CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-9

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. A three digit number is to be formed using the digits 3, 4, 7, 8 and 2 without repetition. The probability that it is an odd number is [1]

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{4}{5}$ (d) $\frac{3}{5}$

Ans : (a) $\frac{2}{5}$

2. If the coordinates of the point of intersection of less than ogive and more than ogive is (13.5,20), then the value of median is [1]

(a) 13.5 (b) 20

(c) 33.5 (d) 7.5

Ans : (a) 13.5

The abscissa of point of intersection gives the median of the data. So, median is 13.5.

3. The volume of a largest sphere that can be cut from cylindrical log of wood of base radius 1 m and height 4 m, is [1]

(a) $\frac{16}{3}\pi\,\text{m}^3$ (b) $\frac{8}{3}\pi\,\text{m}^3$

(c) $\frac{4}{3}\pi\,\text{m}^3$ (d) $\frac{10}{3}\pi\,\text{m}^3$

Ans : (c) $\frac{4}{3}\pi\,\text{m}^3$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3$$

$$= \frac{4}{3}\pi m^3$$

4. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is [1]

(a) 2 (b) 3

(c) 5 (d) 6

Ans : (a) 2

Given, $S_{11} = 33$

$$\frac{11}{2}[2a+10d] = 33 \Rightarrow a+5d = 3$$

i.e., $a_6 = 3 \Rightarrow a_4 = 2$

[Since, Alternate terms are integers and the given sum is possible]

5. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is [1]

(a) 2 (b) -2

(c) 4 (d) -4

Ans : (c) 4

$$\text{Sum of the zeroes} = \frac{3k}{2}$$

$$6 = \frac{3k}{2}$$

$$k = \frac{12}{3} = 4$$

6. Which of the following rational number have non-terminating repeating decimal expansion? [1]

(a) $\frac{31}{3125}$ (b) $\frac{71}{512}$

(c) $\frac{23}{200}$ (d) None of these

Ans : (d) None of these

3125, 512 and 200 has factorization of the form $2^m \times 5^n$ (where m and n are whole numbers). So given fractions has terminating decimal expansion.

7. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is [1]

(a) 2 (b) 3

(c) 5 (d) 15

Ans : (d) 15

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \quad ...(1)$$

and $$\frac{x-1}{y-1} = 7 \quad ...(2)$$

Solving (1) and (2),

We have $x = 15,\ y = 3,$

i.e. $x = 15$

8. $(x^2+1)^2 - x^2 = 0$ has [1]

(a) four real roots (b) two real roots

(c) no real roots (d) one real root

Ans : (c) no real roots

Given equation is,

$$(x^2+1)^2 - x^2 = 0$$

$$x^4 + 1 + 2x^2 - x^2 = 0 \quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$x^4 + x^2 + 1 = 0$$

Let, $x^2 = y$

$$(x^2)^2 + x^2 + 1 = 0$$

$$y^2 + y + 1 = 0$$

On comparing with $ay^2 + by + c = 0$,

we get $a = 1,\ b = 1$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$$= (1)^2 - 4(1)(1)$$

$$= 1 - 4 = -3$$

Since, $D < 0$

$$y^2 + y + 1 = 0$$

i.e., $x^4 + x^2 + 1 = 0$

or $(x^2+1)^2 - x^2 = 0$ has no real roots.

9. C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is $(-2, 4)$, then x and y respectively are [1]

(a) -6 and 1 (b) -6 and 2

(c) 6 and -1 (d) 6 and -2

Ans : (a) -6 and 1

Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.

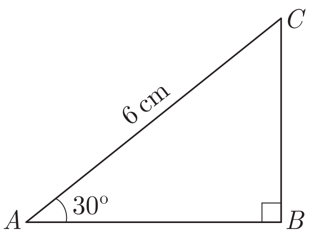
We have, $\frac{4-2}{2} = y$

and $\frac{4+x}{2} = -1$

$$y = 1$$

and $x = -6$

10. In the adjoining figure, the length of BC is [1]



(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm (d) 3 cm

Ans : (d) 3 cm

In ΔABC,

$$\sin 30^\circ = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3\,\text{cm}$$

(Q.11-Q.15) Fill in the blanks.

11. Only two .... can be drawn to a circle from an external point. [1]

Ans : Tangents

12. Point on the X-axis which is equidistant from $(2, -5)$ and $(-2, 9)$ is .... [1]

Ans : $(-7, 0)$

or

Relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear is .... [1]

Ans : $x + 3y = 7$

13. The common point of a tangent to a circle and the circle is called .... [1]

Ans : Point of contact

14. Triangle in which we study trigonometric ratios is called .... [1]

Ans : Right Triangle

15. .... theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. [1]

Ans : Basic proportionality

(Q.16-Q.20) Answer the following

16. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere. [1]

Ans :

No. of spheres $= 12$

Radius of cone, $r = 1$ cm

Height of the cone $= 48$

Volume of 12 spheres = Volume of cone

Let the radius of sphere be R.

$$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (1)^2 \times 48$$

$$R^3 = 1$$

$$R = 1 \text{ cm}$$

or

Three cubes of iron whose edges are 3 cm, 4 cm and 5 cm respectively are melted and formed into a single cube, what will be the edge of the new cube formed ?

Ans :

Let the edge of single cube be x cm.

Volume of single cube= Volume of three cubes

$$x^3 = (3)^3 + (4)^3 + (5)^3$$

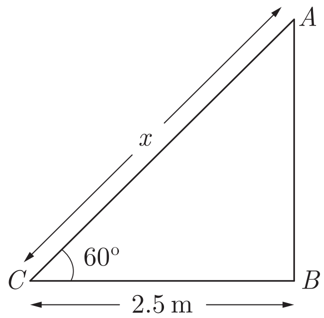
$$= 27 + 64 + 125 = 216$$

$$x = 6 \text{ cm}$$

17. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder. [1]

Ans :

Let the length of ladder be x. As per given in question we have drawn figure below.



In ΔACB, $\angle C = 60^\circ$

$$\cos 60^\circ = \frac{2.5}{AC}$$

$$\frac{1}{2} = \frac{2.5}{AC}$$

$$AC = 2 \times 2.5 = 5 \text{ m}$$

18. The diameter of two circle with centre A and B are 16 cm and 30 cm respectively. If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C. [1]

Ans :

As we know that,
Area of circle $= \pi r^2$,
Let the radius of circle with centre $C = R$
According to question we have,

$$\pi(8)^2 + \pi(15)^2 = \pi R^2$$

$$64\pi + 225\pi = \pi R^2$$

$$289\pi = \pi R^2$$

$$R^2 = 289 \text{ or } R = 17 \text{ cm}$$

Circumference of circle

$$2\pi R = 2\pi \times 17$$

$$= 34\pi \text{ cm}$$

19. In the following frequency distribution, find the median class. [1]

Height (in cm)	104-145	145-150	150-155	155-160	160-165	165-170
Frequency	5	15	25	30	15	10

Ans :

Height	Frequency	c.f.
140-145	5	5
145-150	15	20
150-155	25	45
155-160	30	75
160-165	15	90
165-170	10	100
	$\sum f = 100$	

$$N = 100$$

$$\Rightarrow \quad \frac{N}{2} \text{ th term} = \frac{100}{2} = 50 \text{ th Mean}$$

Hence, Median class in $155 - 160$.

20. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant. [1]

Ans :

In the English language there are 26 alphabets. Consonant are 21. The probability of chosen a consonant

$$P = \frac{21}{26}$$

Section B

21. Using Euclid's algorithm, find the HCF of 240 and 228. [2]

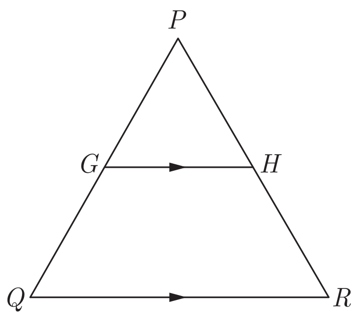
Ans :

We have $240 = 228 \times 1 + 12$

and $228 = 12 \times 19 + 0$

Hence, HCF of 240 and 228 = 12

22. In the given figure, G is the mid-point of the side PQ of ΔPQR and $GH || QR$. Prove that H is the mid-point of the side PR or the triangle PQR. [2]



Ans :

Since G is the mid-point of PQ we have

$$PG = GQ$$

$$\frac{PG}{GQ} = 1$$

According to the question, $GH || QR$, thus

$$\frac{PG}{GQ} = \frac{PH}{HR} \quad \text{(By BPT)}$$

$$1 = \frac{PH}{HR}$$

$$PH = HR. \quad \text{Hence proved.}$$

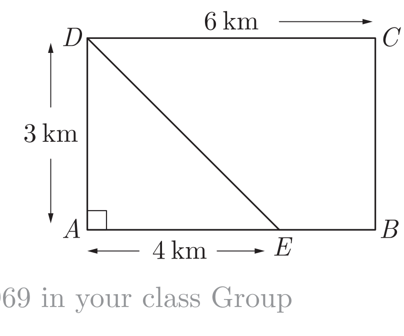
Hence, H is the mid-point of PR.

or

In a rectangle $ABCD, E$ is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Ans :

As per given condition we have drawn the figure below.



We have $AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4$ km

In right triangle ADE,

$$DE^2 = (3)^2 + (4)^2 = 25$$

Thus $DE = 5$ km

23. Solve for x : $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$ [2]

Ans :

$$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$$
$$x^2 - \sqrt{3}x - 1x + \sqrt{3} = 0$$
$$x(x-\sqrt{3}) - 1(x-\sqrt{3}) = 0$$
$$(x-\sqrt{3})(x-1) = 0$$

Thus $x = \sqrt{3}, x = 1$

24. The mean and median of 100 observation are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110. Find the true mean and median. [2]

Ans :

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\Rightarrow \quad 50 = \frac{\sum fx}{100}$$

$$\Rightarrow \quad \sum fx = 5000$$

$$\sum fx' = 5000 - 100 + 110 = 5010$$

$$\therefore \quad \text{Correct Mean} = \frac{5010}{100} = 50.1$$

Median will remain same *i.e.* median = 52.

or

Find the sum of the lower limit of the median class and the upper limit of the modal class :

Classes	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3

Ans :

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3
Cumulative Frequency	1	4	9	18	25	28

$$\text{Median} = \frac{N}{2}\text{th}$$

$$= \frac{28}{2} = 14\text{th term}$$

$\therefore$ Median class : 40-50 $\Rightarrow$ Lower limit $= 40$

and Modal class : 40-50 $\Rightarrow$ Upper limit $= 50$

Their sum $= 40 + 50 = 90$

25. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m and the canvas to be used cost ₹ 100 per sq.m, find the amount, the associations will have to pay. [Use $\pi = \frac{22}{7}$] [2]

Ans :

Given, Height of upper conical part

$$h = 2.8 \text{ m}$$

and radius, $r = \frac{4.2}{2} = 2.1$ m

$$\text{Slant height, } l = \sqrt{h^2 + r^2} = \sqrt{(2.8)^2 + (2.1)^2}$$
$$= \sqrt{7.84 + 4.41} = 3.5 \text{ m}$$

Surface area of tent $= 2\pi rh + \pi rl$

Area of canvas for 1 tent

$$= 2 \times \frac{22}{7} \times 2.1 \times 4 + \frac{22}{7} \times 2.1 \times 3.5$$

$$= 6.6(8 + 3.5) = 6.6 \times 11.5 \text{ m}^2$$

$$\text{Area for 100 tents} = 6.6 \times 11.5 \times 100$$
$$= 66 \times 115 \text{ m}^2 = 7590 \text{ m}^2$$

Cost of 100 tents = ₹ 7590×100

$$50\ \% \text{ cost} = \frac{50}{100} \times 7590 \times 100$$

= ₹ 379500

Value : Helping the flood victims.

26. Read the following passage and answer the questions that follows:

A box contains 8 black beads and 12 white beads. Another box contains 9 black beads and 6 white beads. One bead from each box is taken. [2]

(a) What is the probability that both beads are black?

(b) What is the probability of getting one black bead and one white bead ?

Ans :

Total number of cases, $20 \times 15 = 300$

Both are black $8 \times 9 = 72$

(a) Probability of getting both black $= \frac{72}{300} = \frac{6}{25}$

One black and one white $8 \times 6 + 12 \times 9 = 156$

(b) Probability $= \frac{156}{300} = \frac{13}{25}$

Section C

27. Solve for : [3]

$$x \quad \frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}; x \neq 1,2,3$$

Ans :

We have $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$

$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2x-4}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2}{(x-1)(x-3)} = \frac{2}{3}$$

$$3 = (x-1)(x-3)$$

$$x^2-4x+3=3$$
$$x^2-4x=0$$
$$x(x-4)=0$$

Thus $x=0$ or $x=4$

28. Find the HCF, by Euclid's division algorithm of the numbers 92690, 7378 and 7161. [3]

Ans :

By using Euclid's Division Lemma, we have

$$92690=7378\times 12+4154$$
$$7378=4154\times 1+3224$$
$$4154=3224\times 1+930$$
$$3224=930\times 3+434$$
$$930=434\times 2+62$$
$$434=62\times 7+0$$
$$\text{HCF }(92690,7378)=62$$

Now, using Euclid's Division Lemma on 7161 and 62, we have

$$7161=62\times 115+31$$
$$62=31\times 2+0$$

Thus HCF $(7161,62)=31$

Hence, HCF of 92690, 7378 and 7161 is 31.

or

Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Ans :

Finding prime factor of given number, we have,

$$16=2\times 2\times 2\times 2=2^4$$
$$36=2\times 2\times 3\times 3=2^2\times 3^2$$
$$\text{HCF}(16,\ 36)=2\times 2=4$$
$$\text{LCM }(16,\ 36)=2^4\times 3^2=16\times 9=144$$

To check HCF and LCM by using formula

$$\text{HCF}(a,b)\times\text{LCM}(a,b)=a\times b$$

or, $$4\times 144=16\times 36$$
$$576=576$$

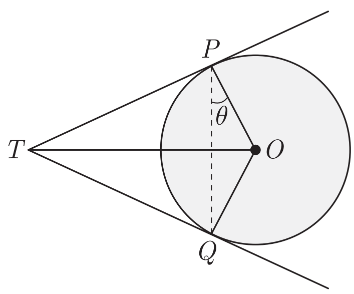
Thus $$\text{LHS}=\text{RHS}$$

29. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that [3]

$$\angle PTO=\angle OPQ$$

Ans :

As per question we draw figure shown below.



Let $\angle TPQ$ be θ. the tangent is perpendicular to the end point of radius,

$$\angle TPO=90^{o}$$

Now $$\angle TPQ=\angle TPO-\theta$$
$$=(90^{o}-\theta)$$

Since, $TP=TQ$ and opposite angels of equal sides are always equal, we have

$$\angle TQP=(90^{o}-\theta)$$

Now in ΔTPQ we have

$$\angle TPQ+\angle TQP+\angle PTQ=180^{o}$$
$$90^{o}-\theta+90^{o}-\theta+\angle PTQ=180^{o}$$
$$\angle PTQ=180^{o}-180^{o}+2\theta=2\theta$$

Hence $\angle PTQ=2\angle OPQ$.

30. The sum of n terms of an A.P. is $3n^2+5n$. Find the A.P. Hence find its 15^{th} term. [3]

Ans :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $$S_n=3n^2+5m$$
$$S_{n-1}=3(n-1)^2+5(n-1)$$
$$=3(n^2+1-2n)+5n-5$$
$$=3n^2+3-6n+5n-5$$
$$=3n^2-n-2$$
$$a_n=S_n-S_{n-1}$$
$$=3n^2+5n-(3n^2-n-2)$$
$$=6n+2$$

Thus A.P. is 8, 14, 20, ...

Now $$a_{15}=a+14d=8+14(6)=92$$

or

Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1^{st} and 4^{rd}) to the product of means (2^{nd} and 3^{rd}) is 5:6.

Ans :

Let the four numbers be $a-3d,a-d,a+d,a+3d$

Now $a-3d+a-d+a+d+a+3d=56$

$$4a=56\Rightarrow a=14$$

Hence numbers are $14-3d,14-d,14+d,14+3d$

Now, according to question,

$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)}=\frac{5}{6}$$
$$\frac{196-9d^2}{196-d^2}=\frac{5}{6}$$
$$6(196-9d^2)=5(196-d^2)$$
$$6\times 196-54d^2=5\times 196-5d^2$$
$$(6-5)\times 196=49d^2$$
$$d^2=\frac{196}{49}=4$$
$$d=\pm 2$$

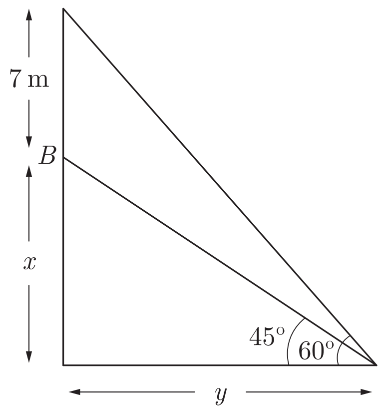
Thus numbers are $$a-3d=14-3\times 2=8$$
$$a-d=14-2=12$$
$$a+d=14+2=16$$
$$a+3d=14+3\times 2=20$$

Thus required A.P is $8, 12, 16, 20$.

31. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct to one place of decimal. (Use $\sqrt{3} = 1.73$) [3]

Ans :

As per given in question we have drawn figure below.



$$\frac{x}{y} = \tan 45^\circ = 1 \Rightarrow x = y$$

$$\frac{x+7}{x} = \tan 60^\circ = \sqrt{3}$$

$$7 = (\sqrt{3} - 1)x$$

$$x = \frac{7}{\sqrt{3} - 1} = \frac{7}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$

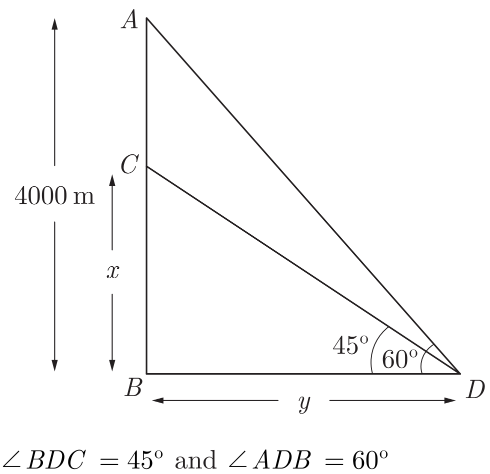
$$x = \frac{7(\sqrt{3} + 1)}{2} = \frac{7(2.73)}{2} = 9.6 \text{ m}$$

or

An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

Ans :

Let the height first plane be $AB = 4000$ m and the height of second plane be $BC = x$ m. As per given in question we have drawn figure below.



Here $\angle BDC = 45^\circ$ and $\angle ADB = 60^\circ$

In ΔCBD, $\frac{x}{y} = \tan 45^\circ = 1 \Rightarrow x = y$

and in ΔABD, $\frac{4000}{y} = \tan 60^\circ = \sqrt{3}$

$$y = \frac{4000\sqrt{3}}{3} = 2309.40 \text{ m}$$

Thus vertical distance between two,

$$4000 - y = 4000 - 2309.40 = 1690.59 \text{ m}$$

32. A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged into water, by how much will the level of water rise in the cylindrical vessel ? [3]

Ans :

Let the rise in level of water be h cm.

Radius of sphere = 3 cm. radius of cylinder

$$= \frac{12}{2} = 6 \text{ cm}$$

Volume of water displaced in cylinder will be equal to the volume of sphere.

Thus $$\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\pi \times 6 \times 6 \times h = \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$h = \frac{4 \times 3 \times 3 \times 3}{3 \times 6 \times 6} = 1 \text{ cm}$$

33. Read the following passage and answer the questions that follows:

Three Students Priyanka, Sania and David are Protesting against killing innocent animals for commercial purposes in a circular park of radius 20 m. They are standing at equal distance on its boundary by holding banners in their hands. [3]

(i) Find the distance between each of them?

(ii) Which mathematical concept is used in it?

Ans :

(i) Let us assume that A, B and C are the position of Priyanka, Sania and David respectively on the boundary of circular park with centre O.

Draw $AD \perp BC$

Since the centre of the circle coincides with the centroid of the equilateral ΔABC.

$$\text{Radius of circumscribed circle} = \frac{3}{2}AD$$

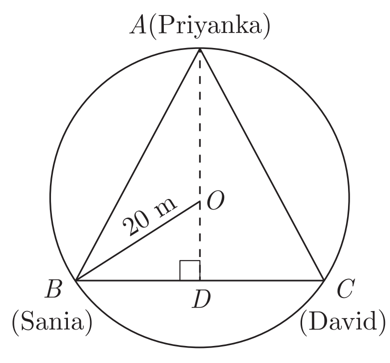
$$20 = \frac{3}{2}AD$$

$$AD = 20 \times \frac{3}{2}$$

$$AD = 30 \text{ m}$$

Now, $AD \perp BC$, and let $AB = BC = CA = x$

$$BD = CD = \frac{1}{2}BC = \frac{x}{2}$$



In rt. ΔBDA, $\angle D = 90°$

By Pythagoras Theorem, we have

$$AB^2 = BD^2 + AD^2$$

$$x^2 = \left(\frac{x}{2}\right)^2 + (30)^2$$

$$x^2 - \frac{x^2}{4} = 900$$

$$\frac{3}{4}x^2 = 900$$

$$x^2 = 900 \times \frac{4}{3} = 1200$$

$$x = \sqrt{1200} = 20\sqrt{3}$$

Hence, distance between each of them is $20\sqrt{3}$.

(ii) Properties of circle, equilateral triangle and Pythagorean theorem.

34. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, Prove that $x^2 + y^2 = 1$. [3]

Ans :

We have $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ (1)

and $x\sin\theta = y\cos\theta$

or, $x = \frac{y\cos\theta}{\sin\theta}$ (2)

Eliminating x from eq. (1) and eq. (2) ,we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$

$$y\cos\theta \times 1 = \sin\theta\cos\theta$$

$$y = \sin\theta \quad ...(3)$$

Substituting this value of y in eq. (2) we have,

$$x = \cos\theta \quad (4)$$

Squaring and adding eq. (3) and eq. (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta$$

$$= 1$$ Hence Proved.

Section D

35. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train. [4]

Ans :

Let the actual speed of the train be x km/hr and actual time taken y hr.

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= xy \text{ km}$$

According to the given condition, we have

$$xy = (x+10)(y-2)$$

$$xy = xy - 2x + 10y - 20$$

$$2x - 10y + 20 = 0$$

$$x - 5y = -10 \quad (1)$$

and $xy = (x-10)(y+3)$

$$xy = xy + 3x - 10y - 30$$

$$3x - 10y = 30 \quad ...(2)$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$3 \times (x - 5y) - (3x - 10y) = -3 \times 10 - 30$$

$$-5y = -60$$

$$y = 12$$

Substituting value of y equation (1),

$$x - 5 \times 12 = -10$$

or, $x = -10 + 60$

or, $x = 50$

Hence, the distance covered by the train

$$= 50 \times 12$$

$$= 600 \text{ km.}$$

36. Show that 3 is a zero of the polynomial $2x^2 - x^2 - 13x - 6$. Hence find all the zeroes of this polynomial. [4]

Ans :

$$\begin{array}{r|l} & 2x^2 + 5x + 2 \\ x-3 & 2x^3 - x^2 - 13x - 6 \\ & \underline{2x^3 - 6x^2} \\ & 5x^2 - 13x - 6 \\ & \underline{5x^2 - 15x} \\ & 2x - 6 \\ & \underline{2x - 6} \\ & 0 \end{array}$$

$$p(x) = 2x^2 - x^2 - 13x - 6$$

$$= 2(3)^3 - (3)^2 - 13(3) - 6$$

$$= 2(27) - 9 - 39 - 6$$

$$= 54 - 54 = 0$$

So, $x - 3$ is a factor of $p(x)$.

by long division

Factorising the quotient, we get

$$= 3x^2 + 4x + x + 2$$

$$= (2x+1)(x+2)$$

$$x = -\frac{1}{2}, -2$$

Hence, All the zeroes of $p(x)$ are $-\frac{1}{2}, -2, 3$

or

Given that $x - \sqrt{5}$ is a factor of the polynomial $x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}$, find all the zeroes of the polynomial.

Ans :

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}x - 15 \\ x-\sqrt{5} & x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5} \\ & \underline{x^3 - \sqrt{5}x^2} \\ & -2\sqrt{5}x^2 - 5x + 15\sqrt{5} \\ & \underline{-2\sqrt{5}x^2 + 10x} \\ & -15x + 15\sqrt{5} \\ & \underline{-15x + 15\sqrt{5}} \\ & 0 \end{array}$$

Factorising the quotient we get

$$x^2-2\sqrt{5}x-15 = x^2-3\sqrt{5}x+\sqrt{5}x-15$$
$$= x(x-3\sqrt{5})+\sqrt{5}(x-3\sqrt{5})$$
$$= (x+\sqrt{5})(x-3\sqrt{5})$$
$$(x+\sqrt{5})(x-3\sqrt{5}) = 0 \Rightarrow x = \sqrt{5},\ 3\sqrt{5}$$

All the zeroes are $\sqrt{5}$, $-\sqrt{5}$ and $3\sqrt{5}$.

37. If $A(-4,8), B(-3,-4), C(0,-5)$ and $D(5,6)$ are the vertices of a quadrilateral $ABCD$, find its area. [4]

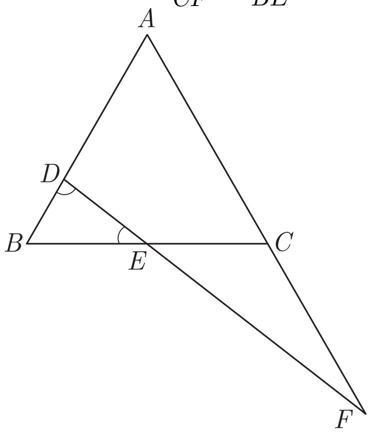
Ans :

We have $A(-4,8)$, $B(-3,-4)$, $C(0,5)$ and $D(5,6)$
Area of quadrilateral

$$= \frac{1}{2}[(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_4-x_4y_3)+(x_4y_1-x_1y_4)]$$

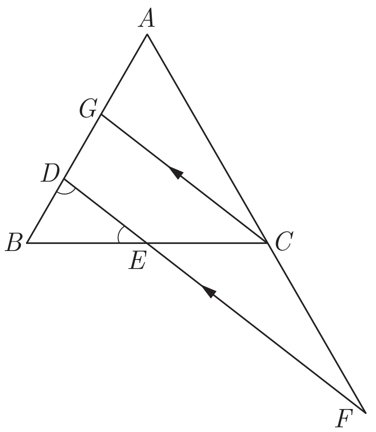
$$\text{Area} = \frac{1}{2}[\{-4\times(-4)-(-3)(8)\}+\{(-3)(-5)-0\times(-4)\}+\{0\times 6-5(-5)\}+[\{5\times 8-(-4)(6)\}]$$
$$= \frac{1}{2}[16+24+15-0+0+25+40+24]$$
$$= \frac{1}{2}[40+15+25+40+24]$$
$$= \frac{1}{2}\times 144 = 72 \text{ sq. units}$$

38. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$. [4]



Ans :

We have redrawn the given figure as below. Here $CG \,||\, FD$.



We have $\angle BED = \angle BDE$

Since E is mid-point of BC,

or, $BE = BD = EC$ ...(1)

In ΔBCG, $DE \,||\, FG$

$$\frac{BD}{DG} = \frac{BE}{EC} = 1 \quad \text{(from (1))}$$

$$BD = DG = EC = BE \quad \text{[using (1)]}$$

In ΔADF, $CG \,||\, FD$

$$\frac{AG}{GD} = \frac{AC}{CF} \quad \text{(By BPT)}$$

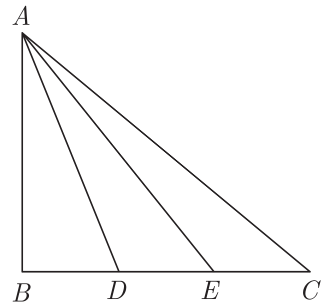
$$\frac{AG+GD}{GD} = \frac{AF+CF}{CF}$$

, $$\frac{AD}{GD} = \frac{AF}{CF}$$

Thus $$\frac{AF}{CF} = \frac{AD}{BE} \quad \text{(using (1))}$$

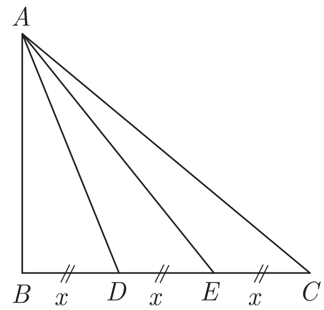
or

In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2+5AD^2$.



Ans :

As per given condition we have drawn the figure below.



Since D and E trisect BC, let $BD = DE = EC$ be x.

Then $BE = 2x$ and $BC = 3x$

In ΔABE, $AE^2 = AB^2+BE^2 = AB^2+4x^2$ ...(1)

In ΔABC, $AC^2 = AB^2+BC^2 = AB^2+9x^2$ ...(2)

In ΔADB, $AD^2 = AB^2+BD^2 = AB^2+x^2$ ...(3)

Multiplying (2) by 3 and (3) by 5 and adding we have

$$3AC^2+5AD^2 = 3(AB^2+9x^2)+(AB^2+x^2)$$
$$= 3AB^2+27x^2+5AB^2+5x^2$$
$$= 8AB^2+32x^2$$
$$= 8(AB^2+4x^2) = 8AE^2$$

Thus $3AC^2+5AD^2 = 8AE^2$ Hence Proved

39. Evaluate : [4]

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$$

Ans :

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$$

$$= \frac{1}{4}\left(\frac{1}{2}\right) + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{3+32+12+1}{24} = \frac{48}{24} = 2$$

or

Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$.

Ans :

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1-\tan\theta)\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$[\therefore a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$

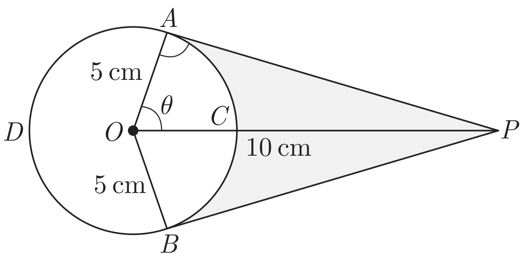
$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

40. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$) [4]



Ans :

Here AP is tangent at point A on circle.

Thus $\angle OAP = 90^\circ$

Now $\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

or, $\theta = 60^\circ$

Reflex $\angle AOB = 360^\circ - 60^\circ - 60^\circ = 240^\circ$

Now arc $ADB = \frac{2 \times 3.14 \times 5 \times 120}{360}$

$= 20.93\text{cm}$

Hence length of elastic in contact $= 20.93$ cm

Now, $AP = 5\sqrt{3}$ cm

Area, $(\Delta\, OAP + \Delta\, OBP) = 25\sqrt{3} = 43.25$ cm^2

Area of sector, $OACB = \frac{25 \times 3.14 \times 120}{360}$

$= 26.16\text{cm}^2$.

Shaded Area $= 43.25 - 26.16$

$= 17.09$ cm^2