CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-13

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The value of k for which the equations $3x-y+8=0$ and $6x-ky=-16$ represent coincident lines is [1]

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) -2

Ans : (c) 2

$$\frac{3}{6}=\frac{-1}{-k}=\frac{8}{16}\Rightarrow k=2$$

2. Which of the following is not a quadratic equation? [1]

(a) $2(x-1)^2=4x^2-2x+1$
(b) $2x-x^2=x^2+5$
(c) $(\sqrt{2}\,x+\sqrt{3})^2+x^2=3x^2-5x$
(d) $(x^2+2x)^2=x^4+3+4x^3$

Ans : (c) $(\sqrt{2}\,x+\sqrt{3})^2+x^2=3x^2-5x$

$$(\sqrt{2}\,x+\sqrt{3})^2+x^2=3x^2-5x$$
$$2x^2+2\sqrt{6}\,x+3+x^2=3x^2-5x]$$
$$(2\sqrt{6}+5)x+3=0, \text{ which is not quadratic.}$$

3. The number of two digit numbers which are divisible by 3 is [1]

(a) 33 (b) 31

(c) 30 (d) 29

Ans : (c) 30

Two digit numbers which are divisible by 3 are 12,15,18,..........99.
These are in A.P. with

$$a=12,\ d=3,\ l=99$$
$$99=12+(n-1)3$$
$$n-1=\frac{87}{3}=29$$
$$n=30$$

4. In the formula $\bar{x}=a+h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$, for finding the mean of grouped frequency distribution $u_i=$ [1]

(a) $\frac{x_i+a}{h}$ (b) $h(x_i-a)$

(c) $\frac{x_i-a}{h}$ (d) $\frac{a-x_i}{h}$

Ans : (c) $\frac{x_i-a}{h}$

$$u_i=\frac{x_i-a}{h}$$

5. A card is selected at random from a pack of 52 cards. The probability of its being a red face card is [1]

(a) $\frac{3}{26}$ (b) $\frac{3}{13}$

(c) $\frac{2}{13}$ (d) $\frac{1}{2}$

Ans : (a) $\frac{3}{26}$

$$\text{Total numbers of outcomes}=52$$
$$\text{Favourable number of outcomes}=6$$
$$\text{Required probability}=\frac{6}{52}=\frac{3}{26}$$

6. If in ΔABC and ΔDEF, $\frac{AB}{DE}=\frac{BC}{FD}$, then they will be similar when [1]

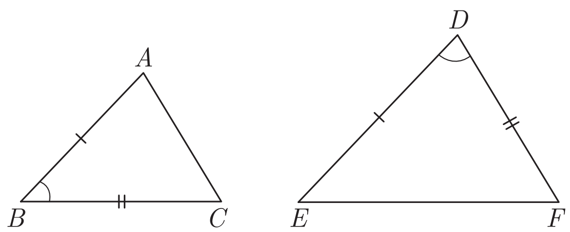
(a) $\angle B=\angle E$ (b) $\angle A=\angle D$

(c) $\angle B=\angle D$ (d) $\angle A=\angle F$

Ans : (c) $\angle B=\angle D$

So, for ΔABC to be similar to ΔDEF,

$\angle B=\angle D$ (by SAS axiom)

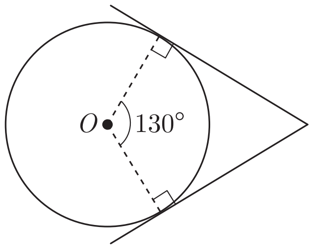


7. If angle between two radii of a circle is 130°, the angle between the tangents at the ends of the radii is [1]

(a) 90° (b) 50°

(c) 70° (d) 40°

Ans : (b) 50°



Angle between the tangents $= 180° - 130°$

$= 50°$

8. The value of $\frac{1-\tan^2 45°}{1+\tan^2 45°}$ is equal to [1]

(a) $\tan 60°$ (b) $\tan 30°$

(c) $\sin 45°$ (d) $\tan 0°$

Ans : (d) $\tan 0°$

$$\frac{1-\tan^2 45°}{1+\tan^2 45°} = \frac{1-1^2}{1+1^2} = 0 = \tan 0° \qquad [\tan 45° = 1]$$

9. If $\cos 9\alpha = \sin\alpha$ and $0° < 9\alpha < 90°$, then the value of $\tan 5\alpha$ is [1]

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) 0

Ans : (c) 1

$$\cos 9\alpha = \sin\alpha$$
$$\cos 9\alpha = \cos(90° - \alpha)$$
$$9\alpha = 90° - \alpha = 10\alpha = 90°$$
$$5\alpha = \tan 45° = 1$$

10. The value of $\frac{\tan 60°}{\cot 30°} - \frac{\sin 47°}{\cos 43°}$ is [1]

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) -1

Ans : (a) 0

$$\frac{\tan 60°}{\cot 30°} - \frac{\sin 47°}{\cos 43°} = \frac{\sqrt{3}}{\sqrt{3}} - \frac{\sin 47°}{\cos(90° - 47°)}$$
$$= 1 - \frac{\sin 47°}{\sin 47°}$$
$$= 1 - 1 = 0$$

(Q.11-Q.15) Fill in the blanks.

11. If $ax + by = a^2 - b^2$ and $bx + ay = 0$, then $(x+y)$ is equal to.......... [1]

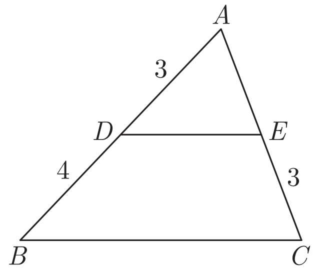
Ans :

$$ax + by = a^2 - b^2$$
$$bx + ay = 0$$

adding equation (i) and (ii), we have

$$(a+b)x + (a+b)y = (a-b)(a+b)$$
$$(a+b)(x+y) = (a-b)(a+b)$$
$$= x + y = a - b$$

12. In the given figure, $DE || BC$ and all measurements are in centimetres. The length of AE is.......... [1]



Ans :

$$DE \parallel BC$$
$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3}{4} = \frac{AE}{3} = AE = \frac{9}{4}$$

13. PQ is a tangent to a circle at point P. Centre of circle is O. If $\Delta\, OPQ$ is an isosceles triangle, then $\angle QOP$ is equal to.......... [1]

Ans : 45

$\Delta\, OPQ$ is an isosceles triangle

and $\angle OPQ = 90°$

So, $\angle QOP = \angle PQO = 45°$

$\angle QOP = 45°$

or

The tangent at any point of a circle is.......... to the radius through the point of contact.

Ans : perpendicular

14. The outcomes which ensure the occurrence is.......... to the radius through the point of contact. [1]

Ans : favourable outcomes

15. The mode of the following frequency distribution is.......... [1]

Marks	18	22	25	37	43	48
Number of students	9	7	18	11	6	3

Ans : 25

Frequency of marks 25 is greatest i.e. 18

Mode $= 25$

(Q.16-Q.20) Answer the following

16. Write the polynomial whose zeroes are $2+\sqrt{3}$ and $2-\sqrt{3}$. [1]

Ans :

Required polynomial $= x^2 - (\text{sum of the zeroes})x$

$+ \text{Product of the zeroes}$

$= x^2 - (2+\sqrt{3}+2-\sqrt{3})x$

$+(2+\sqrt{3})(2-\sqrt{3})$

$= x^2 - 4x + 1.$

17. If the quadratic equation $px^2 - 2\sqrt{5}\,px + 15 = 0$ has two equal real roots, then find the value of p. [1]

Ans :

Given,

$$px^2 - 2\sqrt{5}\,px + 15 = 0$$

For equal roots,

$$\text{D} = (-2\sqrt{5}\,p)^2 - 4 \times p \times 15 = 0$$

$$20p^2 - 60p = 0 \Rightarrow 20p(p-3) = 0$$
$$p = 0 \text{ or } p = 3 \text{ but } p \neq 0 \; p = 3.$$

or

Which term of the arithmetic progression 4,9,14,19 .... is 109?

Ans :

Given 4,9,14,19...109

Here, $a = 4,\ d = 5,\ l = 109$

Now, $l = a + (n-1)d$

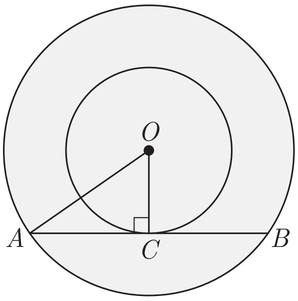
$$109 = 4 + (n-1) \times 5 = 5(n-1) = 105$$
$$n - 1 = 21,\ n = 22.$$

Hence, 22nd term is 109.

18. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle. [1]

Ans :

As per the given question we draw the figure as below.

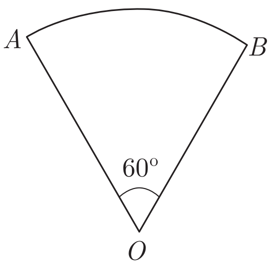


Here AB is the chord of large circle which touch the smaller circle at point C. We can see easily that ΔAOC is right angled triangle.
Here, $AO = 5$ cm, $OC = 3$ cm

$$AC = \sqrt{AO^2 - OC^2}$$
$$= \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9} = \sqrt{16} = 4 \text{ cm}$$

Length of chord, $AB = 8$ cm.

19. If the adjoining figure is a sector of a circle of radius 10.5 cm, find the perimeter of the sector. $\left(Take\ \pi = \frac{22}{7}\right)$. [1]



Ans :

Given, Radius $= 10.5 \text{ cm} = \frac{21}{2}$ cm

$$\text{Length of arc AB} = \left(\frac{60}{360} \times 2 \times \frac{22}{7} \times \frac{21}{2}\right) \text{cm}$$
$$= 11 \text{ cm}$$
$$\text{Perimeter} = \text{AC} + \text{BC} + \text{AB}$$
$$= (10.5 + 10.5 + 11) \text{ cm} = 32 \text{ cm}$$

20. Find the value of a, if the distance between the points $A(-3,-14)$ and $B(a,-5)$ is 9 units. [1]

Ans :

Given, Distance between point A and B

$$\text{AB} = 9$$
$$\sqrt{(a+3)^2 + (-5+14)^2} = 9$$
$$(a+3)^2 + 81 = 81$$

and, $= (a+3)^2 = 0$

$$a + 3 = 0$$
$$a = -3.$$

Section B

21. Show that any positive odd integer is of the form $4q+1$ or $4q+3$, where q is some integer. [2]

Ans :

Let a be any positive odd integer.
Applying Euclid's division lemma with divisor 4, we get

$$a = 4q,\ 4q+1,\ 4q+2 \text{ or } 4q+3$$

where q is some whole number.

22. The coordinates of the mid-point of the line joining the points $(3p,4)$ and $(-2,2q)$ are $(5,p)$. Find the values of p and q. [2]

Ans :

The mid-point of the line segment joining the points $(3p,4)$ and $(-2,2q)$ is

$$\left(\frac{3p-2}{2}, \frac{4+2q}{2}\right) i.e. \left(\frac{3p-2}{2}, 2+q\right)$$

But the mid-point is given as $(5,p)$

$$\frac{3p-2}{2} = 5 \Rightarrow 3p - 2 = 10$$
$$3p = 12$$
$$p = 4$$

and $2 + q = p \Rightarrow 2 + q = 4$

$$q = 2.$$

Hence,

$$p = 4 \text{ and } q = 2.$$

or

If the points $P(5,3)$ and $Q(k,4)$ are on a circle centre $O(2,-1)$, find the value of k.

Ans :

P and Q are points on the circle with centre O

$$\text{OP} = \text{OQ}$$
$$\sqrt{(5-2)^2 + (3+1)^2} = \sqrt{(k-2)^2 + (4+1)^2}$$
$$9 + 16 = (k-2)^2 + 25$$
$$(k-2)^2 = 0$$
$$k - 2 = 0$$
$$k = 2$$

23. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. $\left[Take\ \pi = \frac{22}{7}\right]$. [2]

Ans :

Given, total surface area of hemisphere

$$= 462\,\text{cm}^2$$

$$3\pi r^2 = 462$$

$$3 \times \frac{22}{7} \times r^2 = 462$$

$$r^2 = \frac{462 \times 7}{3 \times 22} \Rightarrow r^2 = 49$$

$$r = 7\,\text{cm}$$

Volume of hemisphere

$$= \frac{2}{3}\pi r^3 = \left(\frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\right)\text{cm}^3$$

$$= \frac{2156}{3}\,\text{cm}^3$$

$$= 718\frac{2}{3}\,\text{cm}^3.$$

24. If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$ [2]

Ans :

Given, $4\cos\theta = 11\sin\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{4}{11}. \quad \text{...(i)}$$

Now, $$\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11 - 7\frac{\sin\theta}{\cos\theta}}{11 + 7\frac{\sin\theta}{\cos\theta}}$$

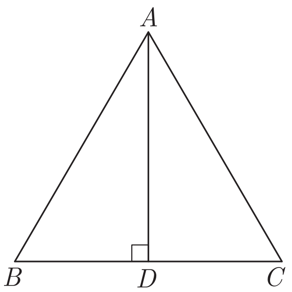
$$= \frac{11 - 7 \times \frac{4}{11}}{11 + 7 \times \frac{4}{11}} \quad \text{(using (i))}$$

$$\frac{\frac{121-28}{11}}{\frac{121+28}{11}} = \frac{93}{149}$$

or

If $\cos\theta = \frac{\sqrt{3}}{2}$, then find the value of $4\cos^3\theta - 3\cos\theta$.

Ans :



Given, $\cos\theta = \frac{\sqrt{3}}{2}$

$$4\cos^3\theta - 3\cos\theta = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3.\frac{\sqrt{3}}{2}$$

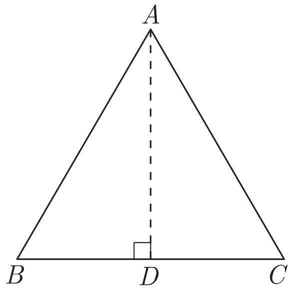
$$= 4.\frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$

25. If an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$. [2]

Ans :



In $\Delta\,ABD$ right angled at D.
By Pythagoras theorem,

$$AB^2 = AD^2 + BD^2$$

$$BC^2 = AD^2 + BD^2 \quad (AB = BC = CA)$$

$$(2BD)^2 = AD^2 + BD^2$$

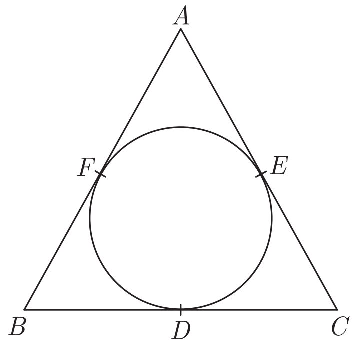
(in an equilateral Δ perpendicular is the median, $BD = \frac{1}{2}BC$)

$$4BD^2 = AD^2 + BD^2$$

$$AD^2 = 3BD^2$$

26. The in circle of an isosceles triangle ABC,in which $AB = AC$, touches the sides BC, CA and AB at D,E and F respectively. Prove that $BD = DC$ [2]

Ans :



Given: A circle inscribed in an isosceles Δ ABC with $AB = AC$, touching the sides BC, CA and AB at D,E and F respectively.

To prove: BD = DC

Proof: Length of tangents from an external point to the circle are equal

$$AF = AE \quad \text{...(i)}$$

$$BF = BD \quad \text{...(ii)}$$

and $$CD = CE \quad \text{...(iii)}$$

$$AB = AC \quad \text{(given)}$$

$$AF + BF = AE + EC$$

$$BF = EC \quad \text{(using (i))}$$

$$BD = CD \quad \text{(using (ii) and (iii))}$$

Section C

27. Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively. [3]

Ans :

Given that required number when divides 2053 and

967 leaves a remainder 5 and 7 respectively.
The required number is HCF of

$$2053 - 5 = 2048 \text{ and } 967 - 7 = 960$$

```
960 ) 2048 ( 2
     - 1920
     --------
       128 ) 960 ( 7
            - 896
            ------
              64 ) 128 ( 2
                  - 128
                  -----
                      0
```

HCF of 2048 and 960 $= 64$

Hence, the required largest number is 64.

28. Solve graphically the pair of linear equations $2x+3y = 11$ and $2x-4y = -24$. Hence, find the value of m, given that the line represented by $y = mx+3$ passes through the intersection of the given pair. [3]

Ans :

The given equation can be written as

$$y = \frac{11-2x}{3} \quad ...(i)$$

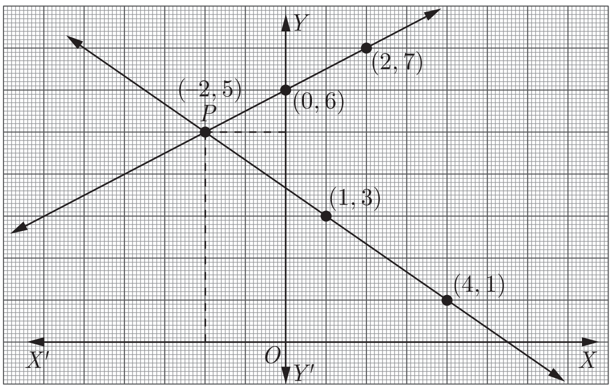
and $$y = \frac{2x+24}{4} \quad ...(ii)$$

Table of values of equation (i)

x	1	4
y	3	1

Table of values of equation (ii)

x	0	2
y	6	7



29. Find the sum of all two digit numbers which when divided by 3 yield 1 as remainder. [3]

Ans :

Two digit numbers which leaves remainder 1 when divided by 3 are 10,13,16,19...,97

These numbers form an AP with

$$a = 10,\ d = 3 \text{ and } l = 97.$$

$$l = a+(n-1)d$$

$$97 = 10+(n-1)\times 3$$

$$3(n-1) = 87$$

$$n-1 = 29$$

$$n = 30$$

Now, the sum of all such numbers

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{30}{2}(10+97)$$

$$= 15\times 107 = 1605.$$

or

The sum of 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44. Find the AP.

Ans :

Let the first term and common difference of AP be a and d respectively.

Given: $T_4+T_8 = 24$ and $T_6+T_{10} = 44$

$$a+3d+a+7d = 24 \text{ and } a+5d+a+9d = 44$$

$$2a+10d = 24 \quad ...(i)$$

and $$2a+14d = 44 \quad ...(ii)$$

Subtracting equation (i) from (ii), we get

$$4d = 20$$

$$d = 5.$$

Putting $d = 5$ in (i), we get

$$2a+10\times 5 = 24$$

$$2a = -26$$

$$a = -13.$$

Hence, the required AP is

$-13, -13+5, -13+2\times 5, -13+3\times 5, ...$

i.e. $-13, -8, -3, 2, ...$

30. Find the mean of the following frequency distribution:

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	8	12	10	11	9

[3]

Ans :

We shall use step-devision method. Taking assumed mean $a = 25$, construct the table as under.

Here, $h = 10$.

Class interval	Class mark x_i	$u_i = \frac{x_i - 25}{10}$	f_i	$f_i u_i$
0-10	5	-2	8	-16
10-20	15	-1	12	-12
20-30	25	0	10	0
30-40	35	1	11	11
40-50	45	2	9	18
Total			$\Sigma f_i = 50$	$\Sigma f_i u_i = 50$

$$\text{Mean} = a+\frac{\Sigma f_i u_i}{\Sigma f_i}\times h$$

$$= 24+\frac{1}{50}\times 10 = 25+\frac{1}{5} = 25.2$$

31. Prove that the points $A(2,3)$, $B(-2,2)$, $C(-1,-2)$ and $D(3,-1)$ are the vertices of a square ABCD.

Ans :

Given A(2,3), B(-2,2), C(-1,-2) and D(3,-1).

$$AB = \sqrt{(-2-2)^2 + (2-3)^2} = \sqrt{16+1} = \sqrt{17}$$

$$BC = \sqrt{(-1+2)^2 + (-2-2)^2} = \sqrt{1+16} = \sqrt{17}$$

$$CD = \sqrt{(3+1)^2 + (-1+2)^2} = \sqrt{16+1} = \sqrt{17}$$

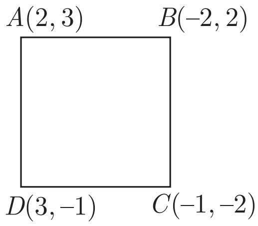
$$DA = \sqrt{(2-3)^2 + (3+1)^2} = \sqrt{1+16} = \sqrt{17}.$$

$$\text{Diagonal } AC = \sqrt{(-1-2)^2 + (-2-3)^2} = \sqrt{9+25} = \sqrt{34}$$

$AB = BC = CD = DA$ *i.e.* all 4 sides are equal

and $AC = BD$ *i.e.* diagonals are equal.

ABCD is a square.



32. Draw a line segment of length 7.8 cm and divide it in the ratio 5:8. Measure the two parts. Also, justify your construction. [3]

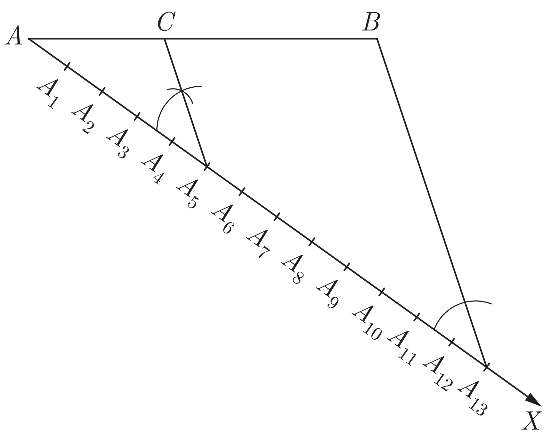
Ans :

Steps of construction:

1. Draw a line segment AB of length 7.8 cm by using ruler.
2. Draw any ray AX making an acute angle with AB.
3. Locate 13(5+8) points $A_1, A_2, A_3 ..., A_{13}$ on AX such that
$$AA_1 = A_1A_2 = A_2A_3 = ... = A_{12}A_{13}$$
4. Join $A_{13}B$.
5. Through A_5, draw a line parallel to $A_{13}B$ (making an angle equal to $\angle AA_{13}B$) to intersect AB at C. Then C is the required point which divides AB in the ratio 5:8.

On measuring, we find that

$$AC = 3\text{ cm and } CB = 4.8\text{ cm}.$$



Justification:

In $\Delta AA_{13}B$,

$$A_5C \parallel A_{13}B.$$

By Basic Proportionality Theorem,

$$\frac{AC}{CB} = \frac{AA_5}{A_5A_{13}} \text{ but } = \frac{AA_5}{A_5A_{13}} = \frac{5}{8}, \text{ so } \frac{AC}{CB} = \frac{5}{8}$$

i.e. C divides AB in the ratio 5:8.

33. Prove the following identity:

$\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\operatorname{cosec} A.$ [3]

Ans :

$$\text{LHS} = \frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A}$$

$$= \tan A\left(\frac{1}{1+\sec A} - \frac{1}{1-\sec A}\right)$$

$$= \tan A\left(\frac{(1-\sec A)-(1+\sec A)}{1-\sec^2 A}\right)$$

$$= \tan A\left(\frac{-2\sec A}{-\tan^2 A}\right) \qquad \left(\begin{matrix}\text{Sec}^2 A - 1 \\ = \tan^2 A\end{matrix}\right)$$

$$= 2\frac{\sec A}{\tan A} = 2.\frac{1}{\cos A}.\frac{\cos A}{\sin A} = \frac{2}{\sin A}$$

$$= 2\operatorname{cosec} A = \text{RHS}$$

or

Prove that $(\operatorname{cosec} A - \sin A)\ (SecA - \cos A)$

$$= \frac{1}{\tan A + \cot A}$$

Ans :

$$\text{LHS} = (\operatorname{cosec} A - \sin A)\ (\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \frac{1-\sin^2 A}{\sin A} \times \frac{1-\cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \cos A \sin A \qquad ...(i)$$

$$\text{RHS} = \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\sin A \cos A}{1} = \sin A \cos A \qquad ...(ii)$$

From equation (i) and (ii)

$$\text{LHS} = \text{RHS}.$$

34. A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm [3]

Ans :

Given, radius of cone = 3.5 cm

Diameter of hemisphere = 7 cm

Radius, $r = \frac{7}{2}\text{ cm} = 3.5\text{ cm}$

Total height of the toy = 15.5 cm

Height of the cone, $h = (15.5 - 3.5)\text{cm} = 12\text{ cm}$

Now, slant height of cone

$$l = \sqrt{r^2+h^2} = \sqrt{(3.5)^2+(12)^2}$$
$$= \sqrt{12.25+144} = \sqrt{156.25} = 12.5\,\text{cm}$$

Total surface area of the toy

$$= \text{C.S.A.of cone} + \text{C.S.A of hemisphere}$$
$$= \pi rl + 2\pi r^2$$
$$= \frac{22}{7}\times 3.5\times 12.5 + 2\times\frac{22}{7}\times 3.5\times 3.5$$
$$= (137.5+77)\text{cm}^2 = 214.5\,\text{cm}^2$$

Hence, the total surface area of the toy is 214.5 cm^2

or

A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer.

Ans :

Given internal diameter of bowl

$$= 36\,\text{cm}$$
$$\text{Radius } (r) = \frac{36}{2}\text{ cm} = 18\,\text{cm}$$

Volume of liquid in the bowl

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\times(18)^3\text{cm}^3$$

Since 10% liquid is wasted in transfer,
90% liquid is transferred in the cylindrical bottles
Volume of liquid transferred in cylindrical bottles

$$= 90\% \text{ of } \frac{2}{3}\pi\times(18)^3$$
$$= \frac{90}{100}\times\frac{2}{3}\pi\times(18)^3$$

Let the height of each bottle be h cm, then
volume of one cylindrical bottle

$$= \pi\times 3^2\times h \qquad (\text{d} = 6\text{cm})$$

volume of 72 cylindrical bottles

$$= 72\times\pi\times 3^2\times h$$

Now, volume of 72 bottles

= volume of liquid transferred in bottles

$$72\times\pi\times 3^2\times h = \frac{90}{100}\times\frac{2}{3}\pi\times(18)^3$$
$$h = \frac{\frac{90}{100}\times\frac{2}{3}\pi\times(18)^3}{72\times\pi\times 3^2}$$
$$h = 5.4\,\text{cm}$$

Section D

35. Ram's mother has given him money to buy some boxes from the market at the rate of $4x^2+3x-2$. The total amount of money given by his mother is represented by $8x^4+14x^3-2x^2+7x-8$. Out of this money he donated some amount of money to a child who was studying in the light of street lamp. Find how much amount of money he donated after purchasing the maximum number of boxes from the market? [4]

Ans :

Total amount of money

$$p(x) = 8x^4+14x^3-2x^2+7x-8.$$

cost of each box $g(x) = 4x^2+3x-2$

Maximum number of boxes purchased $= q(x)$

Amount of money donated to the child $= r(x)$

We need to find $q(x)$ and $r(x)$.

On dividing $p(x)$ by $g(x)$, we have

$$\begin{array}{r|l|l} 4x^2+3x-2 & 8x^4+14x^3-2x^2+7x-8 & 2x^2+2x-1 \\ & \underset{-}{8x^4}+\underset{-}{6x^3}\underset{+}{-4x^2} & \\ \hline & 8x^3+2x^2+7x-8 & \\ & \underset{-}{8x^3}+\underset{-}{6x^2}\underset{+}{-4x} & \\ \hline & -4x^2+11x-8 & \\ & \underset{+}{-4x^2}\underset{+}{-3x}\underset{-}{+2} & \\ \hline & 14x-10 & \end{array}$$

$$q(x) = 2x^2+2x-1 \text{ and } r(x) = 14x-10.$$

Hence, the amount of money he donated to the child

$$= 14x-10$$

and maximum number of boxes, he purchased

$$= 2x^2+2x-1.$$

36. A student scored a total of 32 marks in class tests in mathematics and science. Had he scored 2 marks less in science and 4 more in mathematics, the product of his marks would have been 253. Find his marks in two subjects. [4]

Ans :

Let the student's marks in mathematics be x.
As the total marks marks in mathematics and science is 32, so the student's marks in science

$$= 32-x.$$

Given if he scores 2 less marks in science and 4 more marks in mathematics, then his new marks in science

$$= 32-x-2$$
$$= 30-x$$

and his new marks in mathematics

$$= x+4$$

According to given,

$$(x+4)(30-x) = 253$$
$$30x+120-x^2-4x = 253$$
$$x^2-26x+133 = 0$$
$$x^2-19x-7x+133 = 0$$
$$(x-7)(x-19) = 0 \quad x = 7 \text{ or } x = 19.$$

When, $x = 7,\ 32-x = 25;$

when $x = 19, 32-x = 13.$

Hence, student's marks in mathematics

= 7 and in science = 25

or student's marks in mathematics

= 19 and in science = 13.

or

If -4 is a root of the equation $x^2+2x+4p = 0$, find the values of k for which the equation $x^2+p(1+3k)x+7(3+2k) = 0$ has equal roots.

Ans :

Given $x = -4$ is a root of the equation

$$x^2 + 2x + 4p = 0$$

$$(-4)^2 + 2 \times (-4) + 4p = 0$$

$$16 - 8 + 4p = 0$$

$$4p = -8$$

$$p = -2.$$

Now, the equation

$$x^2 + p(1+3k)x + 7(3+2k) = 0$$

has equal roots

$$D = 0$$

$$\{p(1+3k)\}^2 - 4 \times 1 \times 7(3+2k) = 0$$

$$p^2(1+6k+9k^2) - 28(3+2k) = 0$$

$$(-2)^2(1+6k+9k^2) - 84 - 56k = 0 \quad (p = -2)$$

$$4 + 24k + 36k^2 - 84 - 56k = 0$$

$$36k^2 - 32k - 80 = 0$$

$$9k^2 - 8k - 20 = 0$$

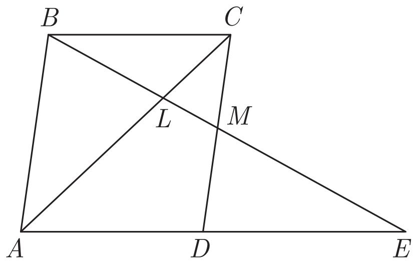
$$(9k+10)(k-2) = 0$$

$$k = \frac{-10}{9}$$

or $k = 2.$

37. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC at L and AD produced in E. Prove that $EL = 2BL$. [4]

Ans :



In Δ BMC and Δ EMD,

$$MC = MD \quad \text{(M is mid-point of CD)}$$

$$\angle CBM = \angle DEM \quad (\text{alt.} \angle s, BC \,||\, AE)$$

$$\angle BMC = \angle EMD \quad (\text{vert. opp.} \angle s)$$

$$\Delta BMC \cong \Delta EMD \quad \text{(AAS criterion of congruence)}$$

$$BC = DE \quad \text{(c.p.c.t.)}$$

Also $AD = BC$ (opp. sides of ||gm ABCD)

$$AE = AD + DE = BC + BC \quad AE = 2BC$$

...(i)

In Δ AEL and Δ CBL,

$$\angle AEL = \angle CBL \quad (\text{alt.} \angle s)$$

$$\angle ALE = \angle CLB \quad (\text{vert.opp.} \angle s)$$

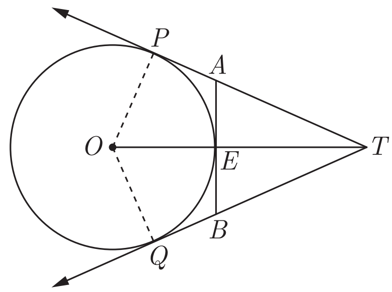
$$\Delta AEL \sim \Delta CBL \quad \text{(AA is similarity criterion)}$$

$$\frac{EL}{BL} = \frac{AE}{BC} = 2 \quad \text{(using (i))}$$

$$EL = 2BL, \text{ as required.}$$

or

In the given figure, O is the centre of a circle or radius 5 cm. T is a point such that $OT = 13$ cm, PT and QT are tangents to the circle at points P and Q respectively. OT intersects the circle at the point E. If AB is tangent to the circle at E, find the length of AB.



Ans :

Since tangent is perpendicular to radius,

$$OP \perp PT.$$

In Δ OPT, $\angle OPT = 90°$. By Pythagoras theorem,

$$PT^2 = OA^2 - OP^2 = 13^2 - 5^2 = 144$$

$$PT = 12\text{cm}$$

As the lengths of tangent drawn from a point to a circle are equal,

$$AP = AE = x \text{ cm (say)}$$

$$AT = PT - AP = (12 - x) \text{ cm}$$

$$ET = OT - OE = 13 \text{ cm} - 5 \text{ cm}$$

$$= 8 \text{ cm}.$$

Given, AB is tangent to the circle at E,

$$OE \perp AB \quad \angle AET = 90°.$$

In Δ AET, $\angle AET = 90°$. By Pythagoras theorem,

$$AT^2 = AE^2 + ET^2$$

$$(12 - x)^2 = x^2 + 8^2$$

$$144 - 24x + x^2 = x^2 + 64$$

$$24x = 80$$

$$x = \frac{10}{3}.$$

$$AE = \frac{10}{3} \text{ cm.}$$

Similarly, $BE = \frac{10}{3}$ cm.

$$AB = AE + BE = \left(\frac{10}{3} + \frac{10}{3}\right) \text{ cm}$$

$$= \frac{20}{3} \text{ cm.}$$

38. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flagstaff are α and β respectively. Prove that the height of the tower is $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$ [4]

Ans :

Let AB be a vertical tower and BP be a flagstaff of height h surmounted on the tower AB, and O be the point of observation on the plane (ground), then

$$BP = h.$$

Let $OA = d$ and $AB = H$.

Given, $\angle BOA = \alpha$ and $\angle POA = \beta$.

From right angled ΔOAB, we get ...(i)

$$\tan\alpha = \frac{H}{d}$$

$$\tan\beta = \frac{H+h}{d}$$

On dividing equation (i) by (ii), we get

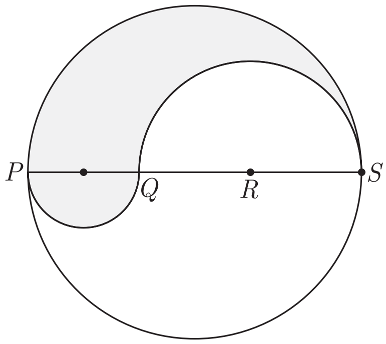
$$\frac{\tan\alpha}{\tan\beta} = \frac{H}{H+h}$$

$$H\tan\alpha + h\tan\alpha = H\tan\beta$$

$$h\tan\alpha = H(\tan\beta - \tan\alpha)$$

$$H = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}, \text{ as required.}$$

39. PS is a diameter of a circle of radius 6 cm. Q and R are points on the diameter that PQ, QR and RS are equal. Semicircles are drawn with PQ and QS as diameters, as shown in the figure. Find the perimeter of the shaded region.
Also find the area of the shaded region. $(\text{USe } \pi = 3.14)$



Ans :

$$PS = 2 \times \text{radius} = (2 \times 6)\text{ cm}$$
$$= 12 \text{ cm.}$$

Given $PQ = QR = RS$

$$PQ = \frac{1}{3} \text{ of PS} = \frac{1}{3} \text{ of 12 cm}$$
$$= 4 \text{ cm}$$

and $QS = 8$ cm.

The semicircles with PQ and QS as diameters have radii 2 cm and 4 cm respectively.

Perimeter of the shaded region

$$= \left(\frac{1}{2} \times 2\pi \times 6 + \frac{1}{2} \times 2\pi \times 4 + \frac{1}{2} \times 2\pi \times 2\right)$$
$$= \pi(6+4+2) \text{ cm} = 12\pi \text{ cm}$$
$$= (12 \times 3.14)\text{ cm} = 37.68 \text{ m}$$

Area of the shaded region

$$= \left(\frac{1}{2}\pi \times 6^2 - \frac{1}{2} \times \pi \times 4^2 + \frac{1}{2} \times \pi \times 2^2\right)\text{cm}^2$$
$$= \frac{\pi}{2}(36 - 16 + 4)\text{ cm}^2$$
$$= 12\,\pi \text{ cm}^2$$
$$= (12 \times 3.14)\text{ cm}^2$$
$$= 37.68 \text{ cm}^2$$

40. A die has six faces marked 0,1,1,1,6,6. Two such dice are thrown together and the total score is recorded. [4]
(i) How many different scores are possible?
(ii) What is the probability of getting a total of 7?

Ans :

(i) For the sum of outcomes on the top faces of two dice i.e. the total score obtained, we construct the table as under:

Sum	0	1	1	1	6	6
0	0	1	1	1	6	6
1	1	2	2	2	7	7
1	1	2	2	2	7	7
1	1	2	2	2	7	7
6	6	7	7	7	12	12
6	6	7	7	7	12	12

The different scores obtained are 0,1,2,6,7,12. They are six in number.
Hence, the number of different possible scores are 6 (in number).

(ii) Total number of outcomes are 6×6 i.e. 36, which are all equally likely.
The table shows that we get score 7 twelve times. So, the number of outcomes favourable to the event 'getting a total of 7' is 12.

$$P(\text{getting a total of 7}) = \frac{12}{36} = \frac{1}{3}.$$

or

Two customers Shyam and Ekta visit a particular shop in the same week from Tuesday to Saturday. Each is equally likely to visit the shop on any day. Find the probability that they visit the shop
(i) on the same day
(ii) on consecutive day
(iii) on different days
(iv) neither on same day nor on consecutive days.

Ans :

Let T,W,Th,F and S represent Tuesday, Wednesday, Thursday, Friday and Saturday respectivly. Since Shyam and Ekta visit a particular shop in the same week from Tuesday to Saturday, the outcomes are:
(T,T), (T,W),(T,Th),(T,F),(T,S)]
(W,T),(W,W),(W,Th),(W,F),(W,S)
(Th,T),(Th,W),(Th,Th),(Th,F),(Th,S)
(F,T),(F,W),(F,Th),(F,F),(F,S)
(S,T),(S.W),(S,Th),(S,F),(S,S)

$$\text{Total number of outcomes} = 5 \times 5 = 25$$

and all outcomes are equally likely.

(i) The outcomes favourable to the event 'visit on the same day' are (T,T),(W,W),(Th,Th),(F,F),(S,S); which are 5 in number.

$$P(\text{visit on the same day}) = \frac{5}{25} = \frac{1}{5}.$$

(ii) The outcomes favourable to the event 'visit on consecutive days' are (T,W),(W,T),(W,Th), (Th ,W),(Th,F),(F,Th),(F,S),(S,F); which are 8 in number.

$$P(\text{visit on consecutive days}) = \frac{8}{25}.$$

(iii) $P(\text{visit on consecutive days})$

$$= 1 - p(\text{visit on same day})$$
$$= 1 - \frac{1}{5} = \frac{4}{5}.$$

(iv) The outcomes favourable to the event 'visit neither on same day nor on consecutive days' are (T,Th),(T,F),(T,S),(W,F),(W,S),(Th,T),(Th,S),(F,T),(F,W),(S,T),(S,W),(S,Th);
which are 12 in number.
P(visit neither on same day nor on consecutive days)

$$= \frac{12}{25}.$$