing frequency

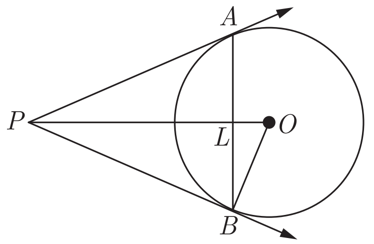
$$= \frac{n}{2} = \frac{100}{2} = 50$$

Through $A$, draw a horizontal line to meet the ogive at $P$. Through $P$, draw a vertical line to meet the $x$

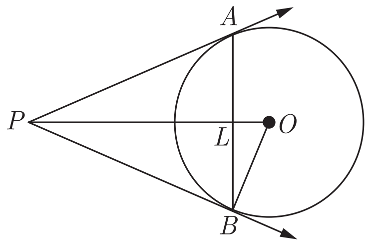
-axis at $M$. The abscissa of the point $M$ represents 70.5 kg.

Hence, the median $= 70.5$ kg of wheat/ha.

**38.** In the given figure, $AB$ is a chord of length 16 cm of a circle with centre $O$ and of radius 10 cm. The tangents at $A$ and $B$ intersect at the point $P$. Find the length of $PA$. [4]




**Ans :**




Given, $AB = 16$ cm

so $AL = BL = 8$ cm

($OP$ is the perpendicular bisector of $AB$)

In $\Delta\, OLB$, $\angle OLB = 90^\circ$ ($OP \perp AB$)

By Pythagoras theorem,

$$OL^2 = OB^2 - BL^2$$

$$= 10^2 - 8^2 = 100 - 64 = 36$$

$$OL = 6 \text{ cm}$$

Let, $LP = x$ cm

and $BP = y$ cm

then $OP = OL + LP = (6 + x)$ cm

Since, tangent is perpendicular to radius,

$$OB \perp PB$$

In $\Delta\, OPB$, $\angle OBP = 90^\circ$

By Pythagoras theorem,

$$OP^2 = BP^2 + OB^2$$

$$(x + 6)^2 = y^2 + 10^2$$

$$x^2 + 12x + 36 = y^2 + 100$$

$$x^2 - y^2 + 12x = 64 \quad ...(i)$$

In $\Delta\, BLP$, $\angle BLP = 90^\circ$ ($OP \perp AB$)

By Pythagoras theorem,

$$BP^2 = LP^2 + LB^2$$

$$y^2 = x^2 + 8^2$$

$$y^2 = x^2 + 64 \quad ...(ii)$$

Substituting the value of $y^2$ from equation (ii) in equation (i), we get

$$x^2 - (x^2 + 64) + 12x = 64$$

$$12x = 128$$

$$x = \frac{32}{3} \quad ...(iii)$$

Substituting the value of $x$ from (iii) in (ii), we get

$$y^2 = \left(\frac{32}{3}\right)^2 + 64$$

$$= \frac{1600}{9}$$

$$y = \frac{40}{3}$$

$$BP = \frac{40}{3} \text{ cm}$$

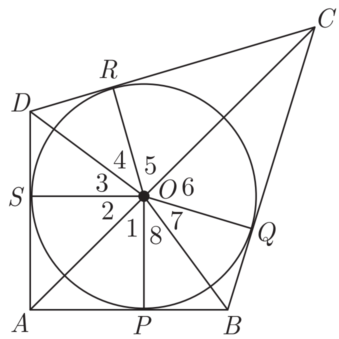
Length of $AP = \frac{40}{3}$ cm

($AP = BP$, lengths of tangents)

**or**

Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Ans :**




Given a quadrilateral $ABCD$ which circumscribes a circle with centre $O$.

We need to prove that

$$\angle AOB + \angle COD = 180^\circ$$

and $\angle BOC + \angle AOD = 180^\circ$

Let the sides $AB$, $BC$, $CD$ and $DA$ of the quadrilateral $ABCD$ touch the circle at the points $P, Q, R$ and $S$ respectively. Mark the angles at $O$ as shown in the adjoining figure.

In $\Delta\, OAP$ and $\Delta\, OAS$,

$OA = OA$

$OP = OS$ (radii of same circle)

and $AP = AS$ (lengths of tangents)

$\Delta\, OAP \cong \Delta\, OAS$

(by SSS rule of congruency)

$\angle 1 = \angle 2$ ...(i)

Similarly, $\angle 3 = \angle 4$ ...(ii)

$\angle 5 = \angle 6$ ...(iii)

and $\angle 7 = \angle 8$ ...(iv)

But, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8$

$= 360^\circ$

(sum of angles at a point)

$2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$

[using equation (i), (ii), (iii)and (iv)]

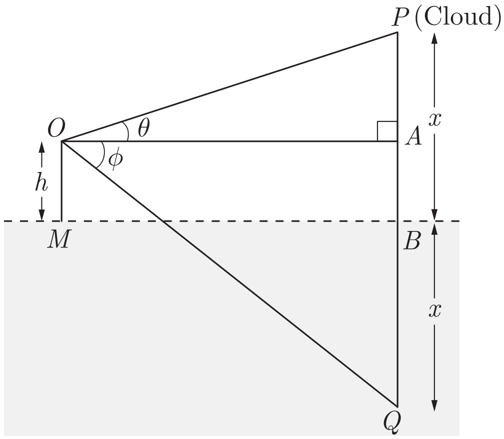
$(\angle 1 + \angle 8) + (\angle 4 + 5) = 180^\circ$

Similarly, $\angle BOC + \angle AOD = 180^\circ$

**39.** The angle of elevation of a cloud from a point $h$ metres above the surface of a lake is $\theta$ and the angle of depression of its reflection in the lake is $\phi$. Prove that the height of the cloud above the lake is $h\left(\frac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta}\right)$. [4]

**Ans :**

Let $O$ be the point of observation $h$ metres above the lake and let the cloud be at the point $P$. If $Q$ is the reflection of the cloud in the lake then $BQ = BP$.
Let the height of the cloud above the lake be $x$ metres. From $O$, draw $OA \perp PQ$.




Then, $AB = OM = h$ metres

$AP = BP - BA$

$= (x-h)$ metres and

$AQ = AB + BQ$

$= (x+h)$ metres

Let $OA = d$ metres

From right angled $\Delta\, OAP$, we get

$$\tan\theta = \frac{AP}{OA}$$

$$\tan\theta = \frac{x-h}{d} \quad \text{...(i)}$$

From right angled $\Delta\, OAQ$, we get

$$\tan\phi = \frac{AQ}{OA}$$

$$\tan\phi = \frac{x+h}{d} \quad \text{...(ii)}$$

Dividing equation (ii) by (i), we get

$$\frac{x+h}{x-h} = \frac{\tan\phi}{\tan\theta}$$

(Apply componendo and dividendo)

$$\frac{(x+h)+(x-h)}{(x+h)-(x-h)} = \frac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta}$$

$$\frac{2x}{2h} = \frac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta}$$

$$x = h\left(\frac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta}\right), \text{ as required}$$

**40.** Draw a circle of radius 3.5 cm. From a point $P$ outside the circle at a distance of 6 cm from the centre of the circle, draw two tangents to the circle. Also measure their lengths. [4]

**Ans :**

**Steps of construction :**

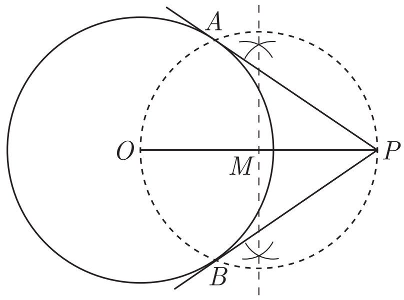
(i) Mark a point $O$. With $O$ as centre and radius 3.5 cm, draw a circle.

(ii) Take a point $P$ at a distance of 6 cm from $O$. $P$ lies outside the circle.

(iii) Join $OP$ and draw its perpendicular bisector to meet $OP$ at $M$.

(iv) Taking $M$ as centre and $OM$ (or $MP$) as radius, draw a circle. Let this circle intersect the previous circle at points $A$ and $B$.

(v) Join $PA$ and $PB$. Then $PA$ and $PB$ are the required tangents. On measuring, we find that $AP = BP = 4.9$ cm (approximately).




**Justification :**

Join $OA$, then $\angle OAP = 90^\circ$

(angle in a semicircle $= 90^\circ$)

As $OA$ is radius and $\angle OAP = 90^\circ$, so $PA$ has to be tangent to the circle.