CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-14

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{4}$ and $BC = 15\,\text{cm}$, then the length of PR is equal to [1]

(a) 10 cm (b) 12 cm

(c) $\frac{20}{3}$ cm (d) 8 cm

Ans : (a) 10 cm

For similar triangles,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left[\frac{BC}{RP}\right]^2 \quad (\Delta ABC \sim \Delta QRP)$$

$$\frac{9}{4} = \left[\frac{15}{RP}\right]^2$$

$$\frac{15}{PR} = \frac{3}{2}$$

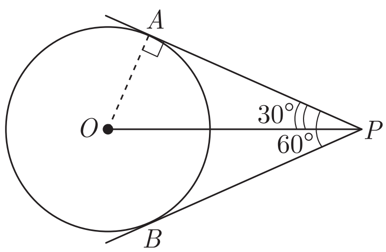
$$PR = 10\text{cm}$$

2. Two tangents inclined at an angle of $60°$ are drawn to a circle of radius $3\,\text{cm}$, then the length of each tangent is [1]

(a) $\frac{3\sqrt{3}}{2}$ cm (b) 6 cm

(c) 3 cm (d) $3\sqrt{3}$ cm

Ans : (d) $3\sqrt{3}$ cm



$$\angle APB = 60°$$

$$\angle APO = 30°$$

In ΔOAP $\tan 30° = \frac{OA}{AP}$

$$\frac{1}{\sqrt{3}} = \frac{3}{AP} \quad (OA = r = 3\text{cm, given})$$

$$AP = 3\sqrt{3}\,\text{cm}$$

So, length of each tangent is $3\sqrt{3}\,\text{cm}$

3. Given that one zero of the cubic polynomial $x^3 - 7x + 6$ is 2. The [1]

(a) 2 (b) -2

(c) 3 (d) -3

Ans : (d) -3

Let the zeroes of polynomial $x^3 - 7x + 6$ be α, β, γ.

One zero is $2 = \alpha\,(say)$,

$$\text{product of zeores} = -\frac{constant\ term}{coeff.\ of\ x^3}$$

$$2.\beta.\gamma = -\frac{6}{1}$$

$$\beta\gamma = -3$$

4. The value of λ for which the pair of equations $\lambda x - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is [1]

(a) 3 (b) -3

(c) -12 (d) no value

Ans : (d) no value

For infinitely many solutions,

$$\frac{\lambda}{6} = \frac{-1}{-2} = \frac{2}{3}$$

$$\frac{\lambda}{6} = \frac{1}{2}$$

$$\lambda = 3$$

and $\frac{\lambda}{6} = \frac{2}{3}$

$$\lambda = 4$$

So, no value of λ

5. If α and β are roots of the equation $2x^2 - 5x + 3 = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is [1]

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$

(c) $-\frac{5}{3}$ (d) $-\frac{3}{5}$

Ans : (a) $\frac{5}{3}$

$$2x^2 - 5x + 3 = 0$$

$$\alpha + \beta = -\frac{(-5)}{2} = \frac{5}{2}$$

and $\alpha\beta = \frac{3}{2}$

So, $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$$= \frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3}$$

6. The quadratic equation $2x^2-\sqrt{5}\,x+1=0$ has [1]
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

Ans : (c) no real roots

$$2x^2-\sqrt{5}\,x+1 = 0$$
$$D = b^2-4ac$$
$$= (-\sqrt{5})^2-4\times 2\times 1$$
$$= 5-8 = -3 < 0$$

no real roots

7. In an AP, if $a=1, a_n=20$ and $S_n=399$, then n is [1]

(a) 19 (b) 21
(c) 38 (d) 42

Ans : (c) 38

$$S_n = \frac{n}{2}(a_1+a_n)$$
$$399 = \frac{n}{2}(1+20)$$
$$n = \frac{399\times 2}{21}$$
$$n = 38$$

8. The volume of two spheres are in the ratio 64:27 [1]
(a) 3:4 (b) 4:3
(c) 9:16 (d) 16:9

Ans : (d) 16:9

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$
$$\left[\frac{r_1}{r_2}\right]^3 = \left[\frac{4}{3}\right]^3$$
$$\frac{r_1}{r_2} = \frac{4}{3}$$

Now, ratio of surface areas

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \left[\frac{r_1}{r_2}\right]^2$$
$$= \left[\frac{4}{3}\right]^2 = \frac{16}{9}$$

9. The number of balls of radius 1cm that can be made form a sphere of radius 10 cm is [1]
(a) 100 (b) 1000
(c) 10000 (d) 100000

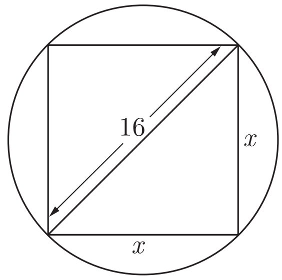
Ans : (b) 1000

$$\text{Number of balls} = \frac{\frac{4}{3}\pi.10^3}{\frac{4}{3}\pi.1^3}$$
$$= 1000.$$

10. The area of the square that can be inscribed in a circle of radius 8 cm is [1]

(a) 256 cm^2 (b) 128 cm^2
(c) $64\sqrt{2}\text{ cm}$ (d) 64 cm^2

Ans : (b) 128 cm^2



Let the side of square be x.

The $16^2 = x^2+x^2$
$$2x^2 = 256$$
$$x^2 = 128$$
$$\text{Area of square} = 128\text{ cm}^2$$

(Q11-Q15) : Fill in the blanks

11. The total surface area of a solid cylinder of radius 7 cm and height 18 cm is .... [1]

Ans :

$$\text{Total surface area} = 2\pi r(h+r)$$
$$= 2\times\frac{22}{7}\times 7(18+7)$$
$$= 44\times 25$$
$$= 1100\text{ cm}^2$$

12. If the corresponding sides of two similar triangles are in the ratio 5:7,then the ratio of their perimeters is .... [1]

Ans :

For similar triangles,
ratio of their perimeters

= ratio of corresponding sides = 5:7

or

If the sides of a rectangle are 12 cm and 5 cm, then the length of its diagonal is ....

Ans :

$$\text{Length of diagonal} = \sqrt{12^2+5^2} = \sqrt{144+25}$$
$$= \sqrt{169} = 13\text{ cm}$$

13. $x=a$ represents a line, which is parallel to .... [1]

Ans : y-axis

14. If the polynomial $x^4-2x^3-8x^2+mx-5$ is exactly divisible by $(x+3)$, the m, is equal to .... [1]

Ans :

If $x^4-2x^3-8x^2+mx-5$ is exactly divisible by $(x+3)$ then $(-3)^4-2(-3)^3-8(-3)^2+m(-3)-5=0$

$$81+54-72-3m-5 = 0$$
$$58-3m = 0$$
$$m = \frac{58}{3}$$

15. The decimal expansion of the rational number $\frac{11}{2^3 \times 5^2}$ well terminate after....... places of decimal. [1]

Ans :

$$\frac{11}{2^3 \times 5^2} = \frac{11 \times 5}{2^3 \times 5^3} = \frac{55}{1000} = 0.055$$

Hence, the decimal expansion of given rational number will terminate after 3 places of decimal.

(Q16-Q20):Answer the following questions:

16. Is $x = -2$ a solution of the equation $x^2 - 2x + 8 = 0$? [1]

Ans :

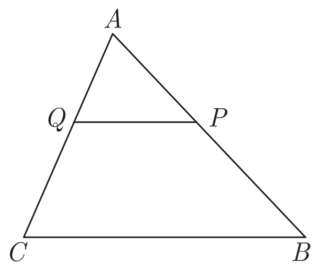
$$(-2)^2 - 2 \times (-2) + 8 = 4 + 4 + 8 = 16 \neq 0$$

$$x = -2$$

is not a solution of the equation

$$x^2 - 2x + 8 = 0.$$

17. In the given figure, P and Q are points on the sides AB and AC respectively of ΔABC such that $AP = 3.5\text{ cm}, PB = 7\text{ cm}, AQ = 3\text{ cm}$ and $QC = 6\text{ cm}$. Prove that $PQ \parallel BC$. [1]



Ans :

$$\frac{AP}{PB} = \frac{3.5}{7} = \frac{1}{2}$$

and $$\frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

By the converse of B.P.T. , $PQ \parallel BC$.

18. If $\tan \theta = \cot (30° + \theta)$, find the value of θ? [1]

Ans :

$$\begin{aligned} \tan \theta &= \cot (30° + \theta) \\ &= \tan [90° - (30° + \theta)] \\ &\tan (60° - \theta) \\ \theta &= 60° - \theta \\ 2\theta &= 60° \\ \theta &= 30° \end{aligned}$$

or

Find the value of $\sin^2 45° + \cos^2 30° - \tan^2 60°$.

Ans :

$$\begin{aligned} \sin^2 45° + \cos^2 30° - \tan^2 60° &= \left[\frac{1}{\sqrt{2}}\right]^2 \left[\frac{\sqrt{3}}{2}\right]^2 - (\sqrt{3})^2 \\ &= \frac{1}{2} + \frac{3}{4} - 3 \\ &= \frac{2 + 3 - 12}{4} = -\frac{7}{4} \end{aligned}$$

19. A cylinder and a cone are of same base radius and of same height. Find the ratio of volume of cylinder to that of the cone. [1]

Ans :

Let r be the base radius of cylinder and cone and h be their heights

$$\frac{\text{Volumeof cylinder}}{\text{Volume of cone}} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{1}$$

Hence, the ratio of volume of cylinder to that of cone

$$= 3:1$$

20. Two coins are tossed simultaneously. Find the probability of getting exactly one head. [1]

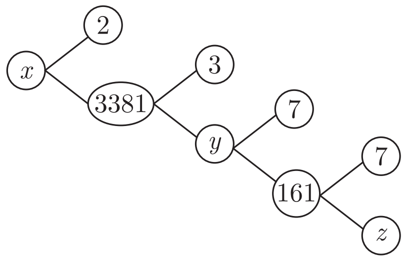
Ans :

when two coins are tossed simultaneously, then the outcomes are HH, HT, TH, TT.
So total number of out comes $= 4$
The outcomes favourable to the event' getting exactly one head' are HT and TH
The number of outcomes favourable to the event$= 2$
P(exactly one head)$= \frac{2}{4} = \frac{1}{2}$.

Section B

Question 21 to 26 carry 2 marks each.

21. Complete the following factor tree and find the numbers x, y and z. [2]



Ans :

$$\begin{aligned} x &= 3381 \times 2 \\ &= 6762 \\ y &= 161 \times 7 = 1127 \end{aligned}$$

and $$z = \frac{161}{7} = 23.$$

or

Show that $2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is irrational.

Ans :

let us assume that $2\sqrt{3}$ is rational say r

The $$2\sqrt{3} = r$$

$$\sqrt{3} = \frac{r}{2}$$

As r is rational, $\frac{r}{2}$ is rational $\sqrt{3}$ is rational.

But this contradicts that $\sqrt{3}$ is irrational.
Hence, our assumption is wrong.
Therefore $2\sqrt{3}$ is an irrational number.

22. If α and β are zeroes of the polynomial $f(x) = x^2 - x - \text{k}$ such that $\alpha - \beta = 9$, find k. [2]

Ans :

Given $f(x) = x^2 - x - k$

$$\alpha + \beta = -\frac{-(-1)}{1} = 1$$

and $\alpha\beta = \frac{-k}{1}$

Now
$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$
$$(9)^2 = (1)^2 - 4 \times (-k)$$
$$81 - 1 = 4k$$
$$k = \frac{80}{4}$$
$$k = 20.$$

23. The coordinates of the vertices of ΔABC are $A(4,1)$, $B(-3,2)$ and $C(0,\text{k})$. Given that the area of ΔABC is $12\,\text{unit}^2$, find the the value of k. [2]

Ans :

Area of ΔABC
$$= \frac{1}{2}[4(2-k) - 3(k-1) + 0(1-2)]$$
$$12 = \frac{1}{2}[8 - 4k - 3k + 3]$$
$$24 = [11 - 7k]$$
$$11 - 7k = \pm 24$$
$$11 - 7k = 24$$
or $11 - 7k = -24$
$$7k = -13$$
or $7k = 35$
$$k = -\frac{13}{7}$$
or $k = \frac{35}{7}$

Hence, $k = -\frac{13}{7}$

or $k = 5$

24. Without using trigonometric tables, evaluate the following $\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}[\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ]$ [2]

Ans :

$$\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}[\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ]$$
$$= \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ} + \sqrt{3}\Big[\tan 10^\circ \frac{1}{\sqrt{3}} \tan 40^\circ \tan(90^\circ - 40)\tan(90^\circ - 10^\circ)\Big]$$
$$= \frac{\cos 72^\circ}{\cos 72^\circ} + \sqrt{3}\Big[\tan 10^\circ \tan \frac{1}{\sqrt{3}} \tan 40^\circ \cot 40^\circ \cot 10^\circ\Big]$$
$$= 1 + \sqrt{3}\Big[\tan 10^\circ . \frac{1}{\sqrt{3}} . \tan 40^\circ . \frac{1}{\tan 40^\circ} . \frac{1}{\tan 10^\circ}\Big]$$
$$= 1 + \sqrt{3}\Big[\frac{1}{\sqrt{3}}\Big] = 1 + 1 = 2.$$

25. Prove that: $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$ [2]

Ans :

$$\text{LHS} = \frac{\cot A - \cos A}{\cot A + \cot A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$
$$= \frac{\cos A\Big[\frac{1}{\sin A} - 1\Big]}{\cos A\Big[\frac{1}{\sin A} + 1\Big]}$$
$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = RHS$$

or

Prove that :
$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}.$$

Ans :

$$\text{LHS} = \frac{1}{\operatorname{cosec} - \cot A} - \frac{1}{\sin A}$$
$$= \frac{\operatorname{cosec} A + \cot A}{(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)} - \operatorname{cosec} A$$
$$= \frac{\operatorname{cosec} A + \operatorname{cosec} A}{\operatorname{cosec}^2 A - \operatorname{cosec}^2 A} - \operatorname{cosec} A$$
$$= \operatorname{cosec} A + \cos A - \operatorname{cosec} A = \cot A$$
$$\text{RHS} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A - \cot A} \times \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A - \cot A}$$
$$= \operatorname{cosec} A - \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec}^2 A - \cot^2 A}$$
$$= \operatorname{cosec} A - \operatorname{cosec} A + \cot A$$
$$= \cot A$$
$$\text{LHS} = \text{RHS}$$

26. Area of a sector of a circle of radius 36 cm is $54\pi\,\text{cm}^2$. Find the length of the corresponding arc of the sector. [2]

Ans :

Let the central angle (in degrees) be θ, then
$$\text{area of sector} = \frac{\theta}{360} \times \pi \times (36)^2$$
$$= 54 \text{ (given)}$$
$$\theta = \frac{54 \times 360}{36 \times 36} = 15.$$

The length of the corresponding arc of the sector
$$= \frac{\theta}{360} \times 2\pi r$$
$$= \frac{15}{360} \times 2\pi \times 36$$
$$= 3\pi$$

Hence, the length of the corresponding arc of the sector$= 3\pi$cm

Section C

27. The HCF of 65 and 117 is expressible in the form $65\text{m} - 117$ using prime factorisation. [3]

Ans :

By Euclid's division algorithm, we have

$$117 = 65 \times 1 + 52;$$
$$65 = 52 \times 1 + 13;$$
$$52 = 13 \times 4 + 0$$
$$\text{HCF of } 65 \text{and } 177 = 13$$
$$65m - 117 = 13$$
$$65m = 130$$
$$m = 2.$$

Prime factorisation of 65 and 117 are as follows:

$$65 = 5 \times 13$$

and $117 = 3 \times 3 \times 13$

LCM of 65 and117

$$= 3 \times 3 \times 5 \times 13 = 585.$$

28. Solve the following pair of linear equation: [3]
$px + qy = p - q; qx - py = p + q.$

Ans :

Given equations are

$$px + qy = p - q \quad ...(i)$$
$$qx - py = p + q \quad ...(ii)$$

Multiplying (i)by p and (ii) by q, we get

$$p^2 x + pqy = p^2 - pq \quad ...(iii)$$

and $q^2 x - pqy = pq + q^2 \quad ...(iv)$

On adding (iii) and (iv),we have

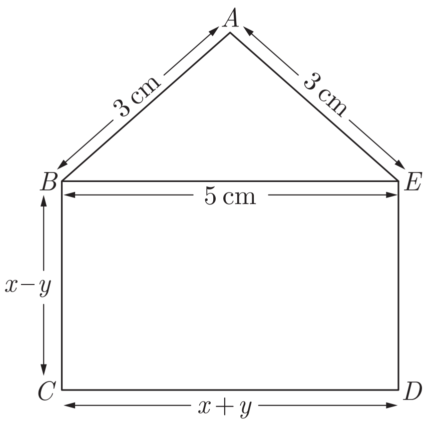
$$(p^2 + q^2) x = p^2 + q^2$$
$$x = \frac{p^2 + q^2}{p^2 + q^2} x = 1$$

Putting, $x = 1$ in equation (i), we have

$$p \times 1 + qy = p - q$$
$$qy = - q$$
$$y = - 1$$
$$x = 1 \ y = - 1$$

or

In the given figure, $ABCDE$ is a pentagon with $BE || CD$ and $BC || ED$. BC is perpendicular to CD. If the perimeter of pentagon $ABCDE$ is $21\,\text{cm}$, find the values of x and y.



Ans :

Given $BE || CD$ and $BC || ED$ also $BC \perp CD$

ECDE is a rectangle

$$CD = BE$$
$$x + y = 5 \quad ...(i)$$

Again, given perimeter of pentagon

$$ABCDE = 21\text{cm}$$
$$AB + BC + CD + DE + EA = 21\text{cm} \quad (BC = ED)$$
$$3 + x - y + x + y + BC + 3 = 21\text{cm}$$
$$2x + x - y = 21 - 6$$
$$3x - y = 15 \quad ...(ii)$$

Adding (i) and (ii), we get

$$4x = 20$$
$$x = \frac{20}{4} \ x = 5$$

Putting $x = 5$ in equation (i),we get

$$5 + y = 5 \ y = 0$$

Hence, $x = 5, \ y = 0$

29. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows. ₹ 200 for the first day,₹ 250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. If the contractor pays ₹27750 as penalty, find for how many days he delayed the work. [3]

Ans :

200,250,300.... is an AP

$$\text{whose first term, } a = 200$$

and $\text{common difference, } d = 50$

Let the number of days for which work is delayed be n, then

$$S_n = 27750 \text{ (given)}$$
$$S_n = \frac{n}{2}[2a(n-1)d]$$
$$27750 = \frac{n}{2}[2 \times 200 + (n-1) \times 50]$$
$$n[200 + 25(n-1)] = 27750$$
$$25n^2 + 175n = 27750$$
$$n^2 + 7n - 1110 = 0$$
$$n^2 + 37n - 30n - 1110 = 0$$
$$n(n + 37) - 30(n + 37) = 0$$
$$(n + 37)(n - 30) = 0$$
$$n = - 37$$

or $n = 30$

But n cannot be negative

$$n = 30$$

Hence, the work was delayed by 30 days.

30. Prove that: [3]
$\sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta) = \sec\theta + \text{cosec}\,\theta.$

Ans :

$$\text{LHS} = \sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta)$$
$$= \sin\theta\Big(1 + \frac{\sin\theta}{\cos\theta}\Big) + \cos\theta\Big(1 + \frac{\cos\theta}{\sin\theta}\Big)$$
$$= \frac{\sin\theta}{\cos\theta}(\cos\theta + \sin\theta) + \frac{\cos\theta}{\sin\theta}$$

$$(\sin\theta+\cos\theta)$$

$$= (\cos\theta+\sin\theta)\left[\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right]$$

$$= (\cos\theta+\sin\theta)\left[\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right]$$

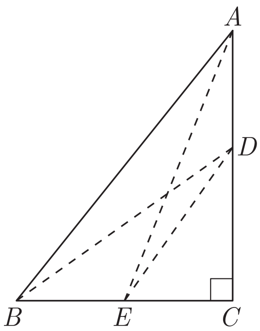
$$= (\cos\theta+\sin\theta)\cdot\frac{1}{\cos\theta\sin\theta}$$

$$= \frac{\cos\theta}{\cos\theta.\sin\theta}+\frac{\sin\theta}{\cos\theta.\sin\theta}$$

$$= \frac{1}{\sin\theta}+\frac{1}{\cos\theta} = \operatorname{cosec}\theta+\sec\theta = \text{RHS}$$

31. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C Prove that $AE^2+BD^2 = AB^2+DE^2$. [3]

Ans :



In ΔABC, $\angle C = 90^\circ$

$$AB^2 = AC^2+BC^2 \quad \text{....(i)}$$

In ΔECD, $\angle ECD = 90^\circ$

$$DE^2 = CD^2+EC^2 \quad \text{....(ii)}$$

In ΔAEC, $\angle AEC = 90^\circ$

$$AE^2 = AC^2+EC^2 \quad \text{...(iii)}$$

In ΔBCD, $\angle BCD = 90^\circ$

$$BD^2 = BC^2+CD^2 \quad \text{...(iv)}$$

Adding (iii) and (iv), we get

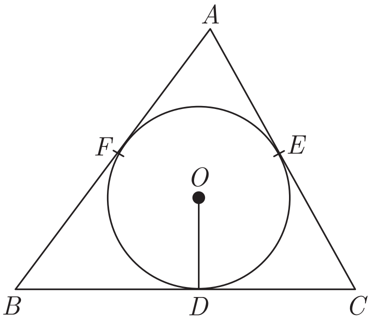
$$AE^2+BC^2 = (AC^2+BC^2)+(CD^2+EC^2)$$

$$= AB^2+DE^2$$

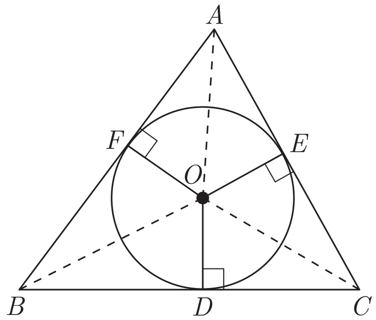
[using (i) and(ii)]

or

In the adjoining figure, a triangle ABC is drawn to circumscribe a circle of radius 2cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 4cm and 3cm respectively. If area of $\Delta ABC = 21\text{ cm}^2$, then find the lengths of sides AB and AC.



Ans :



Since tangents drawn from an external point to a circle are equal.

$$BF = BD = 4\text{ cm (given)}$$

and $$CE = CD = 3\text{ cm (given)}$$

Let $$AF = AE = x\text{ cm}$$

Now, area

$$(\Delta ABC) = \text{area}(\Delta AOB)+\text{area}(\Delta BOC)+\text{area}(\Delta AOC)$$

$$21 = \frac{1}{2}AB\times OF+\frac{1}{2}BC\times OD+\frac{1}{2}AC\times OE$$

$$21 = \frac{1}{2}(AF+BF)\times 2+\frac{1}{2}(BD+CD)\times 2+\frac{1}{2}(AE+CE)\times 2$$

$$(OD = OE = OF = 2\text{ cm})$$

$$21 = (x+4)+(4+3)+(x+3)$$

$$21 = 2x+14$$

$$2x = 7$$

$$x = \frac{7}{2} \Rightarrow x = 3.5$$

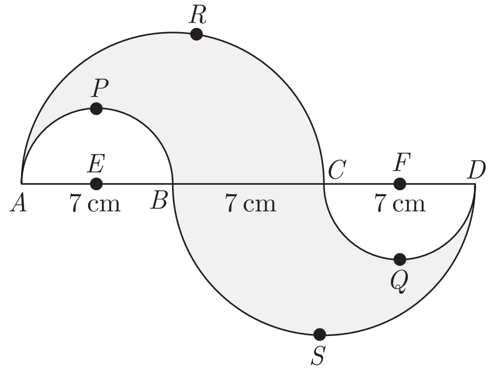
$$AB = x+4$$

$$= (3.5+4)\text{ cm} = 7.5\text{cm}$$

and $$AC = x+3$$

$$= (3.5+3)\text{ cm} = 6.5\text{ cm}$$

32. In the given figure, APB and CQD are semicircles of diameter 7cm each, while ARC and BSD are semicircles of diameter 14cm each. Find the perimeter of the shaded region. $\left[Use \pi = \frac{22}{7}\right]$ [3]



Ans :

Given BC = radius = 7 cm

and $\quad AE = CF = \frac{7}{2}\text{cm} = r(\text{say})$

The perimeter of the shaded region

= perimeter of semicircle $APB+$ perimeter of semicircle $BSD+$ perimeter of semicircle $DQC+$ perimeter of semicircle CRA

$$= \pi \times \frac{7}{2} + \pi \times 7 + \pi \times \frac{7}{2} + \pi \times 7$$

(perimeter of semicircle$= \pi r$)

$$= \pi\left[\frac{7}{2} + 7 + \frac{7}{2} + 7\right] = \frac{22}{7} \times (7 + 14)$$

$$= \left[\frac{22}{7} \times 21\right]\text{cm} = 66\,\text{cm}.$$

33. Two different dice are rolled together. Find the probability of getting:
1. the sum of numbers on two dice as 5.
2. even numbers on both dice. [3]

Ans :

Total possible outcomes$= 6 \times 6 = 36$
1. The possible outcomes favourable to the event'sum of numbers on two dice is 5' are (2,3),(3,2),(1,4) (4,1) *i.e.*4 in number.

Required probability$= \frac{4}{36} = \frac{1}{9}$

2. The possible outcomes favourable to the event' even numbers on both dice' are(2,2),(2,4),(2,6),(4,2) (4,4),(4,6),(6,2),(6,4),(6,6)*i.e.* 9 in number.

Required probability$= \frac{9}{36} = \frac{1}{4}$

or

The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar.

Ans :

We know that the sum of probabilities of all elementary events $= 1$

P(red ball) +P(blue ball)+P(orange ball) $= 1$

$$\frac{1}{4} + \frac{1}{3} + P(\text{orange ball}) = 1$$

$$\text{P(orange ball)} = 1 - \frac{1}{4} - \frac{1}{3}$$

$$= \frac{12 - 3 - 4}{12}$$

$$= \frac{5}{12} \quad ..(i)$$

Let the total number of balls in the jar be x
As the jar contains 10 orange balls,

$$\text{P(orange ball)} = \frac{10}{x} = \frac{5}{12} \quad \text{using(i))}$$

$$x = 24$$

Hence, the total number of balls in the jar=24

34. Find the value of p for the following distribution whose mean is 10: [3]

x_i	5	7	9	11	13	15	20
f_i	4	4	p	7	3	2	1

Ans :

x_i	f_i	$f_i x_i$
5	4	20
7	4	28
9	p	$9p$
11	7	77
13	3	39
15	2	30
20	1	20
Total	$\sum f_i = 21 + p$	$\sum f_i x_i = 214 + 9p$

$$\text{Mean} = a + \frac{\sum f_i x_i}{\sum f_i}$$

$$10 = \frac{214 + 9p}{21 + p}$$

$$210 + 10p = 214 + 9p$$

$$10p - 9p = 214 - 210$$

$$p = 4$$

Section D

35. If pth, qth and rth terms of an AP are a, b and c respectively, then show that $(a-b)r + (b-c)p + (c-a)q = 0$. [4]

Ans :

Let A be the first term and D be the common difference of the AP.

Given, $\quad a = p$th term

$$a = A + (p-1)D \quad ...(i)$$

$$b = q\text{th term}$$

$$b = A + (q-1)D \quad ...(ii)$$

$$c = r\text{th term}$$

$$c = A + (r-1)D \quad ...(iii)$$

Subtracting equation (ii) from (i), we get

$$a - b = (p-q)D \quad ...(iv)$$

Similarly,

$$b - c = (q-r)D \quad ...(v)$$

and $\quad c - a = (r-p)D \quad ..(vi)$

$$(a-b)r + (b-c)p + (c-a)q$$

$$= [(p-q)r + (q-r)p + (r-p)q]D$$

$$= (pr - qr + pq + qr - pq)D$$

$$= 0 \times D = 0.$$

36. If the centroid of ΔABC, in which $A(a,b), B(b,c), C(c,a)$ is at the origin, then calculate the value of $(a^3 + b^3 + c^3)$ [4]

Ans :

Given vertices of ΔABC as A(a,b),B(b,c),C(c,a) and

the centroid of ΔABC is G(0,0).
As we know that centroid of Δ whose vertices are $(x_1, y_1), (x_2, y_2), and (x_3, y_3)$

is $\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$

$$(0,0) = \left[\frac{a+b+c}{3}, \frac{b+c+a}{3}\right]$$

$$a + b + c = 0 \qquad ....(i)$$

We know that

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca) \quad \text{(using(i))}$$

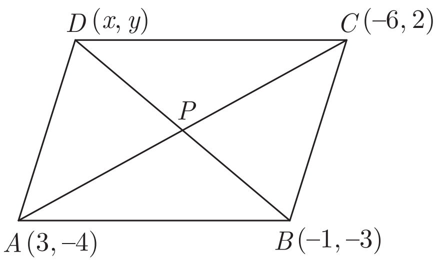
$$a^3 + b^3 + c^3 - 3abc = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

or

The three vertices of a parallelogram $ABCD$ are $A(3, -4), B(-1, -3)$ and $C(-6, 2)$. Find the coordinates of vertex D and find the area of parallelogram $ABCD$.

Ans :



Given, vertices of a parallelogram $ABCD$ are $A(3, -4)$,$B(-1, -3)$ and $(-6, 2)$.
Let the coordinates of D be (x, y).

Mid-point of diagonal $AC\left[\frac{3 + (-6)}{2}, \frac{-4+2}{2}\right]$ *i.e*

$$\left[\frac{-3}{2}, -1\right]$$

and mid-point of diagonal $BD\left[\frac{x + (-1)}{2}, \frac{y + (-3)}{2}\right]$

$$i.e\left[\frac{x-1}{2}, \frac{y-3}{2}\right]$$

Since the diagonals of a parallelogram bisect each other, the mid-points of AC and BD are the same

$$\frac{x-1}{2} = \frac{-3}{2}$$

and $$\frac{y-3}{2} = -1$$

$$x - 1 = -3$$

and $$y - 3 = -2$$

$$x = -2$$

and $$y = 1$$

Hence, coordinates of D are (-2,1)

Now, area $(\Delta ABC) = \frac{1}{2}|3(-3-2) - 1(2+4) - 6(-4+3)|$

$$= \frac{1}{2}|-15 - 6 + 6|$$

$$= \frac{15}{2} \text{ sq. units}$$

Since diagonal of a parallelogram divides it into two equal areas.

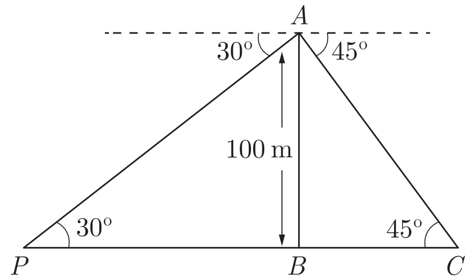
$$\text{area}(||^{\text{gm}} \text{ABCD}) = 2\, area(\Delta ABC)$$

$$= 2 \times \frac{15}{2} = 15 \text{ sq. units}$$

Hence, the area of $||^{\text{gm}}$ $ABCD = 15$ sq. units

37. From the top of a tower 100m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars. ($Use \sqrt{3} = 1.73$) [4]

Ans :



Let AB be a tower 100 m high. P and Q be the position of two cars.
Angles of depressions are shown in adjoining figure.

$$\angle APB = 90°$$

and $$\angle AQB = 45°$$

In ΔAPB, $$\angle ABP = 90°$$

$$\tan 30° = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{PB} \quad PB = 100\sqrt{3} \qquad ...(i)$$

In ΔAQB, $$\angle ABQ = 90°$$

$$\tan 45° = \frac{AB}{PQ}$$

$$1 = \frac{100}{BQ} \quad BQ = 100 \qquad ...(ii)$$

Now, $$PQ = PB + BQ$$

$$= (100\sqrt{3} + 100)\,\text{m}$$

$$= 100(\sqrt{3} + 1)\,\text{m}$$

$$= 100(1.73 + 1)\,\text{m}$$

$$= (100 \times 2.73)\,\text{m}$$

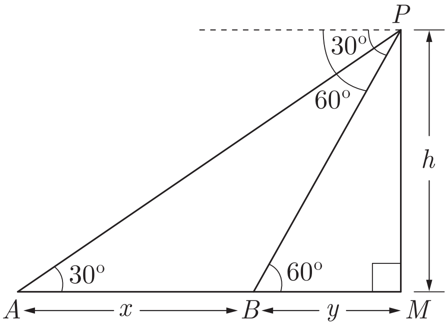
$$= 273\,\text{m}.$$

Hence, the distance between two cars is 273 m

or

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Ans :



Let MP be the tower of height h metres. Let A be the position of the car when its angle of depression (as seen from P) is $30°$ and 6 seconds later, let the car be at the point B when its angle of depressions is $60°$. Let AB be x metres and BM be $y$metres.

From ΔPAM,

$$\tan 30° = \frac{MP}{AM} \quad \frac{1}{\sqrt{3}} = \frac{h}{x+y} \qquad ...(i)$$

From ΔPBM,

$$\tan 60° = \frac{MP}{BM} \quad \sqrt{3} = \frac{h}{y} \qquad ...(ii)$$

On dividing (ii)by (i),we get

$$\sqrt{3} \times \frac{\sqrt{3}}{1} = \frac{h}{y} \times \frac{x+y}{h}$$

$$3 = \frac{x+y}{y}$$

$$3y = x+y \quad y = \frac{x}{2}.$$

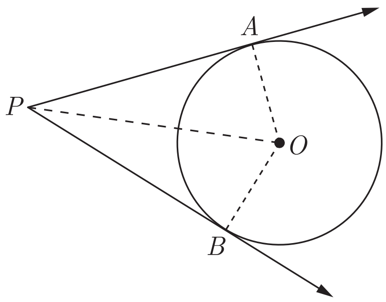
As the car takes 6 seconds in moving from A to B *i.e* to cover x metres,

time taken by the car in moving from B to M *i.e.*$\frac{x}{2}$ metres

$$= \frac{6}{2}\text{seconds} = 3\text{seconds}.$$

38. Prove that the lengths of tangents drawn from an external point to a circle are equal. [4]

Ans :



Given: P is an external point to a circle with centre O PA and PB are two tangents drawn from P to the circle, A and B being points of contact.

To prove: $PA = PB$

Construction: Join OA,$OB$and OP

Proof: Since the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\angle OAP = 90°$$

and $\angle OBP = 90°$

Now, in ΔOAPandΔOBP

$$OA = OB$$

$$OP = OP$$

$$\angle OAP = \angle OBP$$

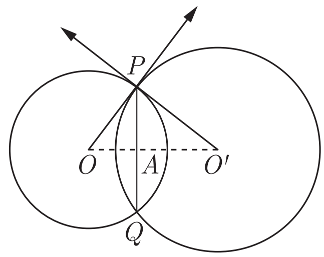
$$\Delta OAP \cong \Delta OBP$$

$$PA = PB$$

or

Two circles with centres O and O' of radii 3cm and 4cm, respectively intersect at two points P and$Q$ such that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ.

Ans :



Since tangent at a point to a circle is perpendicular to its radius,$OP \perp O'P$.

$$OO'^2 = OP^2 + O'P = 3^2 + 4^2 = 25$$

$$OO' = 5\,\text{cm}$$

As the line joining the centres of two intersecting circles is the right bisector of the common chord,

$OO' \perp PQ$

and $AP = AQ$

Let $OA = x\,\text{cm}$,

then $AO = OO' - OA = (5-x)\,\text{cm}$.

Let $AP = AQ = y\,\text{cm}$.

InΔOAP,

$$\angle OAP = 90°$$

By Pythagoras theorem,

$$OA^2 + AP^2 = OP^2$$

$$x^2 + y^2 = 3^2$$

$$x^2 + y^2 = 9 \qquad ..(i)$$

In$\Delta O'AP$, $\angle O'AP = 90°$

By Pythagoras theorem,

$$AO'^2 + AP^2 = O'P^2$$

$$(5-x)^2 + y^2 = 4^2$$

$$x^2 + y^2 - 10x = -9 \qquad ...(ii)$$

Subtracting equation (ii) from (i),we get

$$10x = 18 \Rightarrow x = \frac{9}{5} \qquad ...(iii)$$

Substituting the value of x from equation (iii) in (i),we get

$$\left[\frac{9}{5}\right]^2 + y^2 = 9$$

$$y^2 = 9 - \frac{81}{25} = \frac{144}{25}$$

$$y = \frac{12}{5}$$

$$AP = \frac{12}{5}\,\text{cm} = 2.4\,\text{cm}$$

$$PQ = 2AP = (2 \times 2.4)\text{ cm} = 4.8\text{ cm}$$

Hence, the length of common chord PQ is 4.8 cm

39. Draw an isosceles triangle ABC in which $AB = AC = 6$ cm and $BC = 5$cm. Construct a triangle PQR similar to ΔABC in which $PQ = 8$cm. Also justify the construction. [4]

Ans :

As $\Delta PQR \sim \Delta ABC$, so the side PQ of ΔPQR is the corresponding side AB of ΔABC.

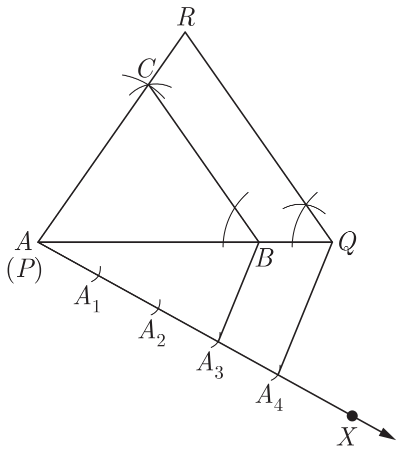
Given $AB = 6$ cm

and we need PQ to be 8 cm, so $\dfrac{PQ}{AB} = \dfrac{8}{6} = \dfrac{4}{3}$

Thus, we are required to construct ΔPQR similar to ΔABC such that sides of ΔPQR are $\dfrac{4}{3}$ times of the corresponding sides of ΔABC.

Steps of construction:

1. Draw $AB = 6$ cm
2. With A as centre and radius 6 cm draw an arc:
3. With B as centre and radius 5 cm, draw an arc to meet the previous arc at C.
4. Join AC and BC then ABC is an isosceles triangle with $AB = AC = 6$ cm and $BC = 5$ cm
5. Take points A and P same.
6. Draw any ray AX making an acute angle with AB on the side opposite to the vertex C.



7. Locate 4(the greater of 4 and 3) points A_1, A_2, A_3 and A_4 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_1A_2$
8. Join A_3B
9. Through A_4, draw a line parallel to A_3B (making an angle equal to $\angle AA_3B$) to intersect the extended line segment AB at Q
10. Through Q, draw a line parallel to BC (making an angle equal to $\angle ABC$) to intersect the extended line segment AC at R.
    Then AQR *i.e.* PQR is the required triangle.

Justification:

In ΔAA_4Q, $A_3B \,||\, A_4Q$

By Basic Proportionality theorem,

$$\frac{AB}{PQ} = \frac{AA_3}{AA_4}$$

but $\dfrac{AA_3}{AA_4} = \dfrac{3}{4}$,

so $\dfrac{AB}{PQ} = \dfrac{3}{4}$

Sin, $BC \,||\, QR$, $\Delta ABC \sim \Delta PQR$

$$\frac{AC}{PR} = \frac{BC}{QR} = \frac{AB}{PQ} = \frac{3}{4}$$

$$\frac{PR}{AC} = \frac{QR}{BC} = \frac{PQ}{AB} = \frac{4}{3}$$

But $AB = 6$ cm,

so $PQ = (\frac{4}{3} \times 6)$ cm $= 8$ cm

Hence, $\Delta PQR \sim \Delta ABC$ and side PQ of $\Delta PQR = 8$cm.

40. Size of agriculture holding in a survey of 200 families is given in the following table. [4]

Size of agriculture holding (in ha)	0-5	5-10	10-15	15-20	20-25	20-30	30-35
Number of families	10	15	30	80	40	20	5

Compute the median and mode size of holding.

Ans :

Agriculture holding (in ha)	No of families (frequently)	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	30	55
15-20	80	135
20-25	40	175
25-30	20	195
30-35	5	200

For median:

The cumulative frequency of the class 15-20 is 135, which is greater then $\frac{n}{2}$ *i.e.* $\frac{200}{2}$ *i.e.* 100 and nearest to it, so the median class is 15-20

Here, $l = 15$,

$c.f. = 55$,

$f = 80$

and $h = 5$

$$\text{Median} = 1 + \frac{\frac{n}{2} - c.f.}{f} \times h$$

$$= 15 + \frac{100 - 55}{80} \times 5$$

$$= 15 + \frac{45}{16} = 17.81\text{(approx.)}$$

For mode:

The class 15-20 has maximum frequency (80), so the modal class is 15-20

Here $l = 15$,

$f_1 = 80$,

$$f_0 = 30,$$
$$f_2 = 40$$
and $h = 5$

Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$= 15 + \frac{80 - 30}{2 \times 80 - 30 - 40} \times 5$$

$$= 15 + \frac{50}{90} \times 5$$

$$= 15 + \frac{25}{3} = 17.77 \text{(approx.)}$$

Hence, the mean size of agriculture holding $= 17.81$ ha (approx.)
and modal size of agriculture holding $= 17.77$ ha (approx.)