CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-5

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. If the difference of mode and median of a data is 24, then the difference of median and mean is [1]

(a) 12 (b) 24
(c) 08 (d) 36

Ans : (a) 12

We have,

Mode − Median = 24

We know that, Mode = 3 (Median) − 2 Mean

Mode − Median = 2 Median − 2 Mean

24 = 2 (Median − Mean)

Median − Mean = 12

2. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and re-solidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to [1]

(a) 3 (b) 3.5
(c) 4 (d) 4.5

Ans : (b) 3.5

Volume of the given ice cuboid $= 8 \times 11 \times 2$
$= 176$

Let the length of the required rod be l.

$$\pi l \frac{8^2}{4} = 176$$

$$l = 3.5 \text{ inches}$$

3. If $f(x) = \cos^2 x + \sec^2 x$, then $f(x)$ [1]

(a) ≥ 1 (b) ≤ 1
(c) ≥ 2 (d) ≤ 2

Ans : (c) ≥ 2

Given, $f(x) = \cos^2 x + \sec^2 x$

$= \cos^2 x + \sec^2 x - 2 + 2$ [adding and subtracting 2]

$= \cos^2 x + \sec^2 x - 2\cos x \cdot \sec x + 2$ $[\cos x \cdot \sec x = 1]$

$= (\cos x - \sec x)^2 + 2$ $[a^2 + b^2 - 2ab = (a-b)^2]$

We know that, square of any expression is always greater than equal to zero.

$f(x) \geq 2$ Hence proved.

4. The 2 digit number which becomes (5/6)th of itself when its digits are reversed. The difference in the digits of the number being 1, then the two digits number is [1]

(a) 45 (b) 54
(c) 36 (d) None of these

Ans : (b) 54

If the two digits are x and y, then the number is $10x + y$.

Now $\frac{5}{6}(10x + y) = 10y + x$

Solving, we get $44x = 55y$

$$\frac{x}{y} = \frac{5}{4} \quad ...(1)$$

Also $x - y = 1$. Solving them, we get $x = 5$ and $y = 4$. Therefore, number is 54.

5. If α and β are zeroes and the quadratic polynomial $f(x) = x^2 - x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ is [1]

(a) $\frac{15}{4}$ (b) $\frac{-15}{4}$
(c) 4 (d) 15

Ans : (a) $\frac{15}{4}$

Given that, $f(x) = x^2 - x - 4$

$\alpha + \beta = 1$ and $\alpha\beta = -4$

We have, $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta = -\frac{1}{4} + 4 = \frac{15}{4}$

6. Two positive numbers have their HCF as 12 and their product as 6336. The number of pairs possible for the numbers, is [1]

(a) 2 (b) 3
(c) 4 (d) 5

Ans : (a) 2

Let the numbers be $12x$ and $12y$ where x and y are co-primes.

Product of these numbers $= 144xy$

Hence, $144xy = 6336 \Rightarrow xy = 44$

Since, 44 can be written as the product of two factors in three ways. i.e. 1×44, 2×22, 4×11. As x and y are co-prime, so (x, y) can be (1,44) or (4,11) but not (2,22).

Hence, two possible pairs exist.

7. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then [1]

(a) $b = c$ (b) $a = b$

(c) $ac = 1$ (d) $a = c$

Ans : (d) $a = c$

If one root is α, then the other $\frac{1}{\alpha}$.

$$\alpha \cdot \frac{1}{\alpha} = \text{product of roots} = \frac{c}{a}$$

$$1 = \frac{c}{a}$$

$$a = c$$

8. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$? [1]

(a) 5 (b) 20

(c) 25 (d) 30

Ans : (c) 25

Given, the common difference of AP i.e., $d = 5$

Now, $a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$

$[\text{Since}, a_n = a + (n-1)d]$

$= a + 17 \times 5 - a - 12 \times 5$

$= 85 - 60 = 25$

9. A bag contains 3 red and 2 blue marbles. If a marble is drawn at random, then the probability of drawing a blue marble is: [1]

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

Ans : (b) $\frac{2}{5}$

There 5 marbles in the bag. Out of these 5 marbles one can be choose in 5 ways.

Hence, Total number of possible outcomes = 5

Since, the bag contains 2 blue marbles. Therefore, one blue marble can be drawn in 2 ways.

Hence, Favourable number of elementary events = 2

Hence, P (getting a blue marble) $= \frac{2}{5}$

10. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240°. If another circle of the area same as the sector is formed, then radius of the new circle is [1]

(a) 79.5 cm (b) 81.5 cm

(c) 83.4 cm (d) 88.5 cm

Ans : (b) 81.5 cm

$$\text{Area of sector} = 240/360 \times \pi(100)^2$$

$$= 20933 \text{ cm}^2$$

Let r be the radius of the new circle, then

$$20933 = \pi r^2$$

$$r = \sqrt{\frac{20933}{\pi}} = 81.6 \text{ cm}$$

(Q.11-Q.15) Fill in the blanks.

11. The volume of a cube with diagonal d is .... [1]

Ans : $\frac{d^3}{3\sqrt{3}}$ cu units.

12. If $a = bq + r$, least value of r is .... [1]

Ans : Zero

13. Area of a rhombus if its vertices are (3, 0), (4, 5), $(-1, 4)$ and $(-2, -1)$ taken in order is .... [1]

Ans : 24. Sq. units

or

Points (3, 2), $(-2, -3)$ and (2, 3) form a .... triangle.

Ans : Right angle

14. In ΔABC, right-angled at B, $AB = 24$ cm, $BC = 7$ cm . $\sin A =$ .... [1]

Ans : 7/25

15. Length of an arc of a sector of a circle with radius r and angle with degree measure θ is .... [1]

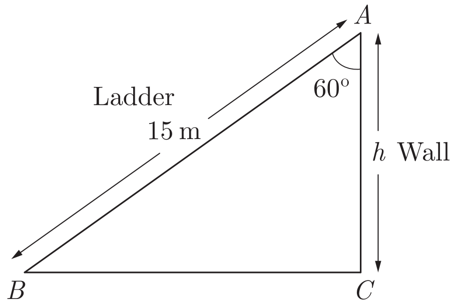
Ans : $\frac{\theta}{360} \times 2\pi r$

(Q.16-Q.20) Answer the following

16. A ladder 15m long leans against a wall making an angle of 60° with the wall. Find the height of the point where the ladder touches the wall. [1]

Ans :

Let the height of wall be h. As per given in question we have drawn figure below.



$$\frac{h}{15} = \cos 60^\circ$$

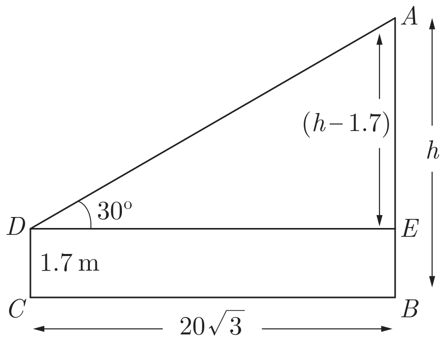
$$h = 15 \times \cos 60^\circ = 15 \times \frac{1}{2} = 7.5 \text{ m}$$

or

An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.

Ans :

Let height of the tower AB be h. As per given in question we have drawn figure below.



Here $AE = h - 1.7$

and $BC = DE = 20\sqrt{3}$

In ΔADE, $\angle E = 90^o$

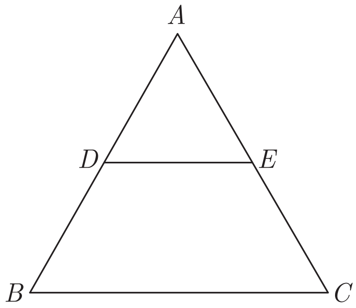
$$\tan 30^o = \frac{h-1.7}{20\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{h-1.7}{20\sqrt{3}}$$

$$h - 1.7 = 20$$

or $h = 20 + 1.7 = 21.7$ m

17. In given figure $DE || BC$. If $AD = 3$ cm, $DB = 4$ cm and $AE = 6cm$, then find EC. [1]



Ans :

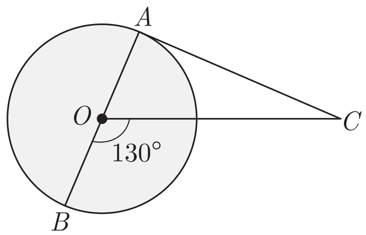
In the given figure $DE \parallel BC$, thus

$$\frac{AD}{AB} = \frac{AE}{EC}$$

$$\frac{3}{4} = \frac{6}{EC}$$

$$EC = 8 \text{ cm}$$

18. In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, the find $\angle ACO$. [1]



Ans :

Here OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have

$$\angle OAC = 90^o$$

From exterior angle property,

$$\angle BOC = OAC + \angle ACO$$

$$130^o = 90° + \angle ACO$$

$$\angle ACO = 130^o - 90^o = 40^o$$

19. To divide a line segment AB in the ratio 2:5, a ray AX is drawn such that $\angle BAX$ is acute. Then points are marked at equal intervals on AX. What is the minimum number of these points ? [1]

Ans :

Minimum number of points marked on, $AX = 2 + 5 = 7$

20. What is the volume of a right circular cylinder of base radius 7 cm and height 10 cm ? (Use $\pi = \frac{22}{7}$) [1]

Ans :

Here $r = 7$ cm, $h = 10$ cm,

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times (7)^2 \times 10$$

$$= 1540 \text{ cm}^3$$

Section B

21. For what value of k, the pair of linear equations $kx - 4y = 3$, $6x - 12y = 9$ has an infinite number of solutions ? [2]

Ans :

We have $kx - 4y - 3 = 0$

and $6x - 12y - 9 = 0$

where, $a_1 = k, b_1 = 4, c_1 = -3$

$a_2 = 6, b_2 = -12, c_2 - 9$

Condition for infinite solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

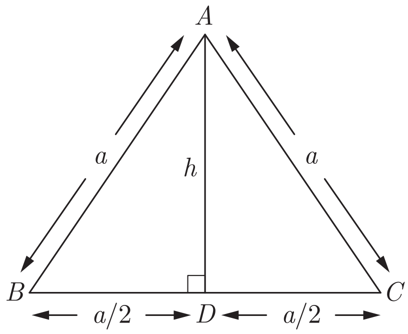
$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$$

Hence, $k = 2$

22. Find the altitude of an equilateral triangle when each of its side is 'a' cm. [2]

Ans :

Let ΔABC be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



In ΔABD, $a^2 = \left(\frac{a}{2}\right)^2 + h^2$

$$h^2 = a^2 - \frac{a^2}{4}$$

$$= \frac{3a^2}{4}$$

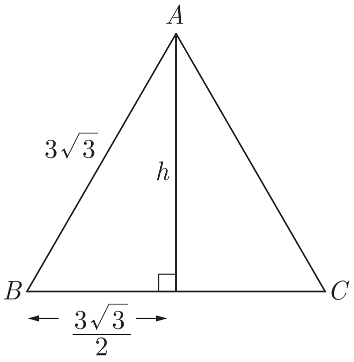
Thus $h = \frac{\sqrt{3}a}{2}$

or

In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

Ans :

Let ΔABC be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

$$27 = h^2 + \frac{27}{4}$$

$$h^2 = 27 - \frac{27}{4} = \frac{81}{4}$$

$$h = \frac{9}{2} = 4.5 \text{ cm}$$

23. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,-5)$ and $R(-3,6)$, find the co-ordinates of P. [2]

Ans :

Let the point $P\ (2y, y)$,

Since $PQ = PR$, we have

$$\sqrt{(2y-2)^2 + (y+5)^2} = \sqrt{(2y+3)^2 + (y-6)^2}$$

$$(2y-2)^2 + (y+5)^2 = (2y+3)^2 + (y-6)^2$$

$$-8y + 4 + 10y + 25 = 12y + 9 - 12y + 36$$

$$2y + 29 = 45$$

$$y = 8$$

Hence, coordinates of point P are $(16, 8)$

24. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere. [2]

Ans :

Diameter of sphere = Radius of hemisphere

= 6 cm

Radius of sphere = 3 cm

Volume, $$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3^3 \text{ cm}^3.$$

$$= 113.14 \text{ cm}^3.$$

or

A cone of height 24 cm and radius of base 6 cm is made up of clay. If we reshape it into a sphere, find the radius of sphere.

Ans :

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\frac{4}{3} \times r_1^3 = (6)^2 \times \frac{24}{3}$$

$$4r_1^3 = 36 \times 24$$

$$r_1^3 = 6^3$$

$$r_1 = 6 \text{ cm}$$

Hence, radius of sphere is 6 cm.

25. A hemisphere and a cone both have same diameter. These two metal solids are joined by putting their bases together. The height of the cone is equal to the diameter of the sphere. This solid is melted and recast into a sphere of a diameter equal to one third of the diameter of the hemisphere. [2]

(a) If radius of the hemisphere is r, find the volume of the combined solid.

(b) Find the number of spheres.

Ans :

(a) Radius of the hemisphere $= r$

Height of the cone $= 2r$

Volume of the solid $$= \frac{1}{3}\pi r^2 \times 2r + \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$$

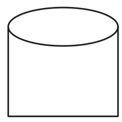
(b) Volume of the sphere $$= \frac{4}{3}\pi\left(\frac{r}{3}\right)^3$$

$$= \frac{4}{3}\pi\frac{r^3}{27}$$

Number of spheres $$= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\frac{r^3}{27}} = 27$$

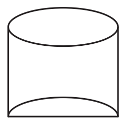
26. Read the following passage and the question that follows:

Ramesh, a juice seller has set up his juice shop. He has three types of glasses of inner diameter 5 cm to serve the customers. The height of the glasses is 10 cm. (Use $\pi = 3.14$). [2]



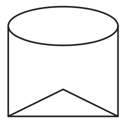
Type A

— A glass with a plane bottom



Type B

— A glass with hemispherical raised bottom



Type C

— A glass with conical raised bottom of height 1.5 cm.

He decided to serve the customer in A" type of glasses.

(i) Find the volume of glass of type A.

(ii) and which glass has the minimum capacity.

Ans :

(i) Given, Diameter = 5 cm

Radius = 2.5 cm

Height = 10 cm

Volume of glass of type A $= \pi r^2 h$

$$= 3.14 \times 2.5 \times 2.5 \times 10$$

$$= 196.25 \text{ cm}^3$$

Volume of hemisphere

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$$

$$= 32.71\,\text{cm}^3$$

Volume of glass of type B

$$= 196.25 - 32.71$$

$$= 163.54\,\text{cm}^3$$

$$\text{Volume of cone } = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times 3.14 \times 2.5 \times 2.5 \times 1.5$$

$$= 3.14 \times 2.5 \times 2.5 \times 0.5$$

$$= 9.81\,\text{cm}^3$$

Volume of glass of type C

$$= 196.25 - 9.81$$

$$= 186.44\,\text{cm}^3$$

(ii) the glass of type B has the minimum capacity.

Section C

27. If 7^{th} term of an A.P. is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term. [3]

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$ (1)

$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7}$ (2)

Subtracting equation (1) from (2), we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} = d$$

$$d = \frac{1}{63}$$

Substituting the value of d in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$

$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus $a_{63} = a + (63-1)\,d$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$

$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$

28. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$. [3]

Ans :

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

(1) $p(x) = 2x^2 - 11x^2 + 17x - 6$

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$$

$$= 16 - 44 + 34 - 6$$

$$= 50 - 50$$

or $p(2) = 0$

(2) $p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$$= 54 - 99 + 51 - 6$$

$$= 105 - 105$$

or $p(3) = 0$

(3) $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

or

Find the zeroes of the quadratic polynomial $5x^2 + 8x - 4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Ans :

We have $p(x) = 5x^2 + 8x - 4 = 0$

$$= 5x^2 + 10x - 2x - 4 = 0$$

$$= 5x(x+2) - 2(x+2) = 0$$

$$= (x+2)(5x-2)$$

Substituting $p(x) = 0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes} = -2 + \frac{2}{5} = \frac{-8}{5}$$

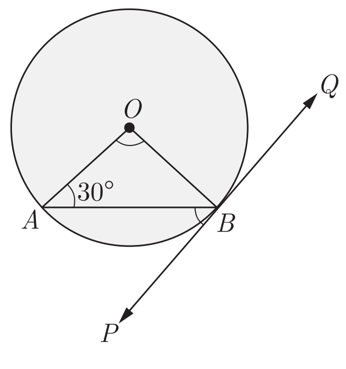
$$\text{Product of zeroes} = (-2) \times \left(\frac{2}{5}\right) = \frac{-4}{5}$$

Now from polynomial we have

$$\text{Sum of zeroes } -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-8}{5}$$

$$\text{Product of zeroes } \frac{c}{a} = \frac{\text{Constan term}}{\text{Coefficient of } x^2} = -\frac{4}{5}$$

29. In the figure, PQ is a tangent to a circle with center O. If $\angle OAB = 30°$, find $\angle ABP$ and $\angle AOB$. [3]



Ans :

Here OB is radius and QT is tangent at B, $OB \perp PQ$

$$\angle OBP = 90°$$

Since the tangent is perpendicular to the end point of radius,

Here OA and OB are radius of circle and equal. Since angles opposite to equal sides are equal,

$$\angle OAB = \angle OBA = 30°$$

Now $\angle AOB = 180° - (30° + 30°) = 120°$

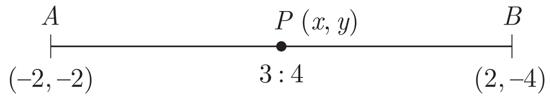
$$\angle ABP = \angle OBP - \angle OBA$$
$$= 90^o - 30^o = 60^o$$

30. If the co-ordinates of points A and B are $(-2,-2)$ and $(2,-4)$ respectively, find the co-ordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB. [3]

Ans :

We have $AP = \frac{3}{7}AB \Rightarrow AP:PB = 3:4$

As per question, line diagram is shown below.



Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula, we get

$$x = \frac{3 \times 2 + 4 \times -2}{3+4} = -\frac{2}{7}$$

$$y = \frac{3 \times -4 + 4 \times -2}{3+4} = -\frac{20}{7}$$

Hence P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$

or

If the distance of $P(x,y)$ from $A(6,2)$ and $B(-2,6)$ are equal, prove that $y = 2x$.

Ans :

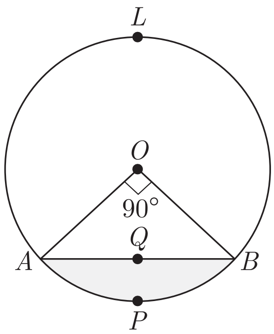
We have $P(x,y), A(6,2), B(-2,6)$

Now $PA = PB$

$$PA^2 = PB^2$$
$$(x-6)^2 + (y-2)^2 = (x+2)^2 + (y-6)^2$$
$$-12x + 36 - 4y + 4 = 4x + 4 - 12y + 36$$
$$-12x - 4y = 4x - 12y$$
$$12y - 4y = 4x + 12x$$
$$8y = 16x$$
$$y = 2x$$

Hence Proved

31. In the given figure, a chord AB of the circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $A\angle LBQA$. (Use $\pi = 3.14$) [3]



Ans :

Area of sector $OAPB$,

$$A_1 = \frac{90}{360}\pi(10)^2 = 25\pi \text{ cm}^2$$

Area of ΔAOB, $A_2 = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$

Area of minor segment $AQBP$,

$$= (25\pi - 50) \text{ cm}^2$$
$$= 25 \times 3.14 - 50$$
$$= 78.5 - 50 = 28.5 \text{ cm}^2 \text{ cm}^2$$

Also area of circle $= \pi(10)^2$

$$= 3.14 \times 100 = 314 \text{ cm}^2$$

Area of major segment $ALBQA = 314 - 28.5$

$$= 285.5 \text{ cm}^2$$

or

Find the area of minor segment of a circle of radius 14 cm, when its centre angle is $60°$. Also find the area of corresponding major segment. Use $\pi = \frac{22}{7}$.

Ans :

Here, $r = 14$ cm, $\theta = 60°$

Area of minor segment $= \pi r^2 \frac{\theta}{360} - \frac{1}{2}r^2 \sin\theta$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60}{360°} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

$$= \left(\frac{308}{3} - 49\sqrt{3}\right) = 17.9 \text{ cm}^2 \text{ approx.}$$

Area of major segment $= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$

$$= \frac{1540}{3} + 49\sqrt{3} = 598.10$$

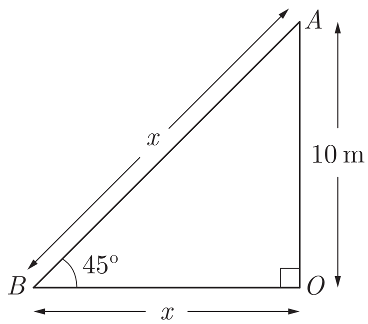
$$= 598 \text{ cm}^2 \text{ approx.}$$

32. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire. [Use $\sqrt{2} = 1.414$] [3]

Ans :

Let OA be the electric pole and B be the point on the ground to fix the pole. Let BA be x.

As per given in question we have drawn figure below.



In ΔABO, we have

$$\sin 45^o = \frac{AO}{AB}$$

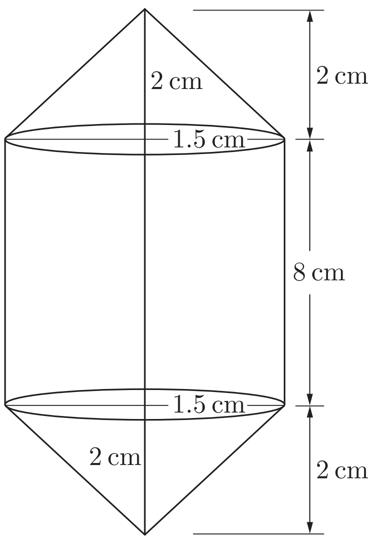
$$\frac{1}{\sqrt{2}} = \frac{10}{x}$$

$$x = 10\sqrt{2} = 10 \times 1.414$$
$$= 14.14 \text{ m}$$

Hence, the length of wire is 14.14 m

33. Read the following passage and the question that follows:
Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm. [3]
(i) Find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same).
(ii) Which mathematical concept is used in the above problem ?

Ans :



(i)Here, radius of two cones and cylinder

$$= \frac{3}{2} \text{ cm}$$

$$= 1.5 \text{ cm}$$

Height of each cone = 2 cm

Height of cylindrical portion = 12 − 2 − 2 = 8 cm

Volume of the air = Volumes of cylindrical part +2×Volume of cone

$$= \pi(1.5)^2 \times 8 + 2\frac{1}{3}\pi(1.5)^2 \times 2$$

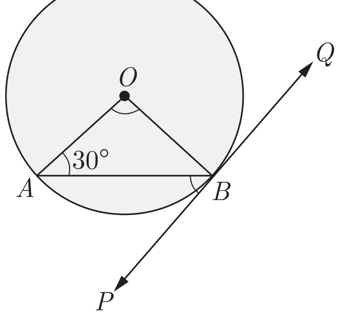
$$= \frac{22}{7} \times (1.5)^2\left[8 + \frac{4}{3}\right]$$

$$= \frac{22}{7} \times 2.25 \times \frac{28}{3}$$

$$= 66 \text{ cm}^3$$

(ii) Volume (Mensuration).

34. In the figure, PQ is a tangent to a circle with center O. If $\angle OAB = 30^\circ$, find $\angle ABP$ and $\angle AOB$. [3]



Ans :

Here OB is radius and QT is tangent at B, $OB \perp PQ$

$$\angle OBP = 90^\circ$$

Since the tangent is perpendicular to the end point of radius,

Here OA and OB are radius of circle and equal. Since angles opposite to equal sides are equal,

$$\angle OAB = \angle OBA = 30^\circ$$

Now $$\angle AOB = 180^\circ - (30^\circ + 30^\circ)$$

$$= 120^\circ$$

$$\angle ABP = \angle OBP - \angle OBA$$

$$= 90^\circ - 30^\circ = 60^\circ$$

Section D

35. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF × LCM of these numbers equal to the product of the given three numbers? [4]

Ans :

Finding prime factor of given number we have,

$$378 = 2\times3^3\times7$$

$$180 = 2^2\times3^2\times5$$

$$420 = 2^2\times3\times7\times5$$

$$\text{HCF}(378, 180, 420) = 2\times3 = 6$$

$$\text{LCM}(378, 180, 420) = 2^2\times3^3\times5\times7$$

$$= 2^2\times3^3\times5\times7 = 3780$$

$$\text{HCF}\times\text{LCM} = 6\times3780 = 22680$$

Product of given numbers

$$= 378\times180\times420 = 28576800$$

Hence, HCF × LCM ≠ Product of three numbers.

36. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of none chair and one table separately. [4]

Ans :

Let cost of 1 chair be Rs x and cost of 1 table be Rs y According to the question,

$$4x + 3y = 2100 \quad ...(1)$$

$$5x + 2y = 1750 \quad ...(2)$$

Multiplying equation (1) by 2 and equation (2) by 3 we get,

$$8x + 6y = 4200 \quad ...(3)$$

$$15x + 6y = 5250 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$7x = 1050$$

$$x = 150$$

Substituting the value of x in (1), $y = 500$

Thus cost of chair and table is Rs 150, Rs 500 respectively.

or

If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Ans :

Let the number of red balls be x and white balls be y.
According to the question,

$$\frac{1}{2}y = \frac{1}{3}x \text{ or } 2x - 3y = 0 \qquad ...(1)$$

and $\quad 3(x+y) - 7y = 6$

or $\quad 3x - 4y = 6 \qquad ...(2)$

Multiplying equation (1) by 3 and equation (2) we have

$$6x - 9y = 0 \qquad ...(3)$$

$$6x - 8y = 12 \qquad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

Substituting $y = 12$ in equation (1),

$$2x - 36 = 0$$

$$x = 18$$

Hence, number of red balls = 18

and number of white balls = 12

37. Evaluate : $\frac{\cos 65^o}{\sin 25^o} - \frac{\tan 20^o}{\cot 70^o} - \sin 90^o + \tan 5^o \tan 35^o$

$\tan 60^o \tan 55^o \tan 85^o$. [4]

Ans :

We have $\quad \frac{\cos 65^o}{\sin 25^o} = \frac{\cos 65^o}{\sin(90^o - 65^o)} = \frac{\cos 65^o}{\cos 65^o} = 1,$

$$\frac{\tan 20^o}{\cot 70^o} = \frac{\tan(90^o - 70^o)}{\cot 70^o} = \frac{\cot 70^o}{\cot 70^o} = 1$$

and $\quad \sin 90^o = 1$

$\tan 5^o \tan 35^o \tan 60^o \tan 55^o \tan 85^o$

$$= \tan(90^o - 85^o)\tan(90^o - 55^o) \tan 55^o \tan 60^o \tan 85^o.$$

$$= \cot 85^o \tan 85^o \cot 55^o \tan 55^o \cdot \sqrt{3}$$

$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$$

Now given expression $= 1 - 1 - 1 + \sqrt{3} = \sqrt{3} - 1$

38. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction. [4]

Ans :

Let the speed while going be x km/h
Speed while returning $= (x+10)$ km/h
According to question we have

$$\frac{150}{x} - \frac{150}{x+10} = \frac{5}{2}$$

$$x^2 + 10x - 600 = 0$$

$$(x+30)(x-20) = 0$$

$$x = 20$$

Speed while going is 20 km/h and speed while returning will be $= 20 + 10 = 30$ km/h

or

A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans :

Let the speed of stream be x km/h
Then the speed of boat upstream $= (18 - x)$ km/h
Speed of boat downstream $= (18 + x)$ km/h
According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\frac{24[(18+x) - (18-x)]}{18^2 - x^2} = 1$$

$$48x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

$$x^2 + 54x - 6x - 324 = 0$$

$$x(x+54) - 6(x+54) = 0$$

$$(x+54)(x-6) = 0$$

$$x + 54 = 0, x - 6 = 0$$

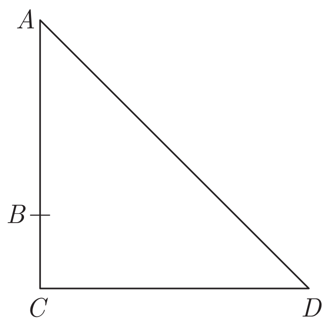
$$x = -54, x = 6$$

Since speed cannot be negative, we reject $x = -54$.

The speed of steam is 6 km/h.

39. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x, BC = h$ and $CD = d$, then find x (in term of h and d). [4]

Ans :



We have $\quad AB + AD = BC + CD$

$$AD = BC + CD - AB$$

$$AD = h + d - x$$

In right angled triangle ΔACD,

$$AD^2 = AC^2 + DC^2$$

$$(h + d - x)^2 = (x+h)^2 + d^2$$

$$(h + d - x)^2 - (x+h)^2 = d^2$$

$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$

$$(d - 2x)(2h + d) = d^2$$

$$2hd + d^2 - 4hx - 2xd = d^2$$

$$2hd = 4hx + 2xd$$

$$= 2(2h + d)x$$

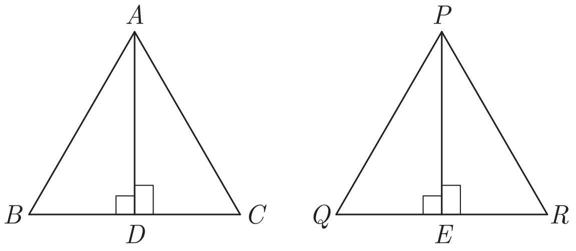
or, $\quad x = \frac{hd}{2h+d}$

or

Prove that ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Ans :

As per given condition we have drawn the figure below. Here $\Delta ABC \sim \Delta PQR$



We have drawn $AD \perp BC$ and $PE \perp QR$

Since $\Delta ABC \sim \Delta PQR$, due to corresponding sides of similar triangles

$$\frac{AB}{PQ}\ \frac{AC}{QR} = \frac{AC}{PR} \qquad ...(1)$$

$$\angle B = \angle Q$$

In ΔADB and ΔPEQ,

$$\angle B = \angle Q \qquad \text{(Proved)}$$

$$\angle ADB = \angle PEQ \qquad \text{[each } 90^o\text{]}$$

$$\Delta ADB \sim \Delta PEQ \qquad (AA \text{ Similarity})$$

Corresponding sides of similar triangle,

$$\frac{AD}{PE} = \frac{AB}{PQ} \qquad ...(2)$$

From eq. (1) and eq. (2),

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PE} \qquad ...(3)$$

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2}\times BC \times AD}{\frac{1}{2}\times QR \times PE}$$

$$= \left(\frac{BC}{QR}\right)\times\left(\frac{AD}{PE}\right) = \frac{BC}{QR}\times\frac{BC}{QR}$$

From equation (3) we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} \qquad ...(4)$$

From equation (3) and equation (4) we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

40. The median of the following data is 525. Find the values of x and y if the total frequency is 100. [4]

Class Interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4
	$N = 100$

Ans :

Class Interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$
	$N = 100$	

Here, $76 + x + y = 100$

$\Rightarrow$ $x + y = 100 - 76 = 24$ ...(i)

Given, Median $= 525$, which lies between class 500-600

$\Rightarrow$ Median class $= 500 - 600$

Now, $$\text{Median} = l + \frac{\frac{n}{2} - c.f.}{f} \times h$$

$\Rightarrow$ $$525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$

$\Rightarrow$ $25 = (50 - 36 - x)5$

$\Rightarrow$ $(14 - x) = \frac{25}{5} = 5$

$\Rightarrow$ $x = 14 - 5 = 9$

Substituting the value of x in equation (i),

$y = 24 - 9 = 15$

Hence, $x = 9$ and $y = 15$