CLASS X (2019-20)
MATHEMATICS BASIC (241)
SAMPLE PAPER-12

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The product and the sum of the zeroes of the polynomial $2x^2 - 2\sqrt{2}\,x + 1$ are respectively [1]

(a) $\frac{1}{2}$ and $-\sqrt{2}$ (b) $-\frac{1}{2}$ and $\sqrt{2}$

(c) $\frac{1}{2}$ and $\sqrt{2}$ (d) $\sqrt{2}$ and $\frac{1}{2}$

Ans : (c) $\frac{1}{2}$ and $\sqrt{2}$

In polynomial $ax^2 + bx + c$,

$$\text{sum of zeroes} = \frac{-b}{a};$$

$$\text{product of zeroes} = \frac{c}{a},$$

$$\text{product} = \frac{1}{2}$$

$$\text{Sum} = -\frac{(-2\sqrt{2})}{2} = \sqrt{2}$$

2. If $k+2, 4k-6$ and $3k-2$ are three consecutive terms of an $A.P.$, then the value of k is [1]

(a) 3 (b) -3

(c) 4 (d) -4

Ans : (a) 3

$$2(4k-6) = k+2+3k-2$$
$$8k-12 = 4k$$
$$4k = 12$$
$$k = 3$$

3. The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2,3)$, $B(6,7)$ and $C(8,3)$ is [1]

(a) (0, 1) (b) (0, -1)

(c) (-1, 0) (d) (1, 0)

Ans : (b) (0, -1)

In parallelogram, diagonals bisect each other.

Mid-point of AC = Mid-point of BD

$$\left(\frac{-2+8}{2}, \frac{3+3}{2}\right) \left(\frac{6+x}{2}, \frac{7+y}{2}\right)$$

(Let fourth vertex be $D(x, y)$)

$$\frac{6+x}{2} = 3$$

and $$\frac{7+y}{2} = 3$$

$$x = 0$$

$y = -1$ so, D is (0, -1)

4. The point on the x-axis which is equidistant from the points $A(-2,3)$ and $B(5,4)$ is [1]

(a) (0, 2) (b) (2, 0)

(c) (3, 0) (d) (-2, 0)

Ans : (b) (2, 0)

Let the point on x-axis be $P(x, 0)$

$$PA = PB \quad \text{(Given)}$$
$$\sqrt{(x+2)^2 + 3^2} = \sqrt{(x-5)^2 + 4^2}$$
$$x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$$
$$14x = 28$$
$$x = 2$$

So, point is (2, 0)

5. If $A = 30°$, then the value of 2 sin A cos A is [1]

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) 1

Ans : (b) $\frac{\sqrt{3}}{2}$

Let $$I = 2\sin A\cos A = 2\sin 30°\cos 30°$$
$$= 2.\frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

6. Sin $(45° + \theta) - \cos(45° - \theta)$ is equal to [1]

(a) $2\cos\theta$ (b) 0

(c) $2\sin\theta$ (d) 1

Ans : (b) 0

$$\sin(45° + \theta) - \cos(45° - \theta)$$
$$= \sin(45° + \theta) - \sin(90° - (45° - \theta))$$

$$(\sin(90^\circ - \theta) = \cos\theta)$$
$$= \sin(45^\circ + \theta) - \sin(45^\circ + \theta) = 0$$

7. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is [1]

(a) 31 cm (b) 25 cm
(c) 62 cm (d) 50 cm

Ans : (d) 50 cm

Let the radius of circle be r cm,

then
$$\pi . r^2 = \pi(24)^2 + \pi(7)^2$$
$$r^2 = 576 + 49$$
$$r^2 = 625$$
$$r = 25$$
Diameter, $d = 2r = 2 \times 25 = 50$ cm

8. If two solid hemispheres of same base radius r joined together along their bases, then the curved surface of this new solid is [1]

(a) $4\pi r^2$ (b) $6\pi r^2$
(c) $3\pi r^2$ (d) $8\pi r^2$

Ans : (a) $4\pi r^2$

New solid will be a sphere of radius r.

So, curved surface area $= 4\pi r^2$

9. The Empirical relation for measuring the mode is [1]

(a) Mode = 3 median − 2 mean
(b) Mode = 3 mean − 2 median
(c) Mode = 2 median − mean
(d) Mode = 2 mean − median

Ans : (a) Mode = 3 median − 2 mean

10. If a fair coin is tossed twice, then the probability of getting two heads is [1]

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) 0

Ans : (a) $\frac{1}{4}$

Total number of outcomes $= 2 \times 2 = 4$

Favourable number of outcomes $= 1 (i.e. HH)$

Required probability $= \frac{1}{4}$

(Q.11-Q.15) Fill in the blanks.

11. The sum of the probabilities of all the elementary events of an experiment is .... [1]

Ans : Sum of the probability of all elementary events of an experiment is 1.

12. If the volume and the surface area of a solid sphere are numerically equal, then its radius is .... [1

Ans :

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$
$$r = 3 \text{ units}$$

13. The perimeter of a sector of a circle of radius r cm and of central angle θ (in degrees) is .... [1]

Ans :

Perimeter of a sector $= \frac{\theta}{360} \times 2\pi r + 2r$

14. If $\sin\alpha = \frac{3}{5}$ and $\alpha + \beta = 90^\circ$, then the value of $\cos\beta$ is .... [1]

Ans :

$$\cos\beta = \cos(90^\circ - \alpha) \qquad (\alpha + \beta = 90^\circ)$$
$$= \sin\alpha = \frac{3}{5} \qquad (\cos(90^\circ - \alpha) = \sin\alpha)$$
$$\cos\beta = \frac{3}{5}$$

or

The value of $\sec^2 18^\circ - \cot^2 72^\circ$ is ....

Ans :

$$\sec^2 18^\circ - \cot^2 72^\circ = \sec^2 18^\circ - \cot^2(90^\circ - 18^\circ)$$
$$= \sec^2 18^\circ - \tan^2 18^\circ = 1$$

15. 9th term of the $A.P. \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}$ .... is .... [1]

Ans :

Here, $a = \frac{3}{4},\ d = \frac{1}{2}$

9th term $= \frac{3}{4} + (9-1) \times \frac{1}{2}$
$$= \frac{3}{4} + 4 = \frac{19}{4}$$

(Q.16-Q.20) Answer the following

16. Find the value of k for which the following pair of linear equations has a unique solution: $2x + 3y = 7; (k-1)x + (k+2)y = 3k$. [1]

Ans :

For unique solution, $\frac{2}{k-1} \neq \frac{3}{k+2}$
$$2k + 4 \neq 3k - 3$$
$$k \neq 7$$

Hence, the given pair of linear equations will have unique solution for all real values of k except 7.

17. Find the nature of the roots of quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$. [1]

Ans :

Given, $2x^2 - \sqrt{5}x + 1 = 0$
$$D = (\sqrt{5})^2 - 4 \times 2 \times 1$$
$$= 5 - 8$$
$$= -3$$
$$D < 0,$$
therefore, given equation has no real roots.

18. What is the probability that a non-leap year has 53 Monday? [1]

Ans :

There are 365 days in a non-leap year.

365 days = 52 weeks +1 day

One day can be M,T,W,Th,F,S,Su = 7 ways

P(53 Mondays in non-leap year) $= \frac{1}{7}$

or

A die is thrown once. Find the probability of getting a prime number.

Ans :

$$\text{Total number of outcomes} = 6(1,2,3,4,5 \text{ or } 6)$$
$$\text{Favourable number of outcomes} = 3(2,3,5)$$
$$\text{P(prime number)} = \frac{3}{6} = \frac{1}{2}.$$

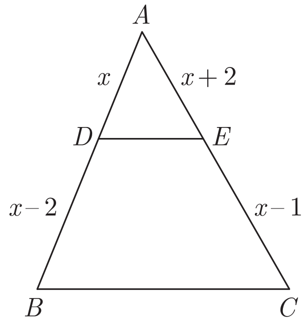
19. Find the mode of the data, whose mean and median are given by 10.5 and 11.5 respectively. [1]

Ans :

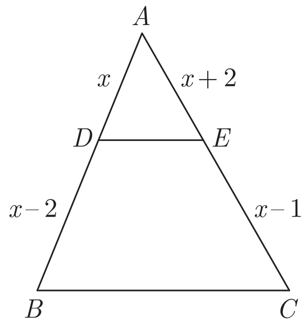
$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$
$$= 3 \times 11.5 - 2 \times 10.5$$
$$= 34.5 - 21 = 13.5$$

Hence, $\text{Mode} = 13.5$

20. In the given figure, $DE || BC$, If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x. [1]



Ans :



$$DE \parallel BC$$

By Basic Proportionality Theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{x}{x-2} = \frac{x+2}{x-1}$$
$$x(x-1) = (x-2)(x+2)$$
$$x^2 - x = x^2 - 4$$
$$-x = -4$$
$$x = 4.$$

Section B

21. Find HCF and LCM of 90 and 144 by method of prime factorisation. [2]

Ans :

$$90 = 2 \times 3 \times 3 \times 5$$

and $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$$\text{HCF} = 2 \times 3 \times 3 = 18$$

and $\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 720$

Hence, HCF $= 18$ and LCM $= 720$.

22. Find the values of a and b for which the following pair of linear equations has infinitely many solutions [2]
$3x - (a+1)y = 2b - 1$; $5x + (1-2a)y = 3b$.

Ans :

Given,

$$3x - (a+1)y - (2b-1) = 0$$

and $5x + (1-2a)y - 3b = 0$

For infinitely many solutions:

$$\frac{3}{5} = \frac{-(a+1)}{+(1-2a)} = \frac{-(2b-1)}{-3b}$$

When $\frac{3}{5} = \frac{-(a+1)}{1-2a}$

$$-5a - 5 = 3 - 6a$$
$$a = 8$$

and when $\frac{3}{5} = \frac{2b-1}{3b}$

$$10b - 5 = 9b$$
$$b = 5.$$

Hence, $a = 8$

and $b = 5$.

or

Solve:

$$\frac{2}{x} + \frac{3}{y} = 13$$
$$\frac{5}{x} - \frac{4}{y} = -2$$

Ans :

Let $\frac{1}{x} = a$ and

$\frac{1}{y} = b$, then given equations are

$$2a + 3b = 13 \quad \text{...(i)}$$
$$5a - 4b = -2 \quad \text{...(ii)}$$

From (ii), $b = \frac{5a+2}{4}$

Putting the value of b in (i), we get

$$2a + 3\left(\frac{5a+2}{4}\right) = 13$$
$$8a + 15a + 6 = 52$$
$$23a = 46$$
$$a = 2$$

So, $b = \frac{5 \times 2 + 2}{4}$

$$b = 3$$
$$x = \frac{1}{a} = \frac{1}{2}$$

and $y = \frac{1}{b} = \frac{1}{3}$

So $x = \frac{1}{2}$,

$$y = \frac{1}{3}$$

23. Without using trigonometric tables, evaluate the following: [2]

$$(\cos^2 25^\circ + \cos^2 65^\circ) + \operatorname{cosec}\theta . \sec(90^\circ - \theta) - \cot\theta \tan(90^\circ - \theta)$$

Ans :

$$(\cos^2 25^\circ + \cos^2 65^\circ) + \operatorname{cosec}\theta . \sec(90^\circ - \theta) - \cot\theta . \tan(90^\circ - \theta)$$

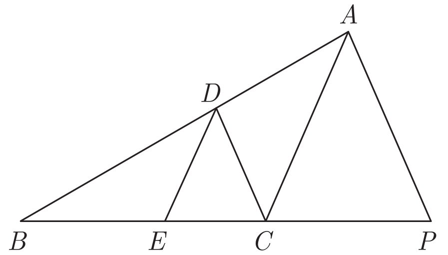
$$= \cos^2 25^\circ + \cos^2(90^\circ - 25^\circ) + \operatorname{cosec}\theta . \operatorname{cosec}\theta - \cot\theta . \cot\theta$$

$$= \cos^2 25^\circ + \sin^2 25^\circ + \operatorname{cosec}^2\theta - \cot^2\theta$$

$$= 1 + 1 = 2.$$

24. In the given figure: $DE || AC$ and $\frac{BE}{EC} = \frac{BC}{CP}$.

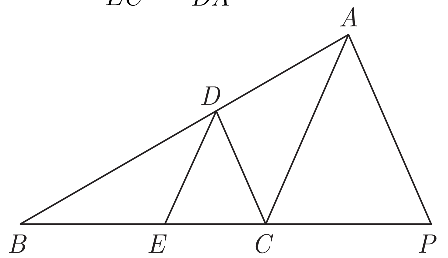
Prove that $DC || AP$. [2]



Ans :

In ΔABC $\quad DE \parallel AC$

$$\frac{BE}{EC} = \frac{BD}{DA} \quad \text{(by B.P.T.)}$$

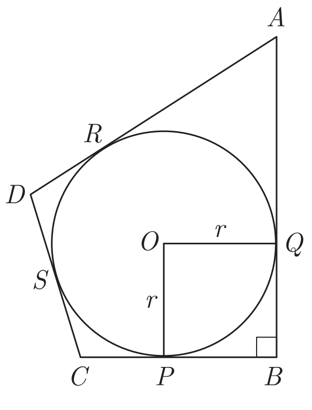


But $$\frac{BE}{EC} = \frac{BC}{CP} \quad \text{(Given)}$$

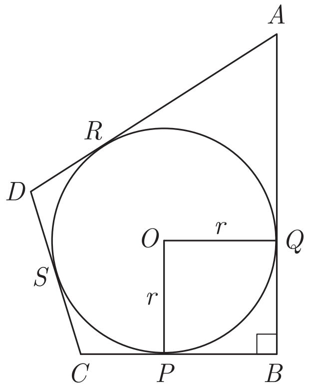
$$\frac{BD}{DA} = \frac{BC}{CP}$$

$$DC \parallel AP \quad \text{(converse of B.P.T.)}$$

25. In the given figure, a circle is inscribed in a quadrilateral $ABCD$ in which $\angle B = 90^\circ$. If $AD = 23$ cm, $AB = 29$ cm and $DS = 5$ cm, find the radius (r) of the circle. [2]



Ans :



$$AR = AQ,$$
$$DR = DS$$
$$BP = BQ \quad \text{(lengths of tangents)}$$

But $\quad DS = 5$ cm

$$DR = 5 \text{ cm}.$$
$$AR = AD - DR$$
$$= 23 \text{ cm} - 5 \text{ cm} = 18 \text{ cm}$$
$$AQ = 18 \text{ cm}.$$
$$BQ = AB - AQ$$
$$= 29 \text{ cm} - 18 \text{ cm} = 11 \text{ cm}.$$

As AB is tangent to the circle at Q,

$$OQ \perp AB$$
$$\angle OQB = 90^\circ.$$

Also $\quad \angle B = 90^\circ$ (Given)

$OQBP$ is a rectangle.

But $\quad BP = BQ$

$OQBP$ is a square.

Radius, $\quad r = OQ = BQ = 11$ cm.

26. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of getting neither a red card nor a queen. [2]

Ans :

Total number of outcomes $= 52$

Number of red card or queen in the pack

$$= 26 + 2$$

(Two red queens are already included in 26 cards)

$$= 28$$

The number of cards in the pack which is neither red nor queen $\quad n = 52 - 28 = 24$

$$P(\text{neither red nor queen}) = \frac{24}{52} = \frac{6}{13}$$

or

Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is a prime number.

Ans :

When two dice are thrown together, the total number of outcomes$= 36$, and all the outcomes are equally likely.

The outcomes favourable to the event 'product of numbers on two dice is prime' are (1, 2), (2, 1), (1, 3), (3, 1), (1, 5) and (5, 1). These are 6 in numbers.

$$P \text{ (neither red nor queen)} = \frac{24}{52} = \frac{6}{13}$$

Section C

27. If two zeroes of the polynomial $x^4 + 3x^3 - 20x^2 - 6x + 36$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeroes of the polynomial. [2]

Ans :

$\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of the given polynomial.

$(x-\sqrt{2})(x+\sqrt{2})$ *i.e.* (x^2-2) is a factor of the given polynomial.

$$\begin{array}{r|l} & x^2+3x-18 \\ \hline x^2-2 & x^4+3x^3-20x^2-6x+36 \\ & x^4 \qquad -2x^2 \\ \hline & -3x^3-18x^2-6x+36 \\ & -3x^3 \qquad -6x \\ \hline & -18x^2 \qquad +36 \\ & -18x^2 \qquad +36 \\ \hline & 0 \end{array}$$

$$x^4+3x^3-20x^2-6x+36 = (x^2-2)(x^2+3x-18)$$

To find the other two zeroes, we proceed as follows:

$$x^2+3x-18 = 0$$
$$(x+6)(x-3) = 0$$
$$x+6 = 0$$
or $$x-3 = 0$$
$$x = -6$$
or $$x = 3$$

Hence, other zeroes are -6 and 3.

28. How many terms of the $A.P. -6, \frac{11}{2}, -5$, ... are needed to give the sum -25? Explain the double answer. [3]

Ans :

Here, $$a = -6,$$
$$d = -\frac{11}{2}-(-6) = -\frac{11}{2}+6 = \frac{1}{2},$$
$$S_n = -25.$$

We are required to find n.

Using $$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$-25 = \frac{n}{2}\left[2\times(-6)+(n-1)\times\frac{1}{2}\right]$$
$$\frac{n}{2}\left[-12+\frac{(n-1)}{2}\right] = -25$$
$$n(-12\times 2+(n-1)) = -25\times 4$$
$$n(-24+n-1) = -100$$
$$n^2-25n+100 = 0$$
$$(n-5)(n-20) = 0$$
$$n = 5, 20$$

Both values of n being positive integers are valid.

We get the double answer because sum from 6th term to 20th term is zero, as some terms are negative and some terms are positive.

or

The 19th term of an AP is equal to three times its 6th term. If its 9th term is 19, find the AP.

Ans :

Let the first term and common difference of AP be a and d respectively.

Given, $$T_{19} = 3T_6$$
and $$T_9 = 19$$
$$a+18d = 3(a+5d)$$
and $$a+8d = 19$$
$$2a = 3d$$
and $$a+8d = 19$$
$$a = \frac{3d}{2}$$

Putting $$a = \frac{3d}{2}$$
in $a+8d = 19$, we get
$$\frac{3d}{2}+8d = 19$$
$$\frac{19d}{2} = 19$$
$$d = 2$$
$$a = \frac{3\times 2}{2} = 3.$$

Hence, AP is 3, 5, 7, 9, ...

29. The father's present age is six times his son's ages. Four years hence the age of the father will be four times his son's age. Find the present ages of the father and son. [3]

Ans :

Let the father's present age be x years and the son's present age be y years.

According to given, $$x = 6y \quad ...(i)$$
4 years later,
$$\text{father's age} = (x+4) \text{ years}$$
$$\text{and son's age} = (y+4) \text{ years}$$
$$(x+4) = 4(y+4)$$
$$x-4y = 12 \quad ...(ii)$$

Putting the value of x from (i) in (ii), we get
$$6y-4y = 12$$
$$2y = 12$$
$$y = 6$$
$$x = 6\times 6 = 36$$

Hence, father's present age = 36 years

and son's present age = 6 years

or

The ratio of incomes of two persons is $9:7$ and the ratio of their expenditures is $4:3$. If each of them manages to save ₹2000 per month, find their monthly incomes.

Ans :

Let the incomes per month of two persons be ₹x and ₹y respectively. As each person saves ₹2000 per month, so their expenditures are ₹$(x-2000)$ and ₹$(y-2000)$ respectively.

According to given, we have

$$\frac{x}{y} = \frac{9}{7} \; i.e. \; 7x - 9y = 0 \quad ...(i)$$

and $$\frac{x-2000}{y-2000} = \frac{4}{3} \; i.e. \; 3x - 4y + 2000 = 0 \quad ...(ii)$$

Multiplying equation (i) by 3 and equation (ii) by 7, we get

$$21x - 27y = 0 \quad ...(iii)$$

and $$21x - 28y + 14000 = 0 \quad ...(iv)$$

Subtracting equation (iv) from equation (iii), we get

$$y - 14000 = 0$$

$$y = 14000.$$

Substituting this value of y in (i), we get

$$7x - 9 \times 14000 = 0$$

$$x = 18000.$$

Hence, the monthly incomes of the two persons are ₹18000 and ₹14000 respectively.

30. ABC is a right triangle, right angled at C. If P is the length of perpendicular from C to AB and a, b, c have usual meanings, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$. [3]

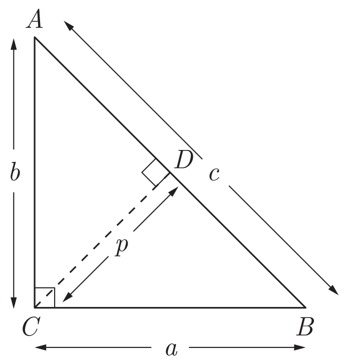
Ans :

In ΔACB and ΔCDB,

$$\angle ACB = \angle CDB \quad \text{(both } 90^\circ)$$

$$\angle B = \angle B \quad \text{(common)}$$

$$\Delta ACB \sim \Delta CDB \quad \text{(by AA similarity criterion)}$$



$$\frac{AC}{CD} = \frac{AB}{CB}$$

$$\frac{b}{p} = \frac{c}{a}$$

$$\frac{1}{p} = \frac{c}{ab}$$

Squaring both the sides, we get

$$\frac{1}{p^2} = \frac{c^2}{a^2 b^2} \quad ...(i)$$

Now, ΔABC is a right angled triangle.

By Pythagoras theorem,

$$AB^2 \; BC^2 + AC^2$$

$$c^2 = a^2 + b^2$$

Putting value of c^2 in (i), we get

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2}$$

$$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

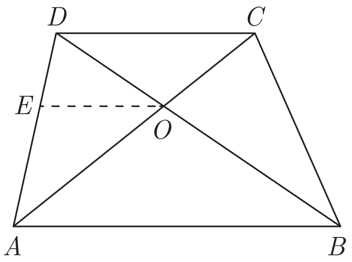
or

If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Ans :

Given, $$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\frac{AO}{CO} = \frac{BO}{DO}$$



Draw $EO \parallel AB$

in ΔDAB, $EO \parallel AB$,

$$\frac{DE}{AE} = \frac{DO}{BO} \quad \text{(by B.P.T.)}$$

$$\frac{AE}{DE} = \frac{BO}{DO}, \text{ But } \frac{BO}{DO} = \frac{AO}{CO}$$

$$\frac{AE}{DE} = \frac{AO}{CO}$$

$$EO \parallel DC \quad \text{(by converse of B.P.T.)}$$

Thus $EO \parallel AB$

and $EO \parallel DC$

$AB \parallel DC$

$ABCD$ is a trapezium.

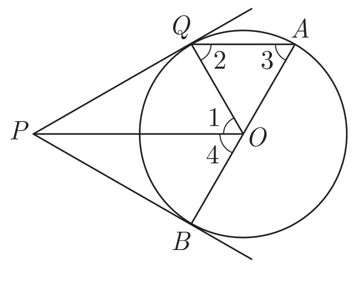
31. PQ is a tangent to a circle with centre O at the point Q. A chord QA is drawn parallel to PO. If AOB is a diameter of the circle, prove that PB is tangent to the circle at the point B. [3]

Ans :

Given a circle with centre O and PQ is tangent to the circle at the point Q from an external point P. Chord QA ia parallel to PO and AOB is a diameter.

We need to prove that PB is tangent to the circle at the point B.

Join OQ and mark the angles as shown in the adjoining figure.



As $QA \parallel PO$,

$$\angle 1 = \angle 2 \quad \text{(alt. } \angle s)$$

and $\angle 4 = \angle 3$ (corres. $\angle s$)

But $\angle 2 = \angle 3$ (In $\Delta OAQ, OA = OQ$ being radii)

$\angle 1 = \angle 4$

In ΔOPB and ΔOPQ,

$OB = OQ$ (radii of same circle)

$\angle 1 = \angle 4$ (proved above)

$OP = OP$ (common)

$\Delta OPB \cong \Delta OPQ$ (SAS congruence rule)

$\angle OBP = \angle OQP$ (c.p.c.t.)

$\angle OBP = 90^\circ$ (tangent is $\perp$ to radius, $OQ \perp PQ$)

$OB \perp PB$ *i.e.*

radius is perpendicular to PB at point B.

Therefore, PB is tangent to the circle at the point B.

32. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm $(r < 14\text{ cm})$. If the volume of bucket is 5390 cm^3, then find the value of r. $\left[\text{Use } \pi = \frac{22}{7}\right]$ [3]

Ans :

Given $h = 15\text{ cm}$,

$R = 14\text{ cm}$,

'r' $= r$ cm

and

volume of bucket $= 5390\text{ cm}^3$

volume of bucket = volume of frustum of cone

$$5390 = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 15(14^2 + r^2 + 14r)$$

$$= 5390$$

$$196 + r^2 + 14r = \frac{5390 \times 3 \times 7}{22 \times 15} = 343$$

$$r^2 + 14r + 196 - 343 = 0$$

$$r^2 + 14r - 147 = 0$$

$$(r + 21)(r - 7) = 0$$

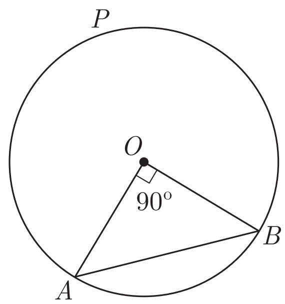
$$r = -21$$

or $r = 7$

r cannot be negative

Radius $r = 7$ cm.

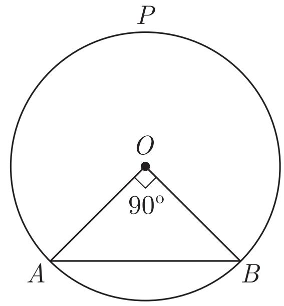
33. Find the area of the major segment APB in adjoining figure, of a circle of radius 35 cm and $\angle AOB = 90^\circ$. $\left[\text{Use } \pi = \frac{22}{7}\right]$. [3]



Ans :

Given $r = 35$ cm

and $\angle AOB = 90^\circ$



Area of minor segment

$$= \text{area of minor sector} - \text{area } (\Delta OAB)$$

$$= \frac{90^\circ}{360^\circ} \times \pi r^2 - \frac{1}{2} OA \times OB$$

$$= \frac{1}{4} \times \frac{22}{7} \times (35)^2 - \frac{1}{2} \times 35 \times 35$$

$$= (35)^2\left(\frac{11}{14} - \frac{1}{2}\right) = 1225 \times \frac{4}{14} = 350\text{ cm}^2$$

Area of circle $= \pi r^2 = \frac{22}{7} \times (35)^2 = 3850\text{ cm}^2$

Now area of major segment

$$= \text{area of circle} - \text{area of minor segment}$$

$$= (3850 - 350)\text{ cm}^2 = 3500\text{ cm}^2$$

Hence, the area of major segment

$$= 3500\text{ cm}^2$$

34. Prove that: $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$ [3]

Ans :

$$\text{LHS} = \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$$

$$= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$$

$$= \frac{\tan A + \sec A - (\sec A + \tan A - (\sec A - \tan A))}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A)(1 - \sec A + \tan A)}{\tan A - \sec A + 1}$$

$$= \tan A + \sec A$$

$$= \frac{\sin A}{\cos A} + \frac{1}{\cos A}$$

$$= \frac{\sin A + 1}{\cos A} = \frac{1 + \sin A}{\cos A} = \text{RHS}$$

Section D

35. Prove that $\sqrt{5}$ is an irrational number and hence show that $2 + \sqrt{5}$ is also an irrational number. [4]

Ans :

Let $\sqrt{5}$ be a rational number, then

$\sqrt{5} = \frac{p}{q}$, where p, q are integers,

$q \neq 0$ and p, q have no common factors (except 1)

$$5 = \frac{p^2}{q^2}$$

$$p^2 = 5q^2$$

As 5 divides $5q^2$, so 5 divides p^2, but 5 is prime.
5 divides p

Let $p = 5\text{m}$, where m is an iterger.

Substituting this value of p in (i), we get

$$(5m)^2 = 5q^2$$

$$25m^2 = 5q^2$$

$$5m^2 = q^2$$

As 5 divides $5m^2$, so 5 divides q^2, but 5 is prime
5 divides q
Thus p and q have a common factor 5. This contradicts that p and q have no common factors (except 1)
Hence, $\sqrt{5}$ is not a rational number.
So, we conclude that $\sqrt{5}$ is an irrational number.
Let $2+\sqrt{5}$ be a rational number, say r

Then,

$$2+\sqrt{5} = r$$

$$\sqrt{5} = r-2$$

As r is rational, $r-2$ is rational
$\sqrt{5}$ is rational
But this contradicts the fact that $\sqrt{5}$ is irrational.
Hence, our assumption is wrong. Therefore, $2+\sqrt{5}$ is an irrational number.

36. If two vertices of an equilateral triangle are (3, 0) and (6, 0), find the third vertex. [4]

Ans :

Given vertices are $A(3,0)$ and $B(6,0)$ and let third vertex be $C(x,y)$, then

$$AB = \sqrt{(6-3)^2+(0-0)^2} = \sqrt{3^2} = 3$$

$$AC = \sqrt{(x-3)^2+(y-0)^2} = \sqrt{x^2+y^2-6x+9} \text{ and}$$

$$BC = \sqrt{(x-6)^2+(y-0)^2} = \sqrt{x^2+y^2-12x+36}$$

As ΔABC is equilateral,

$$AB = AC = BC$$

$$AB = AC$$

and $$AC = BC$$

$$\sqrt{x^2+y^2-6x+9} = 3$$

$$x^2+y^2-6x = 0$$

and

$$\sqrt{x^2+y^2-6x+9} = \sqrt{x^2+y^2-12x+36}$$

$$x^2+y^2-6x+9 = x^2+y^2-12x+36$$

$$6x = 27$$

$$x = \frac{9}{2}$$

Substituting this value of x in equation (i), we get

$$\left(\frac{9}{2}\right)^2+y^2-6\left(\frac{9}{2}\right) = 0$$

$$y^2 = 27-\frac{81}{4}$$

$$y^2 = \frac{27}{4}$$

$$y = \pm\frac{-3\sqrt{3}}{2}$$

Third vertex of equilateral triangle is $\left(\frac{9}{2},\frac{3\sqrt{3}}{2}\right)$ or $\left(\frac{9}{2},\frac{-3\sqrt{3}}{2}\right)$.

or

The mid-points D,E and F of the sides AB,BC and CA of a triangle are (3, 4), (8, 9) and (6, 7) respectively. Find the coordinates of the vertices of the triangle.

Ans :

Let the vertices A,B and C of the triangle ABC be (x_1,y_1), (x_2,y_2) and (x_3,y_3) respectively.
Since points D and F are mid-points of the sides AB and AC respectively, by mid-point theorem,

$$DF \parallel BC$$

and $DF = \frac{1}{2}BC$ but E is mid-point of BC, so

$$DF \parallel BE$$

and $$DF = BE$$

Therefore, $DBEF$ is a parallelogram.
Similarly, $DECF$ and $DEFA$ area parallelograms.
Since the diagonals of a parallelogram bisect each other,
mid-points of diagonals BF and DE are same.

$$\frac{x_2+6}{2} = \frac{3+8}{2}$$

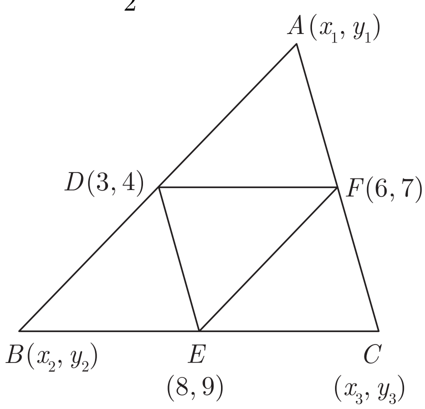
and $$\frac{\frac{1}{2}+7}{2} = \frac{4+9}{2}$$

$$x_2+6 = 11$$

and $$\frac{1}{2}+7 = 13$$

$$x_2 = 5$$

and $$\frac{1}{2} = 6.$$



Therefore, the point B is (5, 6).
As $DECF$ is a parallelogram, mid-points of CD and EF are same.

$$\frac{x_3+3}{2} = \frac{8+6}{2}$$

and $$\frac{\frac{1}{3}+4}{2} = \frac{9+7}{2}$$

$$x_3+3 = 14$$

and $$\frac{1}{3}+4 = 16.$$

$$x_3 = 11$$

and $$\frac{1}{3} = 12.$$

Therefore, the point C is (11, 12).

Also $DEFA$ is a parallelogram, mid-points of AE and DF are same.

$$\frac{x_1+8}{2} = \frac{3+6}{2}$$

$$\frac{\frac{1}{1}+9}{2} = \frac{4+7}{2}$$

$$x_1 + 8 = 9$$

and $$\frac{1}{3} + 4 = 16.$$

$$x_3 = 11$$

and $$\frac{1}{3} = 12.$$

Therefore, the point C is (11, 12).

Also $DEFA$ is a parallelogram, mid-points of AE and DF are same.

$$\frac{x_1+8}{2} = \frac{3+6}{2}$$

and $$\frac{\frac{1}{1}+9}{2} = \frac{4+7}{2}$$

$$x_1 + 8 = 9$$

and $$\frac{1}{1} + 9 = 11$$

$$x_1 = 1$$

and $$\frac{1}{1} = 2.$$

Therefore, the point A is (1, 2).

Hence, the vertices of the triangle are $A(1,2)$, $B(5,6)$ and $C(11,12)$.

37. While boarding an aeroplane, a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalise the injured and so the plane started late by 30 minutes. To reach the destination, 1500 km away in time, the pilot increased the speed by 100 km/h. Find the original speed/hour of the plane. [4]

Ans :

Let the original speed of the aeroplane be be x km/h.

Time taken to cover the distance of 1500 km

$$= \frac{1500}{x} \text{ hours}$$

New speed of the aeroplane $= (x+100)$ km/h

Time taken to cover the distance of 1500 km at new speed $= \frac{1500}{x+100}$ hours

Since the aeroplane takes 30 minutes $i.e. \frac{30}{60}$

$$= \frac{1}{2} \text{ hours less time.}$$

$$\frac{1500}{x} - \frac{1500}{x+100} = \frac{1}{2}$$

$$\frac{1500(x+100) - 1500x}{x+(x+100)} = \frac{1}{2}$$

$$x^2 + 100x = 300000$$

$$x^2 + 100x - 300000 = 0$$

$$x^2 + 600x - 500x - 300000 = 0$$

$$(x-500)(x+600) = 0$$

$$x = 500$$

or $$x = -600$$

But speed cannot be negative.

Hence, the original speed of the aeroplane

$$= 500 \text{ km/h}.$$

38. Draw a pair of tangents to a circle of radius 3 cm which are inclined at an angle of 60°. If the tower is 50 m high, find the height of the building. [4]

Ans :

Steps of construction:

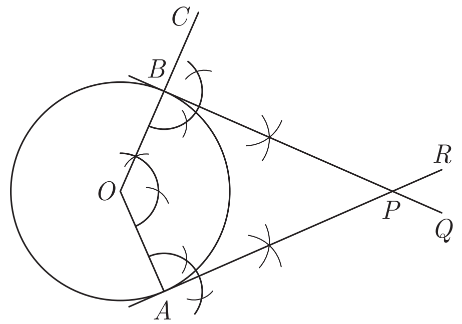
(i) Draw a circle of radius 3 cm with O as its centre.

(ii) Draw any radius OA

(iii) At O, construct $\angle AOC = 120°$ to meet the chircle at B.

(iv) At A construct $\angle OAR = 90°$.

(v) At B, construct $\angle OBQ = 90°$ to meet AR at P.



Then PA and PB are tangents to the circle inclined at an angle of 60° to each other.

Justification:

As $\angle APB$ and $\angle AOB$ are supplementary, so $\angle APB = 60°$.

39. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° If the tower is 50m high, find the height of the building [4]

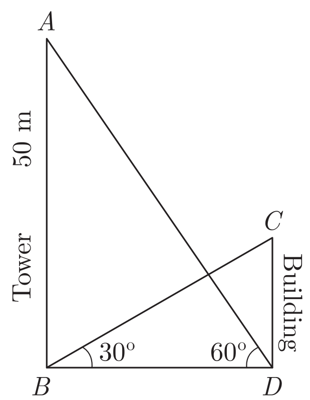
Ans :

Let $CD = h$

meters be the height of the building and AB be the tower, then $AB = 50$ m.

Let $BD = d$

metres be the distance between the foot of the tower and the foot of the building.



Given, $\angle CBD = 30°$

and $\angle ADB = 60^\circ$

From right angled $\Delta\, CBD$, we get

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{d}$$

$$d = \sqrt{3}\,h \qquad ...(i)$$

From right angled $\Delta\, ABD$, we get

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{50}{d}$$

$$d = \frac{50}{\sqrt{3}} \qquad ...(ii)$$

From (i) and (ii), we get

$$\sqrt{3}\,h = \frac{50}{\sqrt{3}}$$

$$h = \frac{50}{3}$$

Hence, the height of the building is $\frac{50}{3}$ m.

or

The angles of depression of two ships from the top of a lighthouse and on the same side of it are found to be 45° and 30°. If the ships are 200 m apart, find the height of the lighthouse.

Ans :

Let the height of the lighthouse AB be h metres and C, D be the positions of two ships. The angles of depressions are shown in the adjoining figure.

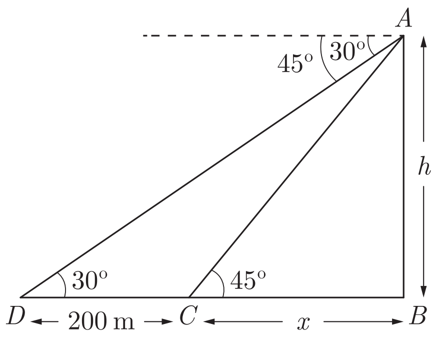
Then $\angle ACB = 45^\circ$

and $\angle ADB = 30^\circ$

Given CD 200 m,

let $BC = x$ metres.

From right angled $\Delta\, ABC$, we get



$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{h}{x}$$

$$x = h \qquad ...(i)$$

From right angled $\Delta\, ADB$, we get

$$\tan 30^\circ = \frac{AB}{DB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{200 + x}$$

$$h\sqrt{3} = 200 + h \qquad ...[\text{from (i)}]$$

$$h(\sqrt{3} - 1) = 200$$

$$h = \frac{200}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$h\sqrt{3} = \frac{200(\sqrt{3}+1)}{3-1} = \frac{200(\sqrt{3}+1)}{2}$$

$$h = 100(\sqrt{3}+1)\,\text{m}$$

Hence, the height of the lighthouse is $100(\sqrt{3}+1)$ m.

40. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequencies f_1 and f_2: [4]

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Ans :

Given, sum of all frequencies $= 50$

$$5 + f_1 + 10 + f_2 + 7 + 8 = 50$$

$$f_1 + f_2 = 20 \qquad ...(i)$$

Here, all classes are of equal size

$$h = 20$$

We shall use step-deviation method. Taking assumed mean $a = 70$,

construct the table as under:

Classes	Mid-value (x_i)	$u_i = \frac{x_i - a}{h}$	Frequency (f_i)	$f_i u_i$
0-20	10	-3	5	-15
20-40	30	-2	f_1	$-2f_1$
40-60	50	-1	10	-10
60-80	70	0	f_2	0
80-100	90	1	7	7
100-120	110	2	8	16
Total			$30 + f_1 + f_2$	$-2f_1 - 2$

$$\text{Mean} = a + h \times \frac{\sum f_i u_i}{\sum f_i} = 62.8 \text{ (given)}$$

$$70 + 20 \times \frac{-2f_1 - 2}{50} = 62.8$$

$$7.2 = \frac{2}{5}(2f_1 + 2)$$

$$2(2f_1 + 2) = 7.2 \times 5$$

$$4(f_1 + 1) = 36$$

$$f_1 + 1 = 9$$

$$f_1 = 8$$

Substituting this value of f_1 in (i), we get

$$8 + f_2 = 20$$

$$f_2 = 12.$$

Hence, $f_1 = 8$

and $f_2 = 12$

or

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mode and mean of the data:

Monthly consumption (in units)	65-85	85-105	105-125	125-145	145-165	165-185	185-205
No. of consumers	4	5	13	20	14	8	4

Ans :

The given distribution is continuous and all classes are of equal size, $h = 20$.

Taking assumed mean $a = 135$,
construct the table as under:

Monthly consumption (in units)	Mid -value (x_i)	$u_i = \frac{x_i - a}{h}$	No. of consumers (f_i)	$f_i u_i$	*c.f.*
65-85	75	-3	4	-12	4
85-105	95	-2	5	-10	9
105-125	115	-1	13	-13	22
125-145	135	0	20	0	42
145-165	155	1	14	14	56
165-185	175	2	8	16	64
185-205	195	3	4	12	68
Total			68	7	

For median:

Cumulative frequency of the class 125-145 is 42, which is greater than $\frac{68}{2}$ *i.e.* 34 and nearest to it, so 125-145 is the median class.

Here $l = 125$,
c.f. of the class preceding the median class is 22,

$f = 20$

and $h = 20$.

Median $= l + \frac{\frac{n}{2} - c.f.}{f} \times h$

$= 125 + \frac{34 - 22}{20} \times 20$

$= 125 + 12 = 137$.

For mode:
The class 125-145 has maximum frequency, so it is the modal class.

Here $l = 125,\ f_1 = 20,\ f_0 = 13,\ f_2 = 14$

and $h = 20$.

Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 125 + \frac{20 - 13}{2 \times 20 - 14 - 14} \times 20$

$= 125 + \frac{7 \times 20}{13}$

$= 135.77$ (approximately)

For mean

Mean $= a + h \times \frac{\sum f_i u_i}{\sum f_i}$

$= 135 + 20 \times \frac{7}{68}$

$= 135 + \frac{35}{17}$

$= 137.06$ (approximately)

Hence, median $= 137$ units,

mode $= 135.77$ units (approx.)

and mean $= 137.06$ units (approx.)