CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-11

Time : 3 Hours **Maximum Marks : 80**

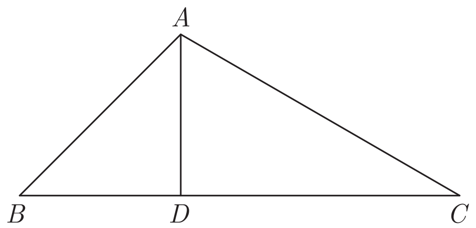
General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. In the adjoining figure, $\angle BAC = 90^\circ$ and $AD \perp BC$, then [1]



(a) $BC \cdot CD = BC^2$ (b) $AB \cdot AC = BC^2$
(c) $BD \cdot CD = AD^2$ (d) $AB \cdot AC = AD^2$

Ans : (c) $BD \cdot CD = AD^2$

$\because$ $\angle BAC = 90^\circ$

and $AD \perp BC$

$\therefore$ $\Delta DBA \sim \Delta DAC$ (AA similarity)

$$\frac{AD}{DC} = \frac{BD}{AD}$$

$$AD^2 = BD \times DC$$

2. In $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true? [1]

(a) $BC \cdot EF = AC \cdot DE$ (b) $AB \cdot AC = BC^2$
(c) $BC \cdot DE = AB \cdot EF$ (d) $BC \cdot DE = AB \cdot FD$

Ans : (c) $BC \cdot DE = AB \cdot EF$

$\because$ $\Delta ABC \sim \Delta EDF$

$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

$$BC \cdot ED \neq AB \cdot EF$$

3. The value of $3\tan^2 26^\circ - 3\,\text{cosec}^2 64^\circ$ is [1]

(a) 0 (b) 3
(c) -3 (d) -1

Ans : (c) -3

$$3\tan^2 26^\circ - 3\sec^2 26^\circ = 3\tan^2 26^\circ - 3\sec^2 26^\circ$$

($\because$ $\text{cosec}\, 64^\circ = \sec 26^\circ$)

$$= -3(\sec^2 26^\circ - 3\sec^2 26^\circ)$$

$$= -3$$

4. If $\cos A = \frac{3}{5}$, then the value of $\tan A$ is [1]

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$
(c) $\frac{4}{3}$ (d) $\frac{5}{4}$

Ans : (c) $\frac{4}{3}$

$$\cos A = \frac{3}{5}$$

$$\sec A = \frac{5}{3}$$

Now, $\tan A = \sqrt{\sec^2 A - 1} = \sqrt{\left(\frac{5}{3}\right)^2 - 1}$

$$= \frac{4}{3}$$

5. If the ratio of the circumferences of two circles is 4 : 9, then the ratio of their areas is [1]

(a) 9 : 4 (b) 4 : 9
(c) 2 : 3 (d) 16 : 81

Ans : (d) 16 : 81

$$\frac{2\pi r_1}{2\pi r_2} = \frac{4}{9}$$

$$\frac{r_1}{r_2} = \frac{4}{9}$$

Now, $\frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

6. If the sector of a circle of diameter 10 cm subtends an angle of 144° at the centre, then the length of the arc of the sector is [1]

(a) 2π cm (b) 4π cm
(c) 5π cm (d) 6π cm

Ans : (b) 4π cm

$$2r = 10\,\text{cm}$$

$$r = 5\,\text{cm}$$

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r = \frac{144}{360} \times 2\pi \times 5$$

$$= 4\pi\,\text{cm}$$

7. If the classes of a frequency distribution are $1-10$, $11-20$, $21-30$, ..., $61-70$, then the upper limit of the class $11-20$ is [1]

(a) 20 (b) 21
(c) 19.5 (d) 20.5

Ans : (d) 20.5

Given classes are discontinuous.

Adjustment factor $= 0.5$

By converting the given data into continuous classes, we get $0.5-10.5$, $10.5-20.5$, $20.5-30.5$, ...

8. If the probability of an event is p, then the probability of its complementary event will be [1]

(a) $p-1$ (b) p
(c) $1-p$ (d) $1-\frac{1}{p}$

Ans : (c) $1-p$

$$\because \quad P(\overline{E}) = 1-P(E)$$
$$P(\overline{E}) = 1-p$$

9. For some integer q, every odd integer is of the form [1]

(a) q (b) $q+1$
(c) $2q$ (d) $2q+1$

Ans : (d) $2q+1$

Every odd integer $= 2q+1$

for $q \in I$

10. If the base area of a cone is 51 cm^2 and its volume is 85 cm^3, then its vertical height is [1]

(a) 3.5 cm (b) 4 cm
(c) 4.5 cm (d) 5 cm

Ans : (d) 5 cm

$$\text{Vertical Height} = \frac{3 \times 85\,\text{cm}^3}{51\,\text{cm}^2} = 5\,\text{cm}$$

(Q.11-Q.15) Fill in the blanks.

11. If the ratio of the corresponding sides of two similar triangles is 7 : 11, then the ratio of their corresponding altitudes is .... [1]

Ans : For two similar triangles,

The ratio of areas = Square of the ratio of their corresponding sides

Also, The ratio of areas = Square of the ratio of their corresponding altitudes

$\therefore$ Ratio of altitudes = Ratio of sides

$= 7:11$

12. The perimeter of a semicircular protactor of diameter 14 cm is .... [1]

Ans :

Given,

$$\text{Diameter,} \quad 2r = 14\,\text{cm}$$
$$r = 7$$
$$\text{Perimeter} = \pi r + 2r$$
$$= \frac{22}{7} \times 7 + 14$$
$$= 36\,\text{cm}$$

or

If the area of a circle is 616 cm^2, then its circumference is ....

Ans :

As we know that,

$$\text{Area of circle} = \pi r^2$$
$$\text{Now,} \quad \pi r^2 = 616\,\text{cm}^2$$
$$r^2 = \frac{616}{22} \times 7$$
$$r^2 = 196$$
$$r = 14\,\text{cm}$$
$$\therefore \quad \text{Circumference} = 2\pi r$$
$$= 2 \times \frac{22}{7} \times 14$$
$$= 88\,\text{cm}$$

13. Ogive is a curve which represents continuous .... frequency distribution graphically. [1]

Ans : Cumulative

14. The value of $\frac{\sec 23^\circ}{\operatorname{cosec} 67^\circ}$ is .... [1]

Ans :

$$\frac{\sec 23^\circ}{\operatorname{cosec} 67^\circ} = \frac{\sec 23^\circ}{\operatorname{cosec}(90^\circ - 23^\circ)}$$
$$= \frac{\sec 23^\circ}{\sec 23^\circ} = 1$$

15. A line intersecting a circle in two points is called a .... [1]

Ans : Secant

(Q.16-Q.20) Answer the following

16. Given that LCM (91, 26) = 182, find HCF (91, 26). [1]

Ans :

$$\because \quad \text{HCF} \times \text{LCM} = 91 \times 26$$
$$\text{HCF} \times 182 = 91 \times 26$$
$$\text{HCF} = \frac{91 \times 26}{182} = 13$$

or

Does the rational number $\frac{441}{2^2 \cdot 5^7 \cdot 7^2}$ has a terminating or a non-terminating decimal representation?

Ans :

$$\frac{\cancel{441}^{9}}{2^2 \cdot 5^7 \cdot \cancel{7^2}_{1}} = \frac{9}{2^2 \cdot 5^7}$$

17. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a-1)x - 1$, then find the value of a. [1]

Ans :

$$\text{Given 1 is a zero of,} \quad p(x) = ax^2 - 3(a-1)x - 1$$
$$p(1) = 0$$
$$a \times 1^2 - 3(a-1) \times 1 - 1 = 0$$
$$a - 3a + 3 - 1 = 0$$
$$-2a = -2$$
$$a = 1$$

18. The n^{th} term of an AP is $7-4n$. Find its common difference. [1]

Ans :

Given, $a_n = 7-4n$

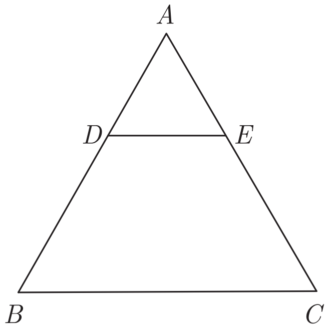
$a_2 = 7-4\times 2 = -1$

and $a_1 = 7-4\times 1 = 3$

$\therefore$ Common difference,

$$d = a_2 - a_1$$
$$= -1-3 = -4$$

19. In the given figure, $DE \parallel BC$ and $AD = 1\,\text{cm}$, $BD = 2\,\text{cm}$. What is the ratio of the area of ΔABC to the area of ΔADE? [1]

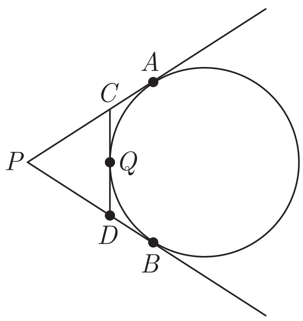


Ans :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\frac{AB}{AD}\right)^2 = \left(\frac{AD+BD}{AD}\right)^2$$
$$= \left(\frac{1+2}{1}\right)^2 = \frac{9}{1}$$
$$(\because \ \Delta ABC \sim \Delta ADE)$$

$\therefore$ Area of ΔABC : Area of $\Delta ADE = 9:1$

20. In the given figure, PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If $PB = 10\,\text{cm}$ and $CQ = 2\,\text{cm}$, what is the length of PC? [1]



Ans :

$$PC = PA - CA = PB - CQ$$
$$= (10-2)\,\text{cm} = 8\,\text{cm}$$

Section B

21. Determine the values of m and n, so that the following system of linear equations has infinite number of solutions: [2]

$(2m-1)x+3y-5;\ 3x+(n-1)y-2 = 0$

Ans :

For infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{-5}{-2}$$
$$\frac{2m-1}{3} = \frac{-5}{-2}$$
$$-4m+2 = -15$$
$$m = \frac{17}{4}$$

and
$$\frac{3}{n-1} = \frac{-5}{-2}$$
$$-5n+5 = -6$$
$$n = \frac{11}{5}$$

Hence, $m = \frac{17}{4}$

and $n = \frac{11}{5}$

22. Find the roots of the quadratic equations: [2]

$4\sqrt{3}x^2+5x-2\sqrt{3} = 0$

Ans :

$$4\sqrt{3}x^2+5x-2\sqrt{3} = 0$$
$$4\sqrt{3}x^2+8x-3x-2\sqrt{3} = 0$$
$$4x(\sqrt{3}x+2)-\sqrt{3}(\sqrt{3}+2) = 0$$
$$(\sqrt{3}x+2)(4x-\sqrt{2}) = 0$$
$$\sqrt{3}x+2 = 0$$

or
$$4x-\sqrt{3} = 0$$
$$x = -\frac{2}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{4}$$

or

Solve for x:

$\sqrt{6x+7}-(2x-7) = 0$

Ans :

$$\sqrt{6x+7}-(2x-7) = 0 \qquad \text{...(i)}$$
$$6x+7 = (2x-7)^2$$
$$6x+7 = 4x^2-28x+49$$
$$4x^2-34x+42 = 0$$
$$2x^2-17x+21 = 0$$
$$(2x-3)(x-7) = 0$$
$$x = \frac{3}{2}, 7$$

Putting, $x = \frac{3}{2}$ in equation (i)

We get,

$$\sqrt{6\times\frac{3}{2}+7}-\left(2\times\frac{3}{2}-7\right) = 0$$
$$4-(-4) = 0$$
$$8 = 0 \text{ i.e. False}$$

Putting, $x = 7$ in equation (i)

We get,

$$\sqrt{6\times 7+7}-(2\times 7-7) = 0$$
$$7-7 = 0 \text{ i.e. True}$$

$\therefore$ $x = 7$

23. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag. Find the probability of getting: [2]
(i) a white ball or a green ball.
(ii) neither a green ball nor a red ball.

Ans :

$$\text{Total number of possible outcomes} = 5+8+7 = 20$$

(i) Number of favourable outcomes

$= $ Number of white balls or number of green balls

$= 7+8 = 15$

$\therefore P$ (a white or a green ball)

$$= \frac{15}{20} = \frac{3}{4}$$

(ii) Number of favourable outcomes

$= $ Number of white balls

$= 7$

$\therefore P$ (neither green nor red ball)

$$= \frac{7}{20}$$

24. Without using trigonometric tables, find the value of: [2]

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \cos 57^\circ \cdot \operatorname{cosec} 33^\circ - 2\cos 60^\circ$$

Ans :

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \cos 57^\circ \cdot \operatorname{cosec} 33^\circ - 2\cos 60^\circ$$

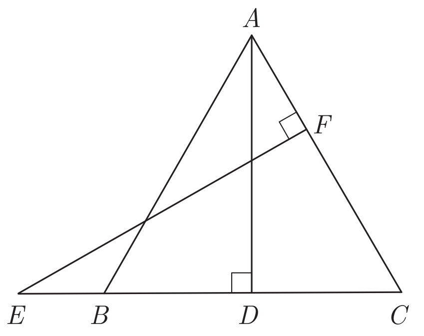
$$= \frac{\cos 70^\circ}{\sin(90^\circ - 70^\circ)} + \cos 57^\circ \cdot \operatorname{cosec}(90^\circ - 57^\circ) - 2\times\frac{1}{2}$$

$$= \frac{\cos 70^\circ}{\cos 70^\circ} + \cos 57^\circ \cdot \sec 57^\circ - 1 \qquad \left(\because \sec\theta = \frac{1}{\cos\theta}\right)$$

$$= 1 + \frac{\cos 57^\circ}{\cos 57^\circ} - 1$$

$$= 1+1-1 = 1$$

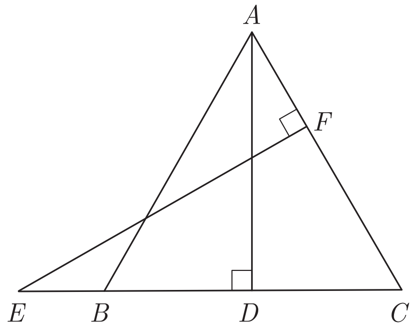
25. In the adjoining figure, E is a point on the side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that: [2]



$$AB \times EF = AD \times EC$$

Ans :

Given, ΔABC is an isosceles triangle with $AB = AC$



$\angle C = \angle B$

(angles opp. equal sides of a Δ are equal)

In ΔABD and ΔECF,

$\angle ABD = \angle EFC$

($\because \angle B = \angle C$, proved above)

and $\angle ADB = \angle EFC$ (each $= 90^\circ$)

$\therefore \quad \Delta ABD \sim \Delta ECF$ (by AA similarity criterion)

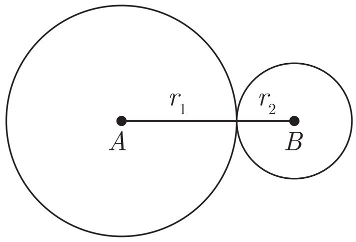
$$\therefore \quad \frac{AB}{EC} = \frac{AD}{EF}$$

$AB \times EF = AD \times EC$, as required.

26. Two circles touch externally. The sum of their areas is 58π cm^2 and the distance between their centres is 10 cm. Find the radii of the two circles. [2]

Ans :

Let r_1 cm and r_2 cm be the radii of two circles,



Then, $r_1 + r_2 = 10$ ...(i)

and $\pi r_1^2 + \pi r_2^2 = 58\pi$

$r_1^2 + r_2^2 = 58$ ...(ii)

Substituting the value of r_2 from (i) in (ii), we get,

$$r_1^2 + (10 - r_1)^2 = 58$$
$$2r_1^2 - 20r_1 + 42 = 0$$
$$r_1^2 - 10r_1 + 21 = 0$$
$$(r_1 - 7)(r_1 - 3) = 0$$
$$r_1 = 7, 3$$

If, $r_1 = 7$,

Then, $r_2 = 3$

and if $r_1 = 3$,

Then, $r_2 = 7$

Hence, the radii of two circles are 7 cm and 3 cm.

or

A student takes a rectangular piece of paper 30 cm long and 21 cm wide. Find the area of the biggest circle that can be cut from the paper. Also find the area of the paper left after cutting out the circle.

Ans :

Given, Length of paper $= 30\text{ cm}$,

Breadth of paper $= 21\text{ cm}$

$\therefore$ Area of paper $= (30 \times 21)\text{ cm}^2$

$= 630\text{ cm}^2$

Now, diameter of the biggest circle which can cut from the given paper is 21 cm.

So, $2r = 21\text{ cm}$

$r = \frac{21}{2}\text{ cm}$

Area of circle $= \pi r^2$

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$= 346.5\text{ cm}^2$

Now, area of the paper left after cutting out the circle

$= (630 - 346.5)\text{ cm}^2$

$= 283.5\text{ cm}^2$

Section C

27. Prove that: [3]

$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\operatorname{cosec} A$$

Ans :

$$\text{LHS} = \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$$

$$= \frac{(\sec A - 1) + (\sec A + 1)}{\sqrt{\sec A + 1} \cdot \sqrt{\sec A - 1}}$$

$$= \frac{2\sec A}{\sqrt{\sec^2 A - 1}} = \frac{2\sec A}{\sqrt{\tan^2 A}}$$

$$= \frac{2\sec A}{\tan A} = 2 \times \frac{1}{\cos A} \times \frac{\cos A}{\sin A}$$

$$= \frac{2}{\sin A} = 2\operatorname{cosec} A = \text{RHS}$$

28. Solve the following pair of linear equations: [3]

$$\frac{a^2}{x} - \frac{b^2}{y} = 0;\ \frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b,\ x \neq 0,\ y \neq 0$$

Ans :

Given, $\frac{a^2}{x} - \frac{b^2}{y} = 0$ ...(i)

and $\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b$ ...(ii)

Multiplying (i) by a, we get,

$\frac{a^3}{x} - \frac{b^2 a}{y} = 0$ ...(iii)

Adding (ii) and (iii), we get,

$$\frac{a^2 b}{x} + \frac{a^3}{x} = a + b$$

$$\frac{a^2}{x}(b + a) = a + b$$

$$\frac{a^2}{x} = \frac{a + b}{a + b}$$

$$\frac{a^2}{x} = 1$$

$$x = a^2$$

Putting $x = a^2$ in (i), we get,

$$\frac{a^2}{a^2} - \frac{b^2}{y} = 0$$

$$1 - \frac{b^2}{y} = 0$$

$$1 = \frac{b^2}{y}$$

$$y = b^2$$

Hence, $x = a^2$

and $y = b^2$

or

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator both increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Ans :

Let the fraction be $\frac{x}{y}$.

Then according to given,

$x + y = 2x + 4$

$x - y + 4 = 0$ ...(i)

and $\frac{x+3}{y+3} = \frac{2}{3}$

$3x + 9 = 2y + 6$

$3x - 2y + 3 = 0$ ...(ii)

Multiplying (i) by 2 and subtracting from (ii), we get,

$$3x - 2y + 3 - 2(x - y + 4) = 0$$

$$3x - 2y + 3 - 2x + 2y - 8 = 0$$

$$x - 5 = 0$$

$$x = 5$$

Putting $x = 5$ in (i), we get,

$$5 - y + 4 = 0$$

$$y = 9$$

Hence, the fraction is $\frac{5}{9}$.

29. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial. [3]

Ans :

Let, $f(x) = 4x^2 + 4x - 3$

Then, $f\left(\frac{1}{2}\right) = 4 \times \left(\frac{1}{2}\right)^2 + 4 \times \frac{1}{2} - 3$

$= 4 \times \frac{1}{4} + 2 - 3$

$= 1 + 2 - 3 = 0$

and $f\left(\frac{-3}{2}\right) = 4 \times \left(\frac{-3}{2}\right)^2 + 4 \times \left(\frac{-3}{2}\right) - 3$

$= 4 \times \frac{9}{4} - 6 - 3$

$= 9 - 9 = 0$

Therefore, $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the given polynomial.

Verification of relationship between zeroes and coefficients:

$$\text{Sum of the zeroes} = \frac{1}{2} + \left(\frac{-3}{2}\right) = -1$$

$$= \frac{-4}{4} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

Product of the zeroes

$$= \frac{1}{2} \times \left(\frac{-3}{2}\right) = \frac{-3}{4}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

or

Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Form a quadratic polynomial whose zeroes 3α and 3β.

Ans :

Given α and β are zeroes of the quadratic polynomial $2x^2 - 3x + 1$,

$$\alpha + \beta = \frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$= -\frac{-3}{2} = \frac{3}{2} \qquad \text{...(i)}$$

and

$$\alpha \cdot \beta = \frac{\text{cons tan t term}}{\text{coefficient of } x^2}$$

$$= \frac{1}{2} \qquad \text{...(ii)}$$

We are to find a quadratic polynomial whose zeroes are 3α and 3β.

Let, $\alpha' = 3\alpha$

and $\beta' = 3\beta$

Then,

$$\alpha' + \beta' = 3\alpha + 3\beta$$

$$= 3(\alpha + \beta)$$

$$= 3 \times \frac{3}{2} = \frac{9}{2} \qquad \text{(using (i))}$$

and

$$\alpha' \cdot \beta' = 3\alpha \cdot 3\beta$$

$$= 9\alpha\beta$$

$$= 9 \times \frac{1}{2}$$

$$= \frac{9}{2} \qquad \text{(using (ii))}$$

A quadratic polynomial whose zeroes are α' and β' is:

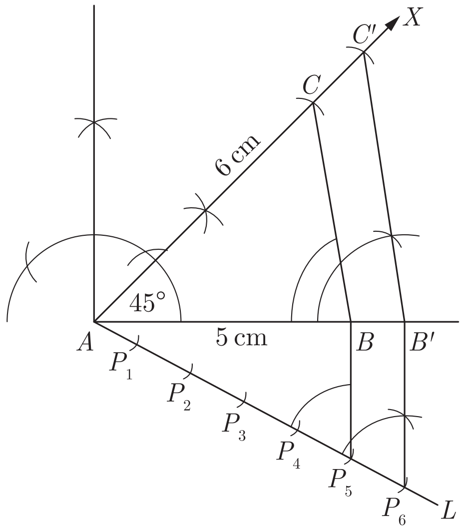
$$x^2 - (\alpha' + \beta')x + \alpha' \cdot \beta' = x^2 - \frac{9}{2}x + \frac{9}{2}$$

$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, a quadratic polynomial whose zeroes are 3α and 3β is $\frac{1}{2}(2x^2 - 9x + 9)$.

30. Construct a ΔABC in which $CA = 6\,\text{cm}$, $AB = 5\,\text{cm}$ and $\angle BAC = 45°$, then construct a triangle similar to the given triangle whose sides are $\frac{6}{5}$ of the corresponding sides of the ΔABC. [3]

Ans :



Steps of construction:

1. Draw $AB = 5\,\text{cm}$.
2. At the point A, draw $\angle BAX = 45°$.
3. From AX cut off $AC = 6\,\text{cm}$.
4. Join BC. ΔABC is formed with given data.
5. Draw $\overrightarrow{AL}$ making acute angle with AB as shown in the figure.
6. Draw 6 arcs P_1, P_2, P_3, P_4, P_5 and P_6 such that
   $AP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = P_5P_6$.
7. Join BP_5.
8. Draw $B'P_6 \parallel BP_5$ meeting AB produced at B'.
9. From B', draw $B'C' \parallel BC$ meeting AX at C'.
   $\Delta AB'C' \sim \Delta ABC$

31. Find the values of k if the points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear. [3]

Ans :

Since the given points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear, the area of triangle formed by them is zero.

$$\frac{1}{2}|(k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)(2k-2k-3)| = 0$$

$$|(k+1)(3-3k) + 3k(3k) + (5k-1)(-3)| = 0$$

$$3(1+k)(1-k) + 9k^2 - 15k + 3 = 0$$

$$3(1-k^2) + 9k^2 - 15k + 3 = 0$$

$$6k^2 - 15k + 6 = 0$$

$$2k^2 - 5k + 2 = 0$$

$$(k-2)(2k-1) = 0$$

$$k = 2, \frac{1}{2}$$

Hence, the values of k are 2, $\frac{1}{2}$.

or

If $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio 3 : 1, find the values of a and b.

Ans :

Given the point P divides the line segment joining points $A(3a+1,-3)$ and $B(8a,5)$ in the ratio 3 : 1 i.e. $AP:PB=3:1$, therefore, the point P is $\left(\frac{3\times 8a+1\times(3a+1)}{3+1},\frac{3\times 5+1\times(-3)}{3+1}\right)$ i.e. $\left(\frac{27a+1}{4},3\right)$. But the point P is $(9a-2,-b)$.

$$9a-2=\frac{27a+1}{4}$$

and $$-b=3$$

$$36a-8=27a+1$$

and $$b=-3$$

$$a=1$$

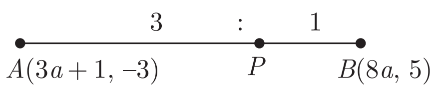
and $$b=-3$$

Hence, $$a=1$$

and $$b=-3$$

32. Prove that the points $A(-3,0)$, $B(1,-3)$ and $C(4,1)$ are the vertices of an isosceles right triangle. [3]

Ans :



$$AB=\sqrt{(1+3)^2+(-3-0)^2}$$
$$=\sqrt{16+9}$$
$$=\sqrt{25}=5$$
$$BC=\sqrt{(4-1)^2+(1+3)^2}$$
$$=\sqrt{9+16}$$
$$=\sqrt{25}=5$$
$$AC=\sqrt{(4+3)^2+(1-0)^2}$$
$$=\sqrt{49+1}$$
$$=\sqrt{50}=5\sqrt{2}$$
$$AB^2+BC^2=5^2+5^2$$
$$=25+25=50$$
$$AC^2=(5\sqrt{2})^2$$
$$=50$$

$\therefore$ $$AB^2+BC^2=AC^2$$

and $$AB=BC$$

33. Cards marked with all 2-digit numbers are placed in a box and are mixed thoroughly. One card is drawn at random. Find the probability that the number on the card is [3]

(i) divisible by 10

(ii) a perfect square number

(iii) a prime number less than 25

Ans :

Cards have numbers 10 to 99 (both inclusive) i.e. 10, 11, 12, ..., 90.

$\therefore$ Total number of cards = 90 (99 − 9 = 90)

(i) The numbers from 10 to 99 which are divisible by 10 are 10, 20, 30, ..., 90.

The number of such numbers = 9

$\therefore$ $$\text{Required probability}=\frac{9}{90}=\frac{1}{10}$$

(ii) The numbers from 10 to 99 which are perfect squares are 16, 25, 36, 49, 64, 81.

The number of such numbers = 6

$\therefore$ $$\text{Required probability}=\frac{6}{90}$$
$$=\frac{1}{15}$$

(iii) The numbers from 10 to 99 which are prime numbers less than 25 are 11, 13, 17, 19, 23.

$$\text{Required probability}=\frac{5}{90}=\frac{1}{18}$$

34. The following frequency distribution shows the number of runs scored by some batsman of India in one-day cricket matches: [3]

Run scored	Number of batsman
2000 – 4000	9
4000 – 6000	8
6000 – 8000	10
8000 – 10000	2
10000 – 12000	1

Find the mode for the above data.

Ans :

The maximum class frequency is 10 and the class corresponding to this frequency is 6000 − 8000. So, the modal class is 6000 − 8000.

Here, $$l=6000,$$
$$h=2000,$$
$$f_1=10,$$
$$f_0=8$$

and $$f_2=2$$

$$\text{Mode}=l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$$
$$=6000+\frac{10-8}{2\times 10-8-2}\times 2000$$
$$=6000+\frac{2}{10}\times 2000$$
$$=6000+400=6400$$

$\therefore$ $$\text{Mode}=6400$$

Section D

35. Prove that n^3-n is divisible by 6 for every positive integer n. [4]

Ans :

Given n is any positive integer. Applying Euclid's lemma with divisor = 6, we get $n=6q$, $6q+2$, $6q+2$, $6q+3$, $6q+4$ or $6q+5$, where q is some whole number. Six cases arise.

Case I :

If $$n=6q$$

Then, $n^3 - n = n(n-1)(n+1)$

$= 6q(6q-1)(6q+1)$

(Which is divisible by 6)

Case II :

If $n = 6q+1$

Then, $n^3 - n = n(n-1)(n+1)$

$= (6q+1)(6q)(6q+2)$

$= 12q(6q+1)(3q+1)$

(Which is divisible by 6)

Case III :

If $n = 6q+2$

Then, $n^3 - n = n(n-1)(n+1)$

$= (6q+2)(6q+1)(6q+3)$

$= 6(3q+1)(6q+1)(2q+1)$

(Which is divisible by 6)

Case IV :

If $n = 6q+3$

Then, $n^3 - n = n(n-1)(n+1)$

$= (6q+3)(6q+2)(6q+4)$

$= 12(2q+1)(3q+1)(3q+2)$

(Which is divisible by 6)

Case V :

If $n = 6q+4$

Then, $n^3 - n = n(n-1)(n+1)$

$= (6q+4)(6q+3)(6q+5)$

$= 6(3q+2)(2q+1)(6q+5)$

(Which is divisible by 6)

Case VI :

If $n = 6q+5$

Then, $n^3 - n = n(n-1)(n+1)$

$= (6q+5)(6q+4)(6q+6)$

$= 12(6q+5)(3q+2)(q+1)$

(Which is divisible by 6)

Thus, in all cases, $n^3 - n$ is divisible by 6.
Hence, for every positive integer n, $n^3 - n$ is divisible by 6.

36. A sum of ₹1600 is to be used to give 10 cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes. [4]

Ans :

Let the first cash prize be ₹ x.

It is given that each prize is ₹20 less than its preceding prize.
These cash prizes form an AP with,

$a = x,$

$d = -20,$

$S_n = 1600$

and $n = 10$

$\therefore \quad S_n = \frac{n}{2}[2a+(n-1)d]$

$1600 = \frac{10}{2}[2x+(10-1)d]$

$\frac{1600}{5} = 2x + 9 \times (-20)$

$320 = 2x - 180$

$2x = 500$

$x = 250$

Therefore, the prizes are $250,\ 250-20,\ 250-2\times 20$

$250-3\times 20,\ 250-4\times 20,\ 250-5\times 20$, i.e. 250,

30, 210, 190, 170, 150, 130, 110, 90, 70
Hence, the values of cash prizes are :
₹250, ₹230, ₹210, ₹190, ₹170, ₹150, ₹130, ₹110,

₹90 and ₹70

37. A motor boat whose speed is 20 km/h in still water takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream. [4]

Ans :

Let the speed of the stream be x km/h.
Given, speed of the motor boat in still water

$= 20$ km/h

Then, the speed of the boat upstream

$= (20-x)$ km/h

and the speed of the boat downstream

$= (20+x)$ km/h

Time taken by the motor boat to cover 48 km upstream

$= \frac{48}{20-x}$ hours

Time taken by the motor boat to cover 48 km downstream

$= \frac{48}{20+x}$ hours

According to given,

$$\frac{48}{20-x} - \frac{48}{20+x} = 1$$

$$48(20+x) - 48(20-x) = (20-x)(20+x)$$

$$48(2x) = 20^2 - x^2$$

$$x^2 + 96x - 400 = 0$$

$$(x-4)(x+100) = 0$$

$$x = 0$$

or $x = -100$

But x being the speed of the stream cannot be negative.

$x = 4$

Hence, the speed of the stream is 4 km/h.

or

A shopkeeper buys some books for ₹80. If he had bought 4 more books for the same amount, each book would have cost ₹1 less. Find the number of books he bought.

Ans :

Let the number of books the shopkeeper bought be x.

Given, Total cost of books = ₹80

Then, Cost of one book $= \frac{80}{x}$

When he had bought 4 more books for same amount i.e. ₹ 80,

Then, Cost of one book $= \frac{80}{x+4}$

According to given,

$$\frac{80}{x} - \frac{80}{x+4} = 1$$

$$80(x+4) - 80x = x(x+4)$$

$$320 = x^2 + 4x$$

$$x^2 + 4x - 320 = 0$$

$$(x+20)(x-16) = 0$$

$$x = 16$$

or $$x = -20$$

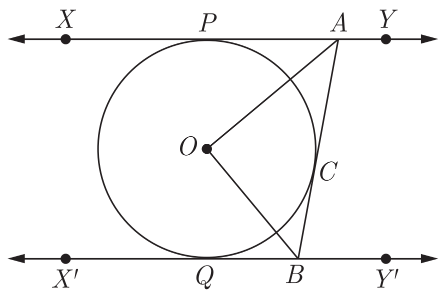
But x being the number of books cannot be negative.

$$x = 16$$

Hence, the number of books he bought

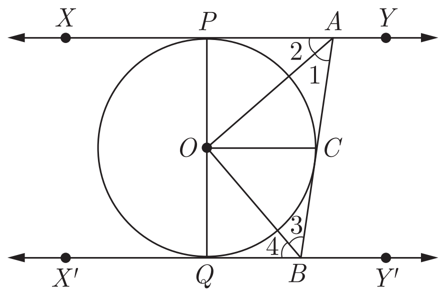
$$= 16$$

38. In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$. [4]



Ans :

Join OP, OQ and OC. Mark the angles as shown in the figure.



In ΔOAC and ΔOAP,

$$OA = OA \quad \text{(common)}$$

$$OC = OP \quad \text{(radii of same circle)}$$

$$AC = AP \quad \text{(tangents drawn from } A\text{)}$$

$\therefore$ $$\Delta OAC \cong \Delta OAP$$

(by SSS rule of congruency)

$\therefore$ $$\angle 1 = \angle 2$$

$$\angle PAC = 2\angle 1 \quad \text{...(i)}$$

Similarly, $\Delta OBC \cong \Delta OBQ$,

$\therefore$ $$\angle 3 = \angle 4$$

$$\angle QBC = 2\angle 3 \quad \text{...(ii)}$$

As $XY \parallel X'Y'$ and AB is a transversal,

$$\angle PAC + \angle QBC = 180° \quad \text{(sum of co-int. } \angle s\text{)}$$

$$2\angle 1 + 2\angle 3 = 180° \quad \text{[using (i) and (ii)]}$$

$$\angle 1 + \angle 3 = 90° \quad \text{...(iii)}$$

In ΔOAB;

$$\angle AOB + \angle 1 + \angle 3 = 180° \quad \text{(sum of angles of a } \Delta\text{)}$$

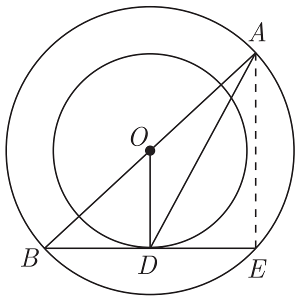
$$\angle AOB + 90° = 180° \quad \text{[using (iii)]}$$

$$\angle AOB = 90°$$

or

The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle touching it at D. Find the length AD.

Ans :



Let the line BD meet the bigger circle at E. Join AE. Let O be the centre of two concentric circles.

As AB is a diameter of the bigger circle, O is mid-point of AB.

BD is tangent to the smaller circle and OD is radius of smaller circle through the point of contact D, $OD \perp BE$.

Since BE is a chord of the bigger circle and $OD \perp BE$,

$$BD = DE$$

($\because$ Perpendicular from the centre to a chord bisects it)

$\therefore$ $$OD = \frac{1}{2}AE$$

($\because$ Segment joining the mid-points of any two sides of a triangle is half of the third side)

$$AE = 2OD$$

$$AE = (2 \times 8)\text{ cm}$$

$$= 16\text{ cm} \quad (\because OD = 8\text{ cm})$$

In ΔOBD,

$$\angle ODB = 90°,$$

By Pythagoras theorem, we get,

$$OB^2 = BD^2 + OD^2$$

$$13^2 = BD^2 + 8^2$$

($\because$ OB is radius of bigger circle, so $OB = 13$ cm)

$$BD^2 = 169 - 64 = 105$$

$$BD = \sqrt{105}\text{ cm}$$

But, $$DE = BD$$

$$DE = \sqrt{105}\text{ cm}$$

Now, $$\angle AEB = 90°$$

($\because$ angle in a semicircle $= 90°$)

$$\angle AED = 90^\circ$$

In $\Delta\, ADE$;

$$\angle AED = 90^\circ,$$

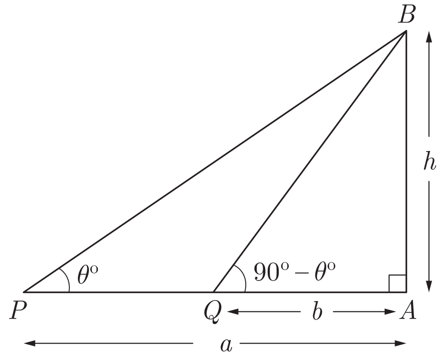
By Pythagoras theorem, we get,

$$AD^2 = AE^2 + DE^2 = 16^2 + (\sqrt{105})^2$$
$$= 256 + 105 = 361$$
$$AD = \sqrt{361} \text{ cm}$$
$$AD = 19 \text{ cm}$$

39. The angles of elevation of the top of a tower from two points P and Q at distances of a and b respectively from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$. [4]

Ans :

Let AB be the tower of height h (units).



$$AP = a,$$
$$QA = b$$

As the angles of elevation are complementary,

If $\angle APB = \theta^\circ$,

Then, $\angle AQB = 90^\circ - \theta^\circ$

From right angled $\Delta\, BPA$, we get,

$$\tan\theta^\circ = \frac{h}{a} \quad ...(i)$$

From right angled $\Delta\, BQA$, we get,

$$\tan(90^\circ - \theta^\circ) = \frac{h}{b}$$
$$\cot\theta^\circ = \frac{h}{b} \quad ...(ii)$$

Multiplying (i) and (ii), we get,

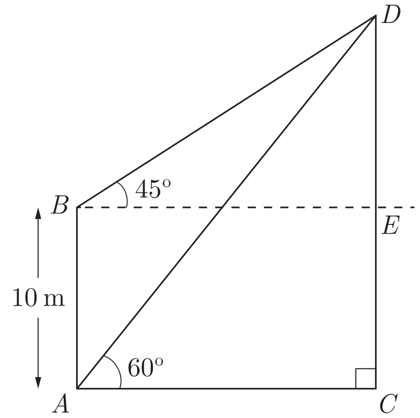
$$\tan\theta^\circ \cot\theta^\circ = \frac{h}{a} \times \frac{h}{b} = \frac{h^2}{ab}$$
$$h^2 = ab$$
$$h = \sqrt{ab} \quad (\because\ h > 0)$$

or

The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 45°. Find the height of the tower.

Ans :

Let CD be a vertical tower of height h metres. From a point A on the ground, the angle of elevation of the top is 60°. B is another point 10 m vertically above A and angle of elevation of the top of the tower from B is 45°.



Let, $AC = d$ metres

From B, draw $BE \perp CD$.

Then, $BE = AC$

and $EC = BA = 10$ m

From right angled $\Delta\, ADC$, we get,

$$\tan 60^\circ = \frac{CD}{AC}$$
$$\sqrt{3} = \frac{h}{d}$$
$$h = \sqrt{3}\, d \quad ...(i)$$

From right angled $\Delta\, BED$, we get,

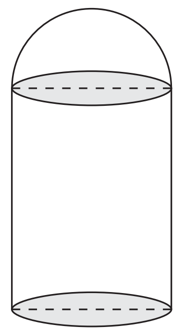
$$\tan 45^\circ = \frac{ED}{BE}$$
$$1 = \frac{CD - CE}{AC} = \frac{h - 10}{d}$$
$$d = h - 10$$

From (i), $h = \sqrt{3}(h - 10)$

$$\sqrt{3}\, h - h = 10\sqrt{3}$$
$$(\sqrt{3} - 1)h = 10\sqrt{3}$$
$$h = \frac{10\sqrt{3}}{\sqrt{3} - 1} = \frac{10\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$
$$= 5(3 + \sqrt{3})$$

Hence, the height of the tower is $5(3 + \sqrt{3})$ m.

40. A building is in the form of cylinder surmounted by a hemispherical dome (shown in the adjoining figure). The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building if it contains $67\frac{1}{21}\, m^3$ of air. [4]



Ans :

Let the radius of the spherical dome be r metres and the total height of the building be h metres.

Since the base diameter of the dome is $\frac{2}{3}$ of the total height of the building, therefore,

$$2r = \frac{2}{3}h$$

$$r = \frac{h}{3}$$

$$\therefore \qquad \text{Height of the cylinder} = \left(h - \frac{h}{3}\right) \text{ m}$$

$$= \frac{2h}{3} \text{ m}$$

Volume of air inside the building

$$= \text{Volume of hemisphere} + \text{Volume of cylinder}$$

$$= \frac{2}{3}\pi r^3 + \pi r^2 H, \quad \text{where } H \text{ is height of cylinder}$$

$$= \left(\frac{2}{3}\pi\right) \times \left(\frac{h}{3}\right)^3 + \pi \times \left(\frac{h}{3}\right)^2 \times \left(\frac{2h}{3}\right) \text{ m}^3$$

$$= \frac{8\pi}{27} h^3 \text{ m}^3$$

But the volume of air inside the building

$$= 67\frac{1}{21} \text{ m}^3$$

$$= \frac{1408}{21} \text{ m}^3$$

$$\therefore \qquad \frac{8}{81} \times \frac{22}{7} \times h^3 = \frac{1408}{21}$$

$$h^3 = 216$$

$$h = 6$$

Hence, the height of the building = 6 m