CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-6

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. Two coins are tossed simultaneously. The probability of getting at most one head is [1]

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) 1

Ans : (c) $\frac{3}{4}$

$$\text{Total outcomes} = \text{HH, HT, TH, TT}$$
$$\text{Favourable outcomes} = \text{HT, TH, TT}$$
$$P(\text{at most one head}) = \frac{3}{4}$$

2. If the perimeter of one face of a cube is 20 cm, then its surface area is [1]

(a) 120 cm^2 (b) 150 cm^2

(c) 125 cm^2 (d) 400 cm^2

Ans : (b) 150 cm^2

$$\text{Edge of cube} = \frac{20}{4}\text{ cm} = 5\text{ cm}$$
$$\text{Surface area} = 6\times 5^2\text{ cm}^2 = 150\text{ cm}^2$$

3. Which of the following will have a terminating decimal expansion? [1]

(a) $\frac{77}{210}$ (b) $\frac{23}{30}$

(c) $\frac{125}{441}$ (d) $\frac{23}{8}$

Ans : (d) $\frac{23}{8}$

For terminating decimal expansion, denominator must have only 2 or only 5 or 2 and 5 as factor.

Here, $\frac{23}{8} = \frac{23}{(2)^3}$

(only 2 as factor of denominator so terminating)

4. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than x^2+y^2 is equal to [1]

(a) 0 (b) 1/2

(c) 1 (d) 3/2

Ans : (c) 1

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$
$$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$$
$$x\sin\theta(\sin^2\theta+\cos^2\theta) = \sin\theta\cos\theta$$
$$x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$$

Now, $x\sin\theta = y\cos\theta$

$$\cos\theta\sin\theta = y\cos\theta$$
$$y = \sin\theta$$

Hence, $x^2+y^2 = \cos^2\theta+\sin^2\theta = 1$

5. One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The correct roots are [1]

(a) $3, -2$ (b) $-3, 2$

(c) $-6, -1$ (d) $6, -1$

Ans : (d) $6, -1$

Let α, β be the roots of the equation.

Then, $\alpha+\beta = 5$

and $\alpha\beta = -6.$

So, the equation is

$$x^2 - 5x - 6 = 0$$

The roots of the equation are 6 and –1.

6. A motor boat takes 2 hours to travel a distance 9 km. down the current and it takes 6 hours to travel the same distance against the current. The speed of the boat in still water and that of the current (in km/hour) respectively are [1]

(a) 3, 1.5 (b) 3, 2

(c) 3.5, 2.5 (d) 3, 1

Ans : (a) 3, 1.5

$$\text{Down-rate} = 9\div 2 = 4.5\text{ km/hr}$$
$$\text{Uprate} = 9\div 6 = 1.5\text{ km/hr}$$
$$\text{Speed of the boat} = (4.5+1.5)\div 2 = 3\text{ km/hr}$$
$$\text{Speed of the current} = (4.5-1.5)\div 2 = 1.5\text{ km/hr}$$

7. Five distinct positive integers are in a arithmetic progression with a positive common difference. If their sum is 10020, then the smallest possible value of the last term is [1]
(a) 2002 (b) 2004
(c) 2006 (d) 2007

Ans : (c) 2006

Let the five integers be $a-2d$, $a-d$, a, $a+d$, $a+2d$.
Then, we have,
$$(a-2d)+(a-d)+a+(a+d)+(a+2d) = 10020$$
$$5a = 10020 \Rightarrow a = 2004$$
Now, as smallest possible value of d is 1.
Hence, the smallest possible value of $a+2d$ is 2004+2 = 2006

8. The value of the polynomial $x^8 - x^5 + x^2 - x + 1$ is [1]
(a) positive for all the real numbers
(b) negative for all the real numbers
(c) 0
(d) depends on value of x

Ans : (a) positive for all the real numbers

Let $f(x) = x^8 - x^5 + x^2 - x + 1$
For $x = 1$ or 0
$f(x) = 1 > 0$
For $x < 0$
each term of $f(x)$ is Positive and so first $f(x) > 0$.

Hence, $f(x)$ is Positive for all real x.

9. If the area of a semi-circular field is 15400 sq m, then perimeter of the field is: [1]
(a) $160\sqrt{2}$ m (b) $260\sqrt{2}$ m
(c) $360\sqrt{2}$ m (d) $460\sqrt{2}$ m

Ans : (c) $360\sqrt{2}$ m

Let the radius of the field be r.

Then, $\frac{\pi r^2}{2} = 15400$
$$\frac{1}{2} \times \frac{22}{7} \times r^2 = 15400$$
$$r^2 = 15400 \times 2 \times \frac{7}{22} = 9800$$
$$r = 70\sqrt{2} \text{ m}$$

Thus, perimeter of the field
$$= \pi r + 2r$$
$$= \frac{22}{7} \times 70\sqrt{2} + 2 \times 70 \times \sqrt{2}$$
$$= 220\sqrt{2} + 140\sqrt{2}$$
$$= \sqrt{2}(220 + 140) = 360\sqrt{2} \text{ m}$$

10. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set [1]
(a) Is increased by 2
(b) Is decreased by 2
(c) Is two times the original median
(d) Remains the same as that of the original set

Ans : (d) Remains the same as that of the original set

Since, $n = 9$
then, median term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ item.

Now, last four observations are increased by 2.
The median is 5^{th} observation, which is remaining unchanged.
There will be no change in median.

(Q.11-Q.15) Fill in the blanks.

11. The total surface area of a solid hemisphere having radius r is .... [1]
Ans : $3\pi r^2$

12. The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2,5)$, $B(6,9)$ and $C(8,5)$ is .... [1]
Ans : $(0,1)$

or

(5, − 2) (6, 4) and (7, − 2) are the vertices of an .... triangle.
Ans : Isosceles

13. The region enclosed by an arc and a chord is called the .... of the circle. [1]
Ans : Segment

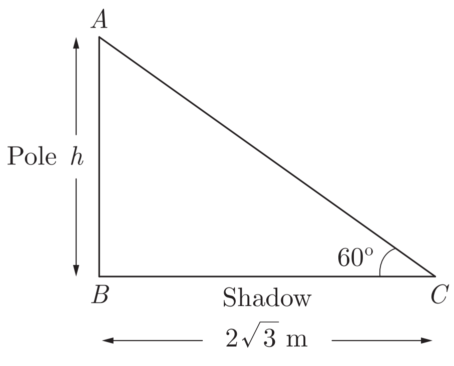
14. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. The value of $\tan P$ is .... [1]
Ans : 12/5

15. An algorithm which is used to find HCF of two positive numbers is .... [1]
Ans : Euclid's division algorithm

(Q.16-Q.20) Answer the following

16. A pole casts a shadow of length $2\sqrt{3}$ m on the ground, when the Sun's elevation is 60°. Find the height of the pole. [1]
Ans :

Let the height of pole be h. As per given in question we have drawn figure below.



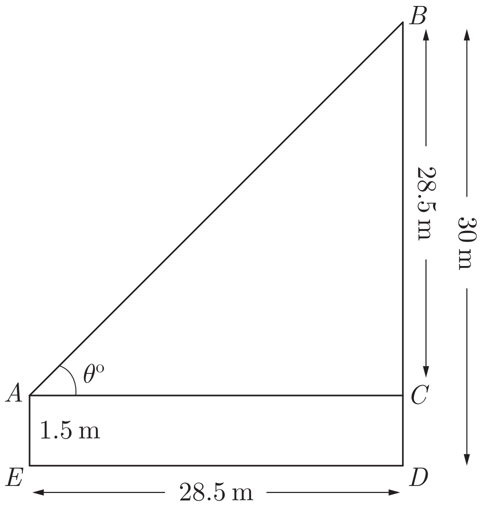
$$\frac{h}{2\sqrt{3}} = \tan 60^\circ$$
$$h = 2\sqrt{3}\tan 60^\circ$$
$$= 2\sqrt{3} \times \sqrt{3} = 6 \text{ m}$$

or

An observer 1.5 m tall is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

Ans :

As per given in question we have drawn figure below.



Here $AE = 1.5$ m is height of observer and $BD = 30$ m is tower.

Now $BC = 30 - 1.5 = 28.5$ m

In ΔBAC, $\tan\theta = \dfrac{BC}{AC}$

$$\tan\theta = \frac{28.5}{28.5} = 1 = \tan 45^\circ$$

$$\theta = 45^\circ$$

Hence angle of elevation is 45°

17. If ratio of corresponding sides of two similar triangles is $5:6$, then find ratio of their areas. [1]

Ans :

Let the triangles be ΔABC and ΔDEF

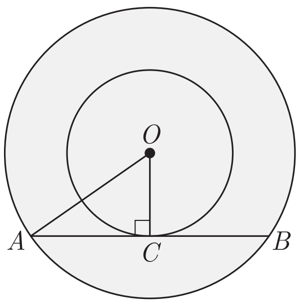
$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

$25:36$

18. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle. [1]

Ans :

As per the given question we draw the figure as below.



Here AB is the chord of large circle which touch the smaller circle at point C. We can see easily that ΔAOC is right angled triangle.

Here, $AO = 5$ cm, $OC = 3$ cm

$$AC = \sqrt{AO^2 - OC^2}$$
$$= \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9} = \sqrt{16} = 4 \text{ cm}$$

Length of chord, $AB = 8$ cm.

19. A line Segment AB is divided at point P such that $\dfrac{PB}{AB} = \dfrac{3}{7}$, then find the ratio $AP : PB$. [1]

Ans :

Here, $AB = 7, PB = 3$

$\therefore$ $AP = AB - PB = 7 - 3 = 4$

$\therefore$ $AP : PB = 4 : 3$

20. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of original cylinder. [1]

Ans :

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h}$$

$$= \frac{1}{4} = 1 : 4$$

Section B

21. For what value of 'k', the system of equations $kx + 3y = 1$, $12x + ky = 2$ has no solution. [2]

Ans :

The given equations can be written as

$kx + 3y - 1 = 0$ and $12x + ky - 2 = 0$

Here, $a_1 = k, b_1 = 3, c_1 = -1$

and $a_2 = 12, b_2 = k, c_2 = -2$

The equation for no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

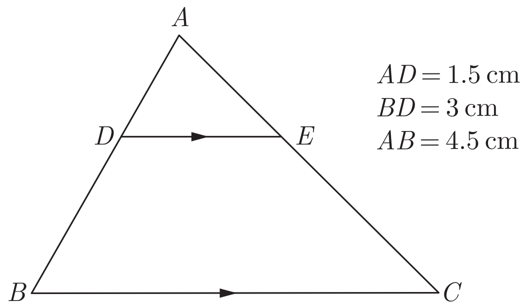
or, $$\frac{k}{12} = \frac{3}{k} \neq \frac{-1}{-2}$$

From $\dfrac{k}{12} = \dfrac{3}{k}$ we have $k^2 = 36 \Rightarrow k \pm 6$

From $\dfrac{3}{k} \neq \dfrac{-1}{-2}$ we have $k \neq 6$

Thus $k = -6$

22. In the given figure, $DE || BC$. If $AD = 1.5$ cm $BD = 2AD$, then find $\dfrac{ar(\Delta ADE)}{ar(\text{trapezium } BCED)}$ [2]



Ans :

We have $AD = 1.5$ cm, $BD = 3$

and $AB = AD + BD = 1.5 + 3.0 = 4.5$ cm

In triangle ADE and ABC, $\angle A$ is common and $DE || BC$

Thus $\angle ADE = \angle ABC$

$\angle AED = \angle ACB$ (corresponding angles)

By AA similarity we have

$$\Delta ADE \sim \Delta ABC$$

Now $$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{(1.5)^2}{(4.5)^2} = \frac{1}{9}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC) - ar(\Delta ADE)} = \frac{1}{9-1}$$

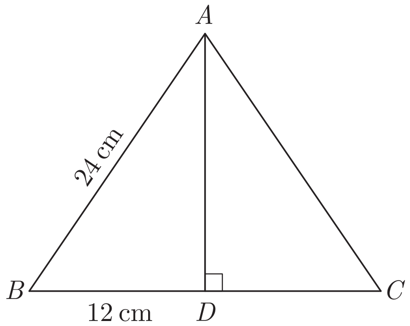
$$\frac{ar(\Delta ADE)}{ar(trapezium\ BCED)} = \frac{1}{8}$$

or

In an equilateral triangle of side 24 cm, find the length of the altitude.

Ans :

Let ΔABC be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $$BD = \frac{BC}{2} = \frac{24}{2} = 12 \text{ cm}$$

$$AB = 24 \text{ cm}$$

$$AD = \sqrt{AB^2 - BD^2} = \sqrt{(24)^2 - (12)^2}$$

$$= \sqrt{576 - 144} = \sqrt{432} = 12\sqrt{3}$$

Thus $AD = 12\sqrt{3}$ cm

$\therefore$ The length of the altitude is $12\sqrt{3}$ cm.

23. Prove that the point $(3,0)$, $(6,4)$ and $(-1,3)$ are the vertices of a right angled isosceles triangle. [2]

Ans :

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now $$AB^2 = (3-6)^2 + (0-4)^2$$

$$= 9 + 16 = 25$$

$$BC^2 = (6+1)^2 + (4-3)^2$$

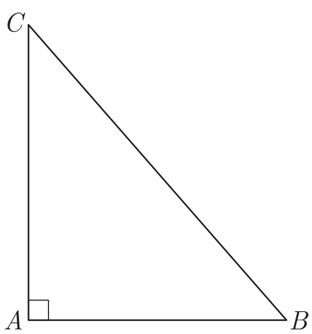
$$= 49 + 1 = 50$$

$$CA^2 = (-1-3)^2 + (3-0)^2$$

$$= 16 + 9 = 25$$

$$AB^2 = CA^2 \text{ or, } AB = CA$$

Hence triangle is isosceles.



Also, $$25 + 25 = 50$$

or, $$AB^2 + CA^2 = BC^2$$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

24. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid. [2]

Ans :

$$\text{Diameter of hemisphere} = \text{Side of cubical block}$$

$$2r = 7$$

or, $$r = \frac{7}{2}$$

Surface area of solid

$= $ Surface area of the cube

$-$ Area of base of hemisphere

$+$ curved surface area of hemisphere

$$= 6l^2 - \pi r^2 + 2\pi r^2$$

$$= 6 \times 49 - 11 \times \frac{7}{2} + 77 = 332.5 \text{ cm}^2$$

or

A metallic solid sphere of radius 4.2 cm is melted and recast into the shape of a solid cylinder of radius 6 cm. Find the height of the cylinder.

Ans :

$$\text{Volume of sphere} = \text{Volume of cylinder}$$

$$\frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\frac{4}{3} \times (4.2)^3 = 6^2 \times h$$

$$h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$

Hence, height of cylinder $h = 2.744$ cm.

25. There are two covers A and B each containing paper slips with natural numbers from 1 to 7 written on them. One slip is drawn from each cover. Using them, a two digit number is formed with a number from A in the units place and the number from B in the tens place. How many such two digit numbers can be formed? What is the probability that a two digit number so formed is even? [2]

Ans :

Number of slips in cover $A = 7$

Number of slips in cover $B = 7$

Numbers formed with a number from cover A in the units place and a number from cover B in the tens place are as follows:

11	12	13	14	15	16	17
31	22	23	24	25	26	27
31	32	33	34	35	36	37
41	42	43	45	45	46	47
51	52	53	54	55	56	57
61	62	63	64	65	66	67
71	72	73	74	75	76	77

Thus, total number of two digit numbers

$$= 7 \times 7 = 49$$

Number of two digit even numbers

$$= 7 \times 3 = 21$$

P (two digit number so formed is even)

$$= \frac{21}{49} = \frac{3}{7}$$

26. Read the following passage and the question that follows:
The radius and height of a wax made cylinder are 6 cm and 12 cm respectively. A cone of same base radius and height has been made from this cylinder by cutting out. [2]
(a) Find the volume of cone
(b) How many candles with 1 cm radius and 12 cm height can be made using the remaining wax.

Ans :

(a) Volume of the cone $= \frac{1}{3}\pi \times (6)^2 \times 12$

$= 144\pi$ cubic centimetre

(b) Volume of the cylinder $= \pi r^2 h = 144\pi \times 3$

$= 432\pi$ cubic centimetre

Volume of the remaining wax $= 288\pi$ cubic centimetre

Volume of one candle $= \pi \times 1^2 \times 12$

$= 12\pi$ cubic centimetres

Number of candles $= \frac{288\pi}{12\pi} = 24$

Section C

27. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6. [3]

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = 9$

$$a + 2d = 9 \quad ...(1)$$

and $a_8 - a_5 = 6$

$$(a+7d) - (a+4d) = 6$$

$$3d = 6$$

$$d = 2$$

Substituting this value of d in equation (1), we get

$$a + 2(2) = 9$$

$$a = 5$$

So, A.P. is 5, 7, 9, 11, ...

28. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'. [3]

Ans :

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha + \beta = -\frac{-5}{a} = \frac{5}{a}$

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10 \quad ...(1)$

and $\frac{c}{a} = 10 \quad ...(2)$

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

or

If α and β are the zeroes of a quadratic polynomial such that $\alpha + \beta = 0$ and $\alpha - \beta = 8$. Find the quadratic polynomial having α and β as its zeroes.

Ans :

We have $\alpha + \beta = 24 \quad ...(1)$

$$\alpha - \beta = 8 \quad ...(2)$$

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

Subtracting (1) from (2) we have

$$2\beta = 24 \Rightarrow \beta = 12$$

Hence, the quadratic polynomial

$$p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$$

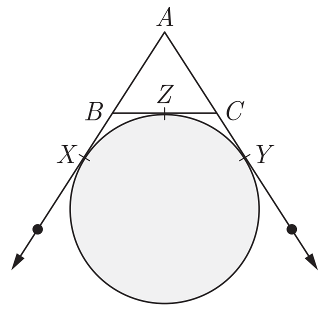
$$= x^2 - (16+8)x + (16)(8)$$

$$= x^2 - 24x + 128$$

29. ABC is a triangle. A circle touches sides AB and AC produced and side BC at BC at X, X, Y and Z respectively. Show that $AX = \frac{1}{2}$ perimeter of ΔABC. [3]

Ans :

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AX = AY$ (1)

At B $BX = BZ$ cm (2)

At C $CY = CZ$ (3)

Perimeter of ΔABC,

$$p = AB + AC + BC$$

$$= (AX - BX) + (AY - CY) + (BZ + ZC)$$

$$= AX + AY - BX + BZ + ZC - CY$$

From eq. (1), (2) and (3), we get

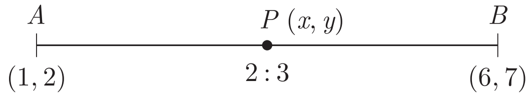
$$= AX + AY = 2AX$$

Thus $AX = \frac{1}{2}p$ Hence Proved

30. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$ [3]

Ans :

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP{:}PB = 2{:}3$

By using section formula,

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{14+6}{5} = 4$$

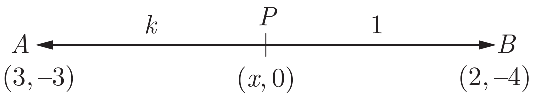
Thus $P(x,y) = (3,4)$

or

Find the ratio in which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Ans :

y co-ordinate of any point on the x will be zero. Let $(x,0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$.

Using section formula for y co-ordinate we have

$$0 = \frac{1(-3) + k(7)}{1+k}$$

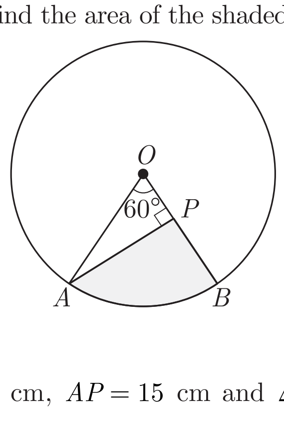
$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have

$$x = \frac{1(3) + k(-2)}{1+k} = \frac{3 - 2 \times \frac{3}{7}}{1 + \frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $(\frac{3}{2}, 0)$.

31. In the given figure, AOB is a sector of angle 60° of a circle with centre O and radius 17 cm. If $AP \perp OB$ and $AP = 15$ cm, find the area of the shaded region. [3]



Ans :

Here $OA = 17$ cm, $AP = 15$ cm and ΔOPA is right triangle

Using Pythagoras theorem, we have

$$OP = \sqrt{17^2 - 15^2} = 8 \text{ cm}$$

Area of the shaded region

$= $ Area of the sector $\Delta\, OAB$

$-$ Area of ΔOPA

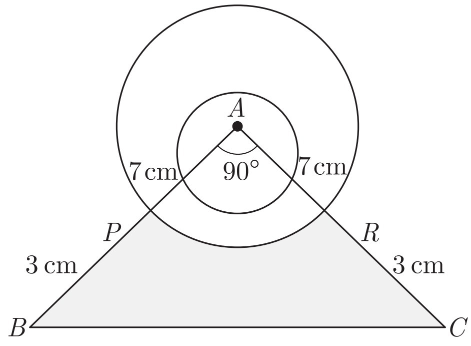
$$= \frac{60}{360} \times \pi r^2 - \frac{1}{2} \times b \times h$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15$$

$$= 151.38 - 60 = 91.38\, cm^2$$

or

A memento is made as shown in the figure. Its base $PBCR$ is silver plate from the Front side. Find the area which is silver plated. Use $\pi = \frac{22}{7}$.



Ans :

From the given figure

Area of right-angled $\Delta ABC = \frac{1}{2} \times 10 \times 10 = 50$

Area of quadrant APR of the circle of radii 7 cm

$$= \frac{1}{4} \times \pi \times (7)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 49 = 38.5 \text{ cm}^2$$

Area of base $PBCR$

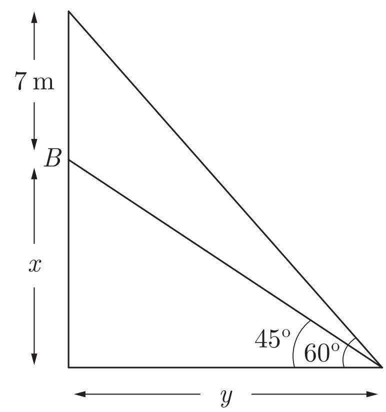
$= $ Area of $\Delta ABC -$ Area of quadrant APR

$= 50 - 38.5 = 11.5$ cm^2

32. A 7m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60^o and 45^o respectively. Find the height of the tower correct to one place of decimal. (Use $\sqrt{3} = 1.73$) [3]

Ans :

As per given in question we have drawn figure below.



$$\frac{x}{y} = \tan 45^o = 1 \Rightarrow x = y$$

$$\frac{x+7}{x} = \tan 60^o = \sqrt{3}$$

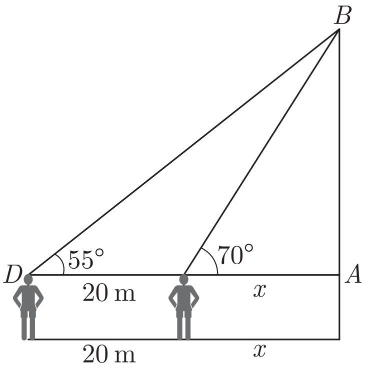
$$7 = (\sqrt{3}-1)x$$

$$x = \frac{7(\sqrt{3}+1)}{2} = \frac{7(2.73)}{2} = 9.6 \text{ m}$$

33. Read the following passage and the question that follows:
One sees the top of a tree on the bank of a river at an elevation of 70° from the other bank. Stepping 20 metres back, he sees the top of the tree at an elevation of 55°. Height of the person is 1.4 metres.
(a) Draw a rough figure and mark the measurements.
(b) Find the height of the tree.
(c) Find the width of the river.
$[\tan 70^\circ = 2.75;\ \tan 55^\circ = 1.43]$ [3]

Ans :

(a)



(b) In $\Delta\, ABC$,

$$\tan 70^\circ = \frac{AB}{x}$$

$$AB = x\tan 70^\circ$$

(c)

In $\Delta\, ABD$, $\tan 55^\circ = \dfrac{AB}{x+20}$

$$AB = (x+20)\tan 55^\circ$$

$$x\tan 70^\circ = (x+20)\tan 55^\circ$$

$$x\tan 70^\circ = x\tan 55^\circ + 20\tan 55^\circ$$

$$x(\tan 70^\circ - \tan 55^\circ) = 20\tan 55^\circ$$

$$x = \frac{20 \times \tan 55^\circ}{\tan 75^\circ - \tan 55^\circ}$$

$$= \frac{20 \times 1.43}{2.75 - 1.43} = \frac{28.6}{1.32}$$

$$= 21.67$$

$$AB = x\tan 70 = 21.67 \times 2.75$$

$$= 59.59 \text{ m}$$

Height of the tree $= 59.59 + 1.4$

$= 60.99$ m

Width of the river $= 21.67$ m

34. Find the area of the rhombus of vertices $(3,0),(4,5)$, $(-1,4)$ and $(-2,-1)$ taken in order. [3]

Ans :

We have $A(3,0), B(4,5), C(-1,4), D(-2,-1)$

Diagonal AC, $d_1 = \sqrt{(3+1)^2 + (0-4)^2}$

$$= \sqrt{16+16} = \sqrt{32}$$

$$= \sqrt{16 \times 2} = 4\sqrt{2}$$

Diagonal BD, $d_2 = \sqrt{(4+2)^2 + (5+1)^2}$

$$= \sqrt{36+36} = \sqrt{72}$$

$$= \sqrt{36 \times 2} = 6\sqrt{2}$$

Area of rhombus $= \dfrac{1}{2} \times d_1 \times d_2$

$$= \frac{1}{2} 4\sqrt{2} \times 6\sqrt{2}$$

$= 24$ sq. unit.

Section D

35. Find the HCF of 256 and 36 using Euclid's Division Algorithm. Also, find their LCM and verify that HCF $\times$ LCM = Product of the two numbers. [4]

Ans :

By using Euclid's Division Lemma, we have

$$256 = 36 \times 7 + 4$$

$$36 = 4 \times 9 + 0$$

Hence, the HCF of 256 and 36 is 4.

LCM : $256 = 2^8$

$$36 = 2^2 \times 3^2$$

$$\text{LCM } (36, 256) = 2^8 \times 3^2 = 256 \times 9 = 2304$$

HCF $\times$ LCM = Product of the two number

$$4 \times 2{,}304 = 256 \times 36$$

$9216 = 9{,}216$ Hence verified.

36. Solve for x and y : [4]

$$2x - y + 3 = 0$$

$$3x - 5y + 1 = 0$$

Ans :

We have $2x - y + 3 = 0$ ...(1)

$3x - 5y + 1 = 0$ ...(2)

Multiplying equation (1) by 5 and subtracting (2) from it we have

$$7x = -14$$

$$x = \frac{-14}{7} = -2$$

Substituting the value of x in equation (1), we get

$$2x - y + 3 = 0$$

$$2(-2) - y + 3 = 0$$

$$-4 - y + 3 = 0$$

$$-y - 1 = 0$$

$$y = -1$$

Hence, $x = -2$ and $y = -1$.

or

A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number.

Ans :

Let the digits of number be x and y, then number will $10x + y$

According to the question, we have

$$8(x+y)-5 = 10x+y$$
$$2x-7y+5 = 0 \qquad ...(1)$$

also
$$16(x-y)+3 = 10x+y$$
$$6x-17y+3 = 0 \qquad ...(2)$$

Comparing the equation with $ax+by+c=0$ we get

$$a_1 = 2, b_1 = -1, c_1 = 5$$
$$a_2 = 6, b_2 = -17, c_2 = 3$$

Now
$$\frac{x}{b_2c_1-b_1c_2} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{c_1b_2-a_2b_1}$$

$$\frac{x}{(-7)(3)-(-17)(5)} = \frac{y}{(5)(6)-(2)(3)}$$
$$= \frac{1}{(2)(-17)-(6)(-7)}$$

$$\frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\frac{x}{8} = \frac{y}{3} = 1$$

Hence, $x = 8, y = 3$

So, required number $= 10\times 8+3 = 83$.

37. In an acute angled triangle ABC, *if* $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$, find $\angle A, \angle B$ and $\angle C$. [4]

Ans :

We have $\sin(A+B-C) = \frac{1}{2} = \sin 30^{\circ}$

or, $A+B-C = 30^{\circ}$ ...(1)

and $\cos(B+C-A) = \frac{1}{\sqrt{2}} = \cos 45^{\circ}$

or, $B+C-A = 45^{\circ}$ ...(2)

Adding equation (1) and (2), we get

$$2B = 75^{\circ}$$

or, $B = 37.5^{\circ}$

Now subtracting equation (2) from equation (1) we get,

$$2(A-C) = -15^{\circ}$$

or, $A-C = 7.5^{\circ}$ ...(3)

Now $A+B+C = 180^{\circ}$

$$A+B+C = 180^{\circ}$$
$$A+C = 180^{\circ}-37.5^{\circ} = 142.5^{\circ} \qquad ...(4)$$

Adding equation (3) and (4), we have

$$2A = 135^{\circ}$$

or, $A = 67.5^{\circ}$

and, $C = 75^{\circ}$

Hence, $\angle A = 67.5^{\circ}, \angle B = 37.5^{\circ}, \angle C = 75^{\circ}$

38. The denominator of a fraction is two more than its numerator. If the sum of the fraction and its reciprocal is $\frac{34}{15}$, find the fraction. [4]

Ans :

Let numerator be x, then denominator will be $x+2$.

and fraction $= \frac{x}{x+2}$

Now
$$\frac{x}{x+2}+\frac{x+2}{x} = \frac{34}{15}$$

$$15(x^2+x^2+4x+4) = 34(x^2+2x)$$
$$30x^2+60x+60 = 34x^2+68x$$
$$4x^2+8x-60 = 0$$
$$x^2+2x-15 = 0$$
$$x^2+5x-3x-15 = 0$$
$$x(x+5)-3(x+5) = 0$$
$$(x+5)(x-3) = 0$$

We reject the $x=-5$. Thus $x = 3$ and fraction $= \frac{3}{5}$

or

A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans :

Let the speed of stream be x km/h
Then the speed of boat upstream $= (24-x)$ km/h
Speed of boat downstream $= (24+x)$ km/h
According to the question,

$$\frac{32}{24-x}-\frac{32}{24+x} = 1$$

$$32\left[\frac{1}{24-x}-\frac{1}{24+x}\right] = 1$$

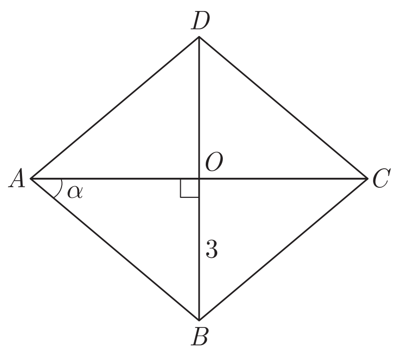
$$32\left[\frac{24+x-24+x}{576-x^2}\right] = 1$$

$$32(24+x-24+x) = 576-x^2$$
$$64x = 576-x^2$$
$$x^2+64x-576 = 0$$
$$x^2+72x-8x-576 = 0$$
$$x(x+72)-8(x+72) = 0$$
$$(x-8)(x+72) = 0$$
$$x = 8, -72$$

Since speed cannot be negative, we reject $x=-72$.

The speed of steam is 8 km/h.

39. $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD. [4]



Ans :

We have $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm

In ΔAOB, $\cos\alpha = \frac{2}{3} = \frac{AO}{AB}$

Let $OA = 2x$ then $AB = 3x$

Now in right angled triangle ΔAOB we have

$$AB^2 = AO^2 + OB^2$$
$$(3x)^2 = (2x)^2 + (3)^2$$
$$9x^2 = 4x^2 + 9$$
$$5x^2 = 9$$
$$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

Hence, $OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}}$ cm

and $AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}}$ cm

Diagonal $BD = 2 \times OB = 2 \times 3 = 6$ cm

and $AC = 2AO$

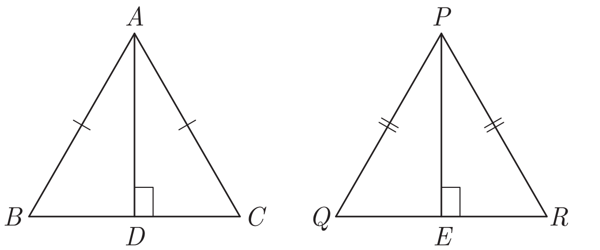
$$= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}} \text{ cm}$$

or

Vertical angles of two isosceles triangles are equal. If their areas are in the ratio $16:25$, then find the ratio of their altitudes drawn from vertex to the opposite side.

Ans :

As per given condition we have drawn the figure below.



Here $\angle A = \angle P$

$\angle B = \angle C, \angle Q = \angle R$

Let $\angle A = \angle P$ be x.

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

$x^2 + \angle B + \angle B = 180^\circ$ $(\angle B = \angle C)$

$2\angle B = 180^\circ - x$

$\angle B = \frac{180^\circ - x}{2}$ ...(1)

Now, in ΔPQR

$\angle P + \angle Q + \angle R = 180^\circ$ $(\angle Q = \angle R)$

$x^2 + \angle Q + \angle Q = 180^\circ$

$2\angle Q = 180^\circ - x$

$\angle Q = \frac{180^\circ - x}{2}$ ...(2)

In ΔABC and ΔPQR,

$\angle A = \angle P$ [Given]

$\angle B = \angle Q$ [From eq. (1) and (2)]

Due to AA similarity,

$\Delta ABC \sim \Delta PQR$

Now $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PE^2}$

$$\frac{16}{25} = \frac{AD^2}{PE^2}$$
$$\frac{4}{5} = \frac{AD}{PE}$$
$$\frac{AD}{PE} = \frac{4}{5}$$

40. Find the median of the following data :

Class Interval	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	6	8	10	12	6	5	3

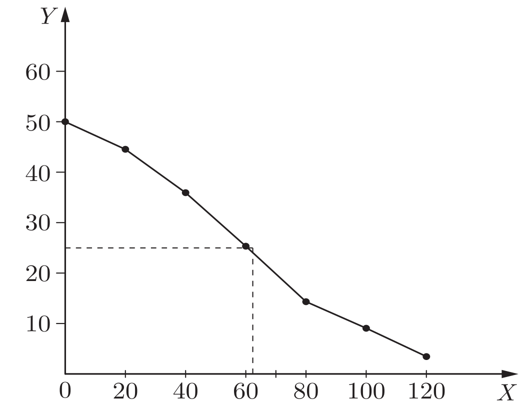
How can we find the median graphically ? [4]

Ans :

(i)

Classes	*c.f.*
More than 0	50
More than 20	44
More than 40	36
More than 60	26
More than 80	14
More than 100	8
More than 120	3

To draw on ogive we take the indices : (0,50),(20, 44), (40, 36), (60, 26), (80, 14), (100, 8), (120, 3)



From graph, $\frac{N}{2} = \frac{50}{2} = 25$

$\therefore$ Median $= 61.6$

(ii) By Formula Method :

Classes	*f*	*c.f.*	
0-20	6	6	
20-40	8	14	
40-60	10	24	
60-80	12	36	Median Class
80-100	6	42	
100-120	5	47	
120-140	3	50	

$$\text{Median} = \frac{N}{2}\text{th term}$$
$$= \frac{50}{2} = 25\text{th term}$$

$$\therefore \quad \text{Median class} = 60 - 80$$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f} \right) \times h$$

$$= 60 + \frac{1}{12} \times 20$$

$$= 60 + \frac{5}{3}$$

$$= \frac{185}{3}$$

$$= 61.67$$