

**CLASS X (2019-20)**  
**MATHEMATICS BASIC(241)**  
**SAMPLE PAPER-8**

**Time : 3 Hours**

**Maximum Marks : 80**

**General Instructions :**

- (i) All questions are compulsory.
- (ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
- (iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
- (iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted.

## Section A

**Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.**

1. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is [1]

- (a)  $\frac{2}{11}$  (b)  $\frac{3}{11}$   
(c)  $\frac{4}{11}$  (d) 0

**Ans :** (c)  $\frac{4}{11}$

$$\begin{aligned} \text{Required probability} &= \frac{1+2+1}{11} \\ &= \frac{4}{11} \end{aligned}$$

2. The mean of discrete observations  $y_1, y_2, \dots, y_n$  is given by [1]

- (a)  $\frac{\sum_{i=1}^n y_i}{n}$  (b)  $\frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n i}$   
(c)  $\frac{\sum_{i=1}^n y_i f_i}{n}$  (d)  $\frac{\sum_{i=1}^n y_i f_i}{\sum_{i=1}^n f_i}$

**Ans :** (a)  $\frac{\sum_{i=1}^n y_i}{n}$

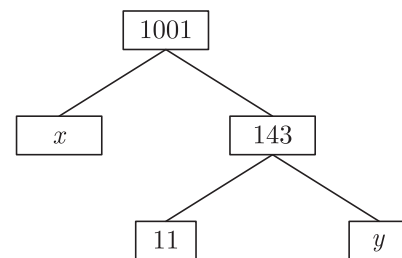
3. Volume of a spherical shell is given by [1]

- (a)  $4\pi(R^2 - r^2)$  (b)  $\pi(R^3 - r^3)$   
(c)  $4\pi(R^3 - r^3)$  (d)  $\frac{4}{3}\pi(R^3 - r^3)$

**Ans :** (d)  $\frac{4}{3}\pi(R^3 - r^3)$

$$\begin{aligned} \text{Volume of spherical shell} &= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(R^3 - r^3) \end{aligned}$$

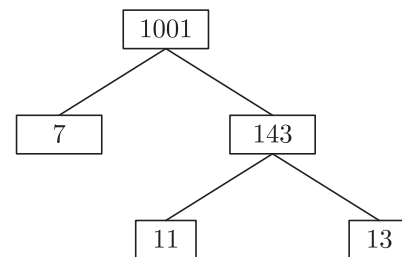
4. The values of  $x$  and  $y$  is the given figure are [1]



- (a) 7, 13 (b) 13, 7  
(c) 9, 12 (d) 12, 9

**Ans :** (a) 7, 13

Given number is 10001. Then, the factor tree of 1001 is given as below



$$1001 = 7 \times 11 \times 13$$

By comparing with given factor tree, we get

$$x = 7, y = 13$$

5. If the sum of the zeroes of the polynomial  $f(x) = 2x^3 - 3kx^2 + 4x - 5$  is 6, then the value of  $k$  is [1]  
(a) 2 (b) -2  
(c) 4 (d) -4

**Ans :** (c) 4

$$\text{Sum of the zeroes} = \frac{3k}{2}$$

$$6 = \frac{3k}{2}$$

$$k = \frac{12}{3} = 4$$

6. If  $3x + 4y : x + 2y = 9 : 4$ , then  $3x + 5y : 3x - y$  is equal to [1]

- (a) 4 : 1 (b) 1 : 4  
(c) 7 : 1 (d) 1 : 7

**Ans :** (c) 7 : 1

$$\frac{3x + 4y}{x + 2y} = \frac{9}{4}$$

Hence,  $12x + 16y = 9x + 18y$

or  $3x = 2y$

$$x = \frac{2}{3}y$$

Substitute  $x = \frac{2}{3}y$  in the required expression.

$$\frac{3\frac{2}{3}y + 5y}{3\frac{2}{3}y - y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

7. The quadratic equation  $2x^2 - \sqrt{5}x + 1 = 0$  has [1]

- (a) two distinct real roots
- (b) two equal real roots
- (c) no real roots
- (d) more than 2 real roots

**Ans :** (c) no real roots

Given equation is,  $2x^2 - \sqrt{5}x + 1 = 0$

On comparing with  $ax^2 + bx + c = 0$ ,

we get  $a = 2, b = -\sqrt{5}$  and  $c = 1$

Discriminant,  $D = b^2 - 4ac$   
 $= (-\sqrt{5})^2 - 4 \times (2) \times (1)$   
 $= 5 - 8 = -3 < 0$

Since, discriminant is negative, therefore quadratic equation  $2x^2 - \sqrt{5}x + 1 = 0$  has no real roots i.e., imaginary roots.

8. There are 60 terms in an A.P. of which the first term is 8 and the last term is 185. The 31<sup>st</sup> term is [1]

- (a) 56
- (b) 94
- (c) 85
- (d) 98

**Ans :** (d) 98

Let  $d$  be the common difference;

$$\text{Then } 60^{\text{th}} \text{ term, } = 8 + 59d = 185$$

$$59d = 177$$

$$d = 3$$

$$a_n = a + (n - 1)d$$

Hence,  $31^{\text{th}} \text{ term} = 8 + 30 \times 3 = 98$

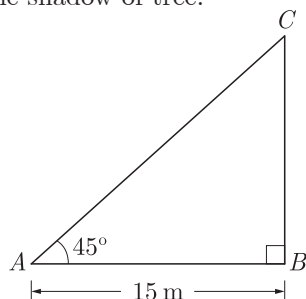
9. The height of a tree, if it casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is  $45^\circ$ , is [1]

- (a) 10 m
- (b) 14 m
- (c) 8 m
- (d) 15 m

**Ans :** (d) 15 m

Let  $BC$  be the tree of height  $h$  meter.

Let  $AB$  be the shadow of tree.



In  $\Delta ABC$ ,  $\angle C = 90^\circ$

$$\frac{BC}{BA} = \tan 45^\circ$$

$$BC = AB = 15 \text{ m}$$

10. The point on the  $X$ -axis which is equidistant from the points  $A(-2, 3)$  and  $B(5, 4)$  is [1]

- (a) (0, 2)
- (b) (2, 0)
- (c) (3, 0)
- (d) (-2, 0)

**Ans :** (b) (2, 0)

Let  $P(x, 0)$  be a point on  $X$ -axis such that,

$$AP = BP$$

$$AP^2 = BP^2$$

$$(x + 2)^2 + (0 - 3)^2 = (x - 5)^2 + (0 + 4)^2$$

$$x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$$

$$14x = 28$$

$$x = 2$$

Hence, required point = (2, 0)

**(Q.11-Q.15) Fill in the blanks.**

11. The value of  $\sin A$  or  $\cos A$  never exceeds ..... [1]

**Ans :** 1

12. Points (1, 5), (2, 3) and (-2, -11) are ..... [1]

**Ans :** Non-collinear

**or**

The value of the expression  $\sqrt{x^2 + y^2}$  is the distance of the point  $P(x, y)$  from the .....

**Ans :** Origin

13. Tangent is perpendicular to the ..... through the point of contact. [1]

**Ans :** radius

14. Two circles are drawn with same centre then the ..... circle have bigger radius. [1]

**Ans :** Outer

15. Two polygons of the same number of sides are similar, if all the corresponding angles are ..... [1]

**Ans :** equal

**(Q.16-Q.20) Answer the following**

16. Volume of two spheres are in the ratio 64 : 27, find the ratio of their surface areas. [1]

**Ans :**

$$\frac{\text{Volume of I}^{\text{st}} \text{ sphere}}{\text{Volume of II}^{\text{nd}}} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

Ratio of their surface areas

$$\frac{4A_1/A_2 \pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

or

Find the volume (in  $\text{cm}^3$ ) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

**Ans :**

Given,

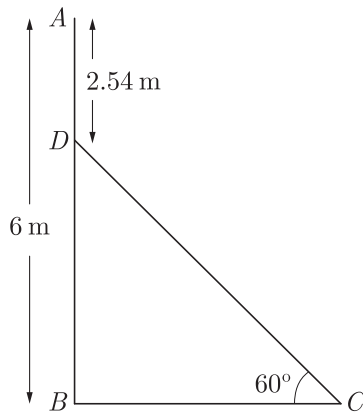
Edge of the cube = 4.2 cm.

Height of the cone,  $h = 4.2$  cm.

Radius of the cone,  $r = \frac{4.2}{2} = 2.1$  cm.

$$\begin{aligned} \text{Volume of the cone} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2 \\ &= 19.4 \text{ cm}^3 \end{aligned}$$

17. In the given figure,  $AB$  is a 6 m high pole and  $DC$  is a ladder inclined at an angle of  $60^\circ$  to the horizontal and reaches up to point  $D$  of pole. If  $AD = 2.54$  m, find the length of ladder. ( use  $\sqrt{3} = 1.73$ ) [1]



**Ans :**

We have  $AD = 2.54$  m  
 $DB = 6 - 2.54 = 3.46$  m

In  $\Delta BCD$ ,  $\angle B = 90^\circ$   
 $\sin 60^\circ = \frac{BD}{DC}$   
 $\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$   
 $DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46}{1.73} = 4$

Thus length of ladder is 4 m.

18. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring. [1]

**Ans :**

Circumference of the outer circle  $2\pi r_1 = 88$  cm

$$r_1 = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

Circumference of the inner circle  $2\pi r_2 = 66$  cm

$$r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm} = 10.5 \text{ cm}$$

Width of the ring

$$r_1 - r_2 = 14 - 10.5 \text{ cm} = 3.5 \text{ cm}$$

19. Following distribution gives cumulative frequencies of 'more than type' : [1]

Marks obtained	Marks obtained 5	More than of equal to 10	More than or equal to 15	More than of equal to 20
Number of student (cummulative frequency)	30	23	8	2

Change the above data to a continuous grouped frequency distribution.

**Ans :**

C.I	5-10	10-15	15-20	20-25
$f$	7	15	6	2

20. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of getting neither a red card nor a queen. [1]

**Ans :**

Given, Total number of cards = 52  
 Number of red cards = 26  
 Number of queens which are not red = 2  
 $\therefore$  Cards which are neither red nor queen  
 $= 52 - [26 + 2]$   
 $= 24$   
 $\therefore$  Required Probability =  $\frac{24}{52} = \frac{6}{13}$

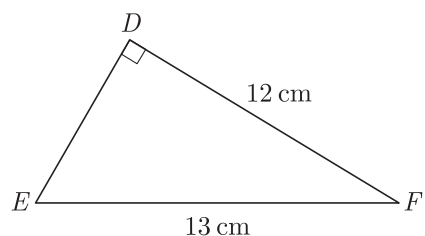
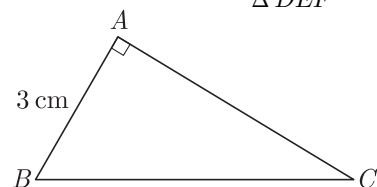
## Section B

21. Find the HCF and LCM of 90 and 144 by the method of prime factorization. [2]

**Ans :**

We have  $90 = 9 \times 10 = 2 \times 3^2 \times 5$   
 and  $144 = 16 \times 9 = 2^4 \times 3^2$   
 HCF =  $2 \times 3^2 = 18$   
 LCM =  $2^4 \times 3^2 \times 5 = 720$

22. Given  $\Delta ABC \sim \Delta DEF$ , find  $\frac{\Delta ABC}{\Delta DEF}$  [2]



**Ans :**

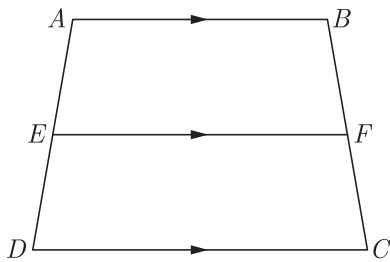
In  $\Delta DEF$ , we have

$$DE = \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144} = \sqrt{25} = 5$$

Thus 
$$\frac{ar \Delta ABC}{ar \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

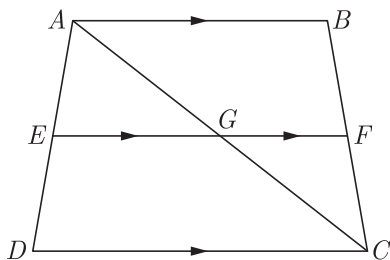
**or**

In the given figure, if  $ABCD$  is a trapezium in which  $AB \parallel CD \parallel EF$ , then prove that  $\frac{AE}{ED} = \frac{BF}{FC}$



**Ans :**

We draw,  $AC$  intersecting  $EF$  at  $G$  as shown below.



In  $\Delta CAB, GF \parallel AB$ , thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \quad \dots(1)$$

In  $\Delta ADC, EG \parallel DC$ , thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \quad \dots(2)$$

From equations (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC} \quad \text{Hence Proved.}$$

**23.** Find the roots of the quadratic equation  $\sqrt{3}x^2 - 2x - \sqrt{3}$ . [2]

**Ans :**

We have 
$$\begin{aligned} \sqrt{3}x^2 - 2x - \sqrt{3} &= 0 \\ \sqrt{3}x^2 - 3x + x - \sqrt{3} &= 0 \\ \sqrt{3}x(x - \sqrt{3}) + 1(x - \sqrt{3}) &= 0 \\ (x - \sqrt{3})(\sqrt{3} + 1) &= 0 \end{aligned}$$

Thus 
$$x = \sqrt{3}, \frac{-1}{\sqrt{3}}$$

**24.** Find the value of  $\lambda$ , if the mode of the following data is 20 :

15, 20, 25, 18, 13, 15, 25, 15, 18, 17, 20, 25, 20,  $\lambda$ , 18. [2]

**Ans :**

Writing the data as discrete frequency distribution, we get

$x_i$	$f_i$
13	1
15	3
17	1
18	3
20	3
$\lambda$	1
25	3

For 20 to be mode of the frequency distribution,  $\lambda = 20$ .

**or**

Find the unknown values in the following table :

Class Interval	Frequency	Cumulative Frequency
0-10	5	5
10-20	7	$x_1$
20-30	$x_2$	18
30-40	5	$x_3$
40-50	$x_4$	30

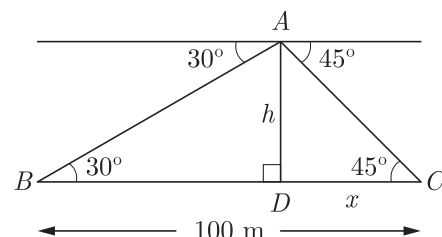
**Ans :**

$$\begin{aligned} x_1 &= 5 + 7 = 12 \\ x_2 &= 18 - 12 = 6 \\ x_3 &= 18 + 5 = 23 \\ \text{and} \\ x_4 &= 30 - 23 = 7 \end{aligned}$$

**25.** Two ships are approaching a light-house from opposite directions. The angle of depression of two ships from top of the light-house are  $30^\circ$  and  $45^\circ$ . If the distance between two ships is 100 m, find the height of light-house. [2]

**Ans :**

Let  $AD$  be the height ( $h$  meter) of the light-house and  $BC$  is the distance between the ships.



Given,  $BC = 100$  m  
In right  $\Delta ADC$ ,

$$\tan 45^\circ = \frac{AD}{DC}$$

$$1 = \frac{h}{DC}$$

$$DC = h \quad \dots(1)$$

In right  $\Delta ADB$ ,

$$\tan 30^\circ = \frac{AD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{100 - DC} = \frac{h}{100 - h}$$

$$100 - h = h\sqrt{3}$$

$$100 = h + h\sqrt{3} = h(1 + \sqrt{3})$$

$$h = \frac{100}{1 + \sqrt{3}} = \frac{100}{2.732} = 36.60$$

∴ Height of tower = 36.60 m

26. Read the following passage and the question that follows:

There are two small boxes *A* and *B*. In *A*, there are 9 white beads and 8 black beads. In *B*, there are 7 white and 8 black beads. We want to take a bead from a box. [2]

- (a) What is the probability of getting a white bead from a box?
- (b) A white bead and a black bead are added to box *B* and then a bead is taken from it. What is the probability of getting a white bead from it ?

**Ans :**

Total number of beads in box *A* =  $9W + 8B = 17$

Total number of beads in box *B* =  $7W + 8B = 15$

(a)  $P$  (white bead from box *A*) =  $\frac{9}{17}$

$P$  (white bead from box *B*) =  $\frac{7}{15}$

∴  $P$  (a white bead from each box)

$$= \frac{9}{17} \times \frac{7}{15} = \frac{21}{85}$$

(b) When a white bead and a black bead are added to box *B*, then

No. of white beads in box *B* =  $7W + 1W = 8W$

No. of black beads in box *B* =  $8B + 1B = 9B$

∴ Total number of beads in box *B* =  $8W + 9B = 17$

Hence,  $P$  (white bead from box *B*) =  $\frac{8}{17}$

### Section C

27. Solve for  $x$  :  $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}, x \neq 0, \frac{2}{3}, 2.$  [3]

**Ans :**

We have  $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}$

$$\frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$$

$$\frac{4x-3}{x(2x-3)} = \frac{1}{x-2}$$

$$(x-2)(4x-3) = 2x^2 - 3x$$

$$4x^2 - 11x + 6 = 2x^2 - 3x$$

$$2x^2 - 8x + 6 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

Thus  $x = 1, 3$

28. Use Euclid division lemma to show that the square of any positive integer cannot be of the form  $5m + 2$  or  $5m + 3$  for some integer  $m$ . [3]

**Ans :**

Let  $a$  be any positive integer, then by Euclid's division

algorithm  $a$  can be written as

$$a = bq + r, 0 \leq r < b \text{ and } q \in \omega$$

Take  $b = 5$ , then  $0 \leq r < 5$  because  $0 \leq r < b$

Thus  $a = 5q, 5q + 1, 5q + 2, 5q + 3$  and  $5q + 4$ ,

Now  $a^2 = (5q)^2 = 25q^2 = 5(5q^2) = 5m$

$$a^2 = (5q + 1)^2 = 25q^2 + 10q + 1 = 5m + 1$$

$$a^2 = (5q + 2)^2 = 25q^2 + 20q + 4 = 5m + 4$$

Similarly  $a^2 = (5q + 3)^2 = 5m + 4$

and  $a^2 = (5q + 4)^2 = 5m + 1$

Thus square of any positive integer cannot be of the form  $5m + 2$  or  $5m + 3$ .

**or**

Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

**Ans :**

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3 \times 3 = 3^2$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$15 = 3 \times 5$$

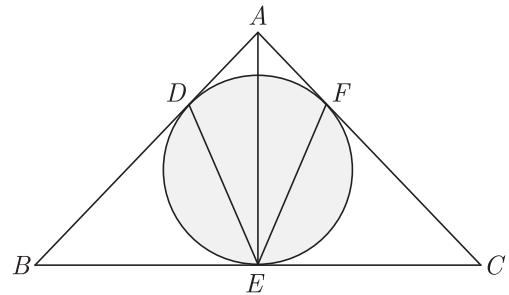
$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5 = 180 \text{ minutes}$$

The bells will toll next together after 180 minutes.

29. In  $\Delta ABD, AB = AC$ . If the interior circle of  $\Delta ABC$  touches the sides  $AB, BC$  and  $CA$  at  $D, E$  and  $F$  respectively. Prove that  $E$  bisects  $BC$ . [3]

**Ans :**

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At *A*,  $AF = AD$  (1)

At *B*,  $BE = BD$  (2)

At *C*,  $CE = CF$  (3)

Now we have  $AB = AC$

$$AD + DB = AF + FC$$

$$BD = FC \quad (AD = AF)$$

$$BE = EC \quad (BD = BE, CE = CF)$$

Thus  $E$  bisects  $BC$ .

30. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8<sup>th</sup> term, we get 6. [3]

**Ans :**

Let the first term be  $a$ , common difference be  $d$  and  $n$ th term be  $a_n$ .

We have  $a_3 = 9$   
 $a + 2d = 9$  ... (1)  
 and  $a_8 - a_5 = 6$   
 $(a + 7d) - (a + 4d) = 6$   
 $3d = 6$   
 $d = 2$

Substituting this value of  $d$  in equation (1), we get

$$a + 2(2) = 9$$

$$a = 5$$

So, A.P. is 5, 7, 9, 11, ...

or

If 7<sup>th</sup> term of an A.P. is  $\frac{1}{9}$  and 9<sup>th</sup> term is  $\frac{1}{7}$ , find 63<sup>rd</sup> term.

Ans :

Let the first term be  $a$ , common difference be  $d$  and  $n$ th term be  $a_n$ .

We have  $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$  (1)

$$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7}$$
 (2)

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} = \frac{1}{63}$$

Substituting the value of  $d$  in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$

$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus

$$a_{63} = a + (63 - 1)d$$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$

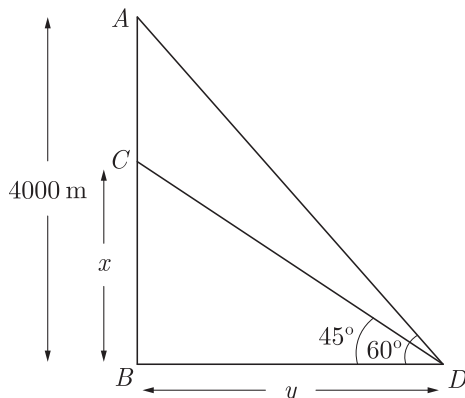
$$= \frac{63}{63} = 1$$

Hence,  $a_{63} = 1$

31. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use  $\sqrt{3} = 1.73$ ) [3]

Ans :

Let the height first plane be  $AB = 4000$  m and the height of second plane be  $BC = x$  m. As per given in question we have drawn figure below.



Here  $\angle BDC = 45^\circ$  and  $\angle ADB = 60^\circ$

In  $\Delta CBD$ ,  $\frac{x}{y} = \tan 45^\circ = 1 \Rightarrow x = y$

and in  $\Delta ABD$ ,  $\frac{4000}{y} = \tan 60^\circ = \sqrt{3}$

$$y = \frac{4000\sqrt{3}}{3} = 2306.67 \text{ m}$$

Thus vertical distance between two,

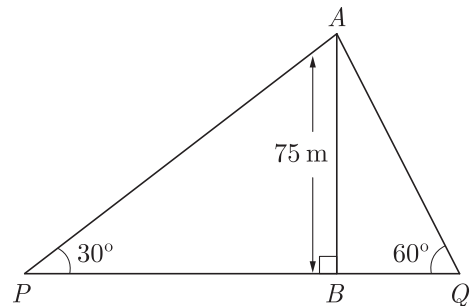
$$4000 - y = 4000 - 2306.67 = 1693.33 \text{ m}$$

or

Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. find the distance between the two men. (Use  $\sqrt{3} = 1.73$ )

Ans :

Let  $AB$  be the building and the two men are at  $P$  and  $Q$ . As per given in question we have drawn figure below.



In  $\Delta ABP$ ,  $\tan 30^\circ = \frac{AB}{BP}$

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$

$$BP = 75\sqrt{3} \text{ m}$$

In  $\Delta ABQ$ ,  $\tan 60^\circ = \frac{AB}{BQ}$

$$\sqrt{3} = \frac{75}{BQ}$$

$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

Distance between the two men,

$$PQ = BP + BQ = 75\sqrt{3} + 25\sqrt{3}$$

$$= 100\sqrt{3} = 100 \times 1.73 = 173$$

32. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use  $\pi = \frac{22}{7}$  [3]

Ans :

Given,

Height of cylinder = 2.1 m

Radius of cylinder = radius of cone =  $\frac{3}{2}$  m

Slant height of cone = 2.8 m

Surface area of tent

$$= C.S.A \text{ of cone} + C.S.A \text{ of cylinder.}$$

$$= \pi r l + 2\pi r h = \pi r(l + 2h)$$

Area of canvas required will be surface area of tent.

$$\begin{aligned} \text{Thus } \pi r(l + 2h) &= \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1) \\ &= \frac{33}{7} \times 7 = 33 \text{ m}^2 \end{aligned}$$

$$\text{Total Cost} = 33 \times 500 = 16,500$$

**33.** Read the following passage and the question that follows:

Roja, Renu and Reena are three friends. They decided to sweep a circular park near their homes. They divided the park into three parts by two equal chords  $AB$  and  $AC$  for convenience. [3]

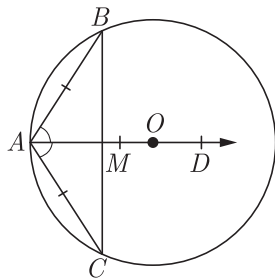
- (i) Prove that the centre of the park lies on the angle bisector of  $\angle BAC$ .
- (ii) Which mathematical concept is used in the above problem?

**Ans :**

(i) Given : A circle  $C(O, r)$  and chord  $AB = \text{chord } AC$ .  $AD$  is bisector of  $\angle CAB$ .

**To prove :** Centre  $O$  lies on the bisector of  $\angle BAC$ .

**Construction:** Join  $BC$ , meeting bisector  $AD$  of  $\angle BAC$ , at  $M$ .



**Proof :** In triangles  $BAM$  and  $CAM$ ,

$$AB = AC \quad (\text{given})$$

$$\angle BAM = \angle CAM \quad (\text{given})$$

and  $AM = AM \quad (\text{common})$

$$\Delta BAM \cong \Delta CAM \quad (\text{SAS})$$

$$BM = CM$$

and  $\angle BMA = \angle CMA$

As  $\angle BMA + \angle CMA = 180^\circ \quad (\text{linear pair})$

$$\angle BMA = \angle CMA = 90^\circ$$

$AM$  is the perpendicular bisector of the chord  $BC$ .

$AM$  passes through the centre  $O$ .

[Perpendicular bisector of chord of a circle passes through the centre of the circle]

Hence, the centre of the park lies on the angle bisector of  $\angle BAC$ .

**34.** 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. Use  $\pi = \frac{22}{7}$  [3]

**Ans :**

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

Volume of metal in 504 cones

$$= 504 \times \frac{\pi}{3} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3$$

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times r^3$$

= Volume of 504 cones

$$\frac{4\pi}{3} \times r^3 = 504 \times \frac{\pi}{3} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3$$

$$r^3 = 126 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 7 \times 9 \times 2 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 3 \times 3 \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \times 3$$

$$r = 3 \times \frac{7}{2} = 10.5 \text{ cm}$$

Thus diameter is 21 cm.

$$\text{Surface area } 4\pi r^2 = 4 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= 1386 \text{ cm}^2$$

## Section D

**35.** Solve the following pairs of linear equations by elimination method. [4]

(a)  $x + y = 5$  and  $2x - 3y = 4$

(b)  $3x + 4y = 10$  and  $2x - 2y = 2$

(c)  $3x - 5y - 4 = 0$  and  $9x = 2y + 7$

**Ans :**

(a) We have,  $x + y = 5 \quad \dots(1)$

and  $2x - 3y = 4 \quad \dots(2)$

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x + y) + (2x - 3y) = 3 \times 5 + 4$$

or,  $3x + 3y + 2x - 3y = 15 + 4$

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

Substituting  $x = \frac{19}{5}$  in equation (1),

$$\frac{19}{5} + y = 5$$

$$y = 5 - \frac{19}{5} = \frac{25 - 19}{5} = \frac{6}{5}$$

Hence,  $x = \frac{19}{5}$  and  $y = \frac{6}{5}$

(b) We have,  $3x + 4y = 10 \quad \dots(1)$

and  $2x - 2y = 2 \quad \dots(2)$

Multiplying equation (2) by 2 and adding in (1),

$$(3x + 4y) + 2(2x - 2y) = 10 + 2 \times 2$$

or,  $3x + 4y + 4x - 4y = 10 + 4$

or,  $7x = 14$

$$y = 1$$

Hence,  $x = 2$  and  $y = 1$ .

(c)

We have,  $3x - 5y = 4$  ... (1)

and  $9x = 2y + 7$  ... (2)

Multiplying equation (1) by 3 and rewriting equation (2) we have

$9x - 15y = 12$  ... (3)

$9x - 2y = 7$  ... (4)

Subtracting equation (4) from equation (3),

$-13y = 5$

$y = -\frac{5}{13}$

Substituting value of  $y$  in equation (1),

$3x - 5\left(-\frac{5}{13}\right) = 4$

$3x = 4 - \frac{25}{13}$

$x = \frac{27}{13 \times 3} = \frac{9}{13}$

Hence  $x = \frac{9}{13}$  and  $y = -\frac{5}{13}$

36. Find the other zeroes of the polynomial  $x^4 - 5x^3 + 2x^2 + 10x - 8$  if it is given that two zeroes are  $-\sqrt{2}$  and  $\sqrt{2}$ . [4]

Ans :

We have two zeroes  $\sqrt{2}$  and  $-\sqrt{2}$ .

Two factors are  $(x + \sqrt{2})$  and  $(x - \sqrt{2})$

$g(x) = (x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2$  is a factor of the given polynomial

$$\begin{array}{r} x^2 - 2 \overline{) x^4 - 5x^3 + 2x^2 + 10x - 8} \\ \underline{3x^4 \phantom{- 10x^3} - 2x^2} \phantom{+ 10x - 8} \\ -5x^3 + 4x^2 + 10x - 8 \\ \underline{-5x^3 \phantom{+ 4x^2} - 10x} \phantom{- 8} \\ 4x^2 - 8 \\ \underline{4x^2 \phantom{- 8}} \\ 0 \end{array}$$

Quotient =  $x^2 - 5x + 4 = (x - 4)(x - 1)$

Hence other zeroes are 4 and 1.

or

Find all the zeros of the polynomial

$3x^4 + 6x^3 - 2x^2 - 10x - 5$  if two of its zeroes are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$

Ans :

$$\begin{array}{r} x^2 + 2x + 1 \phantom{+ 1} \\ 3x^2 - 5 \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{3x^4 \phantom{+ 6x^3} - 5x^2} \phantom{- 10x - 5} \\ 6x^3 + 3x^2 - 10x - 5 \\ \underline{-6x^3 \phantom{+ 3x^2} - 10x} \phantom{- 5} \\ 3x^2 - 5 \\ \underline{3x^2 \phantom{- 5}} \\ 0 \end{array}$$

Since  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$  are two zeroes of the given polynomial.

So,  $(x - \sqrt{\frac{5}{3}}), (x + \sqrt{\frac{5}{3}})$  will be its two factors

$(x - \sqrt{\frac{5}{3}})(x + \sqrt{\frac{5}{3}}) = \frac{1}{3}(3x^2 - 5)$

is a factor of given polynomial

Now, dividing it by  $3x^2 - 5$ .

$x^2 + 2x + 1 = (x + 1)^2 = (x + 1)(x + 1)$

two other zeroes = -1 and -1

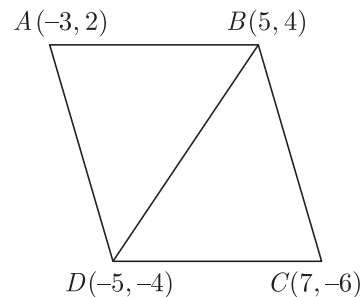
Hence all the zeroes of given polynomial

$= \sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}, -1$  and  $-1$

37. Find the area of a quadrilateral  $ABCD$ , the coordinates of whose vertices are  $A(-3, 2), B(5, 4), C(7, -6)$  and  $D(-5, -4)$ . [4]

Ans :

As per question the quadrilateral is shown below.



Area of triangle  $ABD$

$\Delta_{ABD} = \frac{1}{2} |-3(8) + 5(-6) + -5(2 - 4)|$   
 $= 22$  sq. units

Area of triangle  $BCD$

$\Delta_{BCD} = \frac{1}{2} |5(-2) + 7(-8) - 5(10)|$   
 $= 58$  sq. units

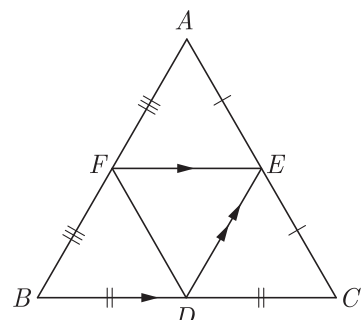
Area  $_{ABCD} = \Delta_{ABD} + \Delta_{BCD}$

$= 22 + 58 = 80$  sq. units

38. In  $\Delta ABC$ , the mid-points of sides  $BC, CA$  and  $AB$  are  $D, E$  and  $F$  respectively. Find ratio of  $ar(\Delta DEF)$  to  $ar(\Delta ABC)$ . [4]

Ans :

As per given condition we have drawn the figure below. Here  $F, E$  and  $D$  are the mid-points of  $AB, AC$  and  $BC$  respectively.





Hence,  $FE \parallel BC, DE \parallel AB$  and  $DF \parallel AC$

By mid-point theorem,

If  $DE \parallel BA$  then  $DE \parallel BF$

and if  $FE \parallel BC$  then  $FE \parallel BD$

Therefore  $FEDB$  is a parallelogram in which  $DF$  is diagonal and a diagonal of Parallelogram divides it into two equal Areas.

Hence  $ar(\Delta BDF) = ar(\Delta DEF) \dots(1)$

Similarly  $ar(\Delta CDE) = ar(\Delta DEF) \dots(2)$

$(\Delta AFE) = ar(\Delta DEF) \dots(3)$

$(\Delta DEF) = ar(\Delta DEF) \dots(4)$

Adding equation (1), (2), (3) and (4), we have

$$ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF) = 4ar(\Delta DEF)$$

$$ar(\Delta ABC) = 4ar(\Delta DEF)$$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

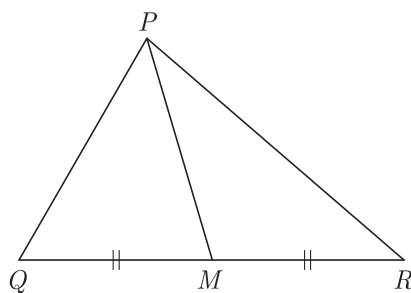
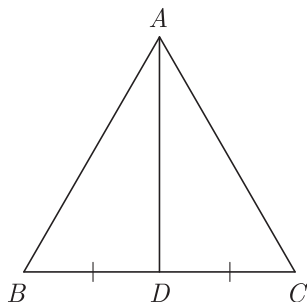
or

In  $\Delta ABC$ ,  $AD$  is the median to  $BC$  and in  $\Delta PQR$ ,  $PM$  is the median to  $QR$ . If  $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ . Prove that  $\Delta ABC \sim \Delta PQR$ .

Prove that  $\Delta ABC \sim \Delta PQR$ .

Ans :

As per given condition we have drawn the figure below.



In  $\Delta ABC$ ,  $AD$  is the median, therefore

$$BC = 2BD$$

and in  $\Delta PQR$ ,  $PM$  is the median,

$$QR = 2QM$$

Given,  $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$

or,  $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$

In triangles  $ABD$  and  $PQM$ ,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In  $\Delta ABC$  and  $\Delta PQR$ ,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus  $\Delta ABC \sim \Delta PQR$ . Hence Proved.

39. Given that  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ ,

find the values of  $\tan 75^\circ$  and  $\tan 90^\circ$  by taking suitable values of  $A$  and  $B$ . [4]

Ans :

We have  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

(i) or  $\tan 75^\circ = \tan(45^\circ + 30^\circ)$   
 $= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$   
 $= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$   
 $= \frac{(\sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$   
 $= \frac{3 + 2\sqrt{3} + 1}{(\sqrt{3})^2 - (1)^2} = \frac{4 + 2\sqrt{3}}{2}$

Hence  $\tan 75^\circ = 2 + \sqrt{3}$

(ii)  $\tan 90^\circ = \tan(60^\circ + 30^\circ)$   
 $= \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ}$   
 $= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$

Hence,  $\tan 90^\circ = \infty$

or

In an acute angled triangle  $ABC$ , if  $\sin(A + B - C) = \frac{1}{2}$  and  $\cos(B + C - A) = \frac{1}{\sqrt{2}}$ , find  $\angle A, \angle B$  and  $\angle C$ .

Ans :

We have  $\sin(A + B - C) = \frac{1}{2} = \sin 30^\circ$

or,  $A + B - C = 30^\circ \dots(1)$

and  $\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

or,  $B + C - A = 45^\circ \dots(2)$

Adding equation (1) and (2), we get

$$2B = 75^\circ$$

or,  $B = 37.5^\circ$

Now subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$

or,  $A - C = 7.5^\circ \dots(3)$

Now  $A + B + C = 180^\circ$   
 $A + B + C = 180^\circ$   
 $A + C = 180^\circ - 37.5^\circ = 142.5^\circ \dots(4)$

Adding equation (3) and (4), we have

$$2A = 135^\circ$$

or,  $A = 67.5^\circ$

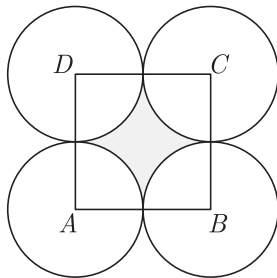
and,  $C = 75^\circ$

Hence,  $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$

40. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circle is  $\frac{24}{7}$  cm<sup>2</sup>. Find the radius of each circle. [4]

**Ans :**

As per question statement the figure is shown below.



Let  $r$  cm be the radius of each circle.

$$\text{Area of square} - \text{Area of 4 sectors} = \frac{24}{7} \text{ cm}^2$$

$$(2r)^2 - 4\left(\frac{90}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$$

$$4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$$

$$\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

$$6r^2 = 24$$

$$r^2 = 4$$

$$r = \pm 2$$

Thus radius of each circle is 2 cm.

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