CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-4

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is [1]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{4}{9}$ (d) $\frac{2}{5}$

Ans : (b) $\frac{1}{4}$

One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.

2. If the equation $(m^2+n^2)x^2-2(mp+nq)x+p^2+q^2=0$ has equal roots, then [1]

(a) $mp=nq$ (b) $mq=np$

(c) $mn=pq$ (d) $mq=\sqrt{np}$

Ans : (b) $mq=np$

$$\begin{aligned} b^2 &= 4ac \\ 4(mp+nq)^2 &= 4(m^2+n^2)(p^2+q^2) \\ m^2q^2+n^2p^2-2mnpq &= 0 \\ (mq-np)^2 &= 0 \\ mq-np &= 0 \\ mq &= np \end{aligned}$$

3. If $x=p\sec\theta$ and $y=q\tan\theta$, then [1]

(a) $x^2-y^2=p^2q^2$ (b) $x^2q^2-y^2p^2=pq$

(c) $x^2q^2-y^2p^2=\frac{1}{p^2q^2}$ (d) $x^2q^2-y^2p^2=p^2q^2$

Ans : (d) $x^2q^2-y^2p^2=p^2q^2$

We know, $\sec^2\theta-\tan^2\theta = 1$

and $\sec\theta = \frac{x}{p}$

$$\tan\theta = \frac{y}{q}$$

$$x^2q^2-y^2p^2 = p^2q^2$$

4. The value of x, for which the polynomials x^2-1 and x^2-2x+1 vanish simultaneously, is [1]

(a) 2 (b) -2

(c) -1 (d) 1

Ans : (d) 1

The expressions $(x-1)(x+1)$ and $(x-1)(x-1)$ which vanish if $x=1$

5. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is [1]

(a) 2 : 1 (b) 1 : 2

(c) 1 : 3 (d) 3 : 1

Ans : (a) 2 : 1

$$\begin{aligned} \pi rl &= 2\pi rh \\ \frac{l}{h} &= \frac{2}{1} \end{aligned}$$

6. For finding the popular size of ready-made garments, which central tendency is used? [1]

(a) Mean

(b) Median

(c) Mode

(d) Both Mean and Mode

Ans : (c) Mode

For finding the popular size of ready made garments, mode is the best measure of central tendency.

7. If n is an even natural number, then the largest natural number by which $n(n+1)(n+2)$ is divisible, is [1]

(a) 6 (b) 8

(c) 12 (d) 24

Ans : (d) 24

Since n is divisible by 2 therefore $(n+2)$ is divisible by 4, and hence $n(n+2)$ is divisible by 8.
Also, $n, n+1, n+2$ are three consecutive numbers.
So, one of them is divisible by 3.
Hence, $n(n+1)(n+2)$ must be divisible by 24.

8. If the common difference of an AP is 5, then what is $a_{18}-a_{13}$? [1]

(a) 5 (b) 20

(c) 25 (d) 30

Ans : (c) 25

Given, the common difference of AP i.e., $d = 5$

Now, $a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$

$[\text{Since}, a_n = a + (n-1)d]$

$= a + 17 \times 5 - a - 12 \times 5$

$= 85 - 60 = 25$

9. X's salary is half that of Y's. If X got a 50% rise in his salary and Y got 25% rise in his salary, then the percentage increase in combined salaries of both is [1]

(a) 30 (b) $33\frac{1}{3}$

(c) $37\frac{1}{2}$ (d) 75

Ans : (b) $33\frac{1}{3}$

Let $y's$ initial salary $= a$

$X's$ initial salary $= \frac{a}{2}$

Total salary $= a + \frac{a}{2} = \frac{3a}{2}$

After increases,

$$X's \text{ salary} = \frac{a}{2} + \left(\frac{50}{100} \times \frac{a}{2}\right) = \frac{a}{2} + \frac{a}{4} = \frac{3a}{4}$$

$$Y's \text{ salary} = \frac{a}{2} + \left(\frac{25}{100} \times 4\right) = a + \frac{a}{4} = \frac{5a}{4}$$

$$\text{Total salary of both} = \frac{3a}{4} + \frac{5a}{4} = \frac{8a}{4} = 2a$$

$$\text{Increment in salary} = 2a - \frac{3a}{2} = \frac{4a-3a}{2} = \frac{a}{2}$$

$$\text{Percentage increment} = \frac{\frac{a}{2}}{\frac{3a}{2}} \times 100$$

$$= \frac{100}{3} = 33\frac{1}{2}\%$$

10. The area of a circular ring formed by two concentric circles whose radii are 5.7 cm and 4.3 cm respectively is (Take $\pi = 3.1416$) [1]

(a) 43.98 sq. cm. (b) 53.67 sq. cm.

(c) 47.24 sq. cm. (d) 38.54 sq. cm.

Ans : (a) 43.98 sq. cm.

Let the radii of the outer and inner circles be r_1 and r_2 respectively, we have

$$\text{Area} = \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$$

$$= \pi(r_1 - r_2)(r_1 + r_2)$$

$$= \pi(5.7 - 4.3)(5.7 + 4.3)$$

$$= \pi \times 1.4 \times 10 \text{ sq. cm}$$

$$= 3.1416 \times 14 \text{ sq. cm.}$$

$$= 43.98 \text{ Sq. cm.}$$

(Q.11-Q.15) Fill in the blanks.

11. If the volume of a cube is 64 cm^3, then its surface area is .... [1]

Ans : 96 cm^2

12. (1, 2), $(4, y)$, $(x, 6)$ and (3, 5) are the vertices of a parallelogram taken in order, then the value of x and y are .... [1]

Ans : (6, 3)

or

If $x - y = 2$ then point (x, y) is equidistant from (7,1) and (...........)

Ans : (3, 5)

13. L.C.M. of 96 and 404 is .... [1]

Ans : 9696

14. In a right triangle ABC, right angled at B, if $\tan A = 1$, $\sin A \cos A =$ .... [1]

Ans : $\frac{1}{2}$

15. If the area of a circle is 154 cm^2, then its circumference is .... [1]

Ans : 44 cm

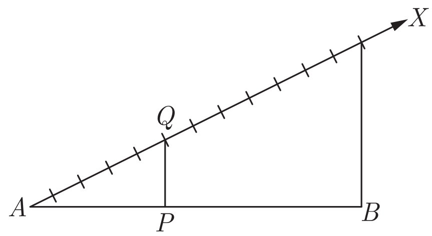
(Q.16-Q.20) Answer the following

16. To divide a line segment AB in the ratio 5:7, first AX is drawn, so that $\angle BAX$ is an acute angle and then at equal distance, points are marked on the ray AX, find the minimum number of these points. [1]

Ans :

Minimum number of points marked on

$$AX = 5 + 7 = 12$$



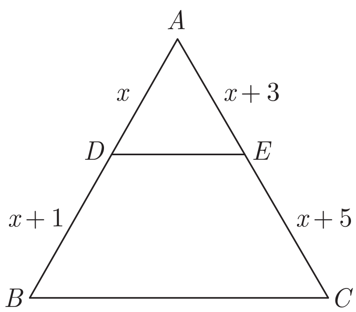
or

To divide a line segment AB in the ratio 2:5, a ray AX is drawn such that $\angle BAX$ is acute. Then points are marked at equal intervals on AX. What is the minimum number of these points ?

Ans :

Minimum number of points marked on $AX = 2 + 5 = 7$

17. In $\Delta ABC, DE || BC$, find the value of x. [1]



Ans :

In the given figure $DE || BC$, thus

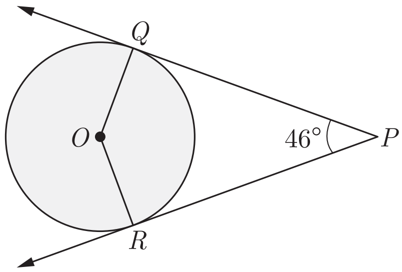
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

18. If PQ and PR are two tangents to a circle with center O. If $\angle QPR = 46^o$ then find $\angle QOR$. [1]



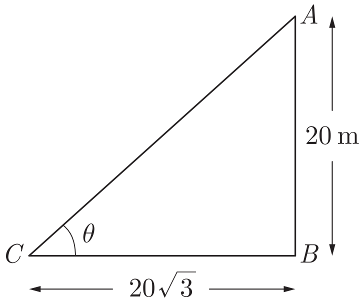
Ans :

We have $\angle QPR = 46^o$

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^o$$
$$\angle QOR + 46^o = 180^o$$
$$\angle QOR = 180^o - 46^o = 134^o$$

19. In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. find the Sun's altitude. [1]



Ans :

Let the $\angle ACB$ be θ.

$$\tan\theta = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $\theta = 30^\circ$

20. The radius of sphere is r cm. It is divided into two equal parts. Find the whole surface of two parts. [1]

Ans :

$$\text{Whole surface of each part} = 2\pi r^2 + \pi r^2 = 3\pi r^2$$
$$\text{Total surface of two parts} = 2 \times 3\pi r^2 = 6\pi r^2$$

Section B

21. Is the system of linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ consistent? Justify your answer. [2]

Ans :

For the equation $2x + 3y - 9 = 0$, we have

$$a_2 = 2, b_1 = 3 \text{ and } c_1 = -9$$

and for the equation, $4x + 6y - 18 = 0$, we have

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$

Here $\frac{a_1}{a_1} = \frac{2}{4} = \frac{1}{2}$

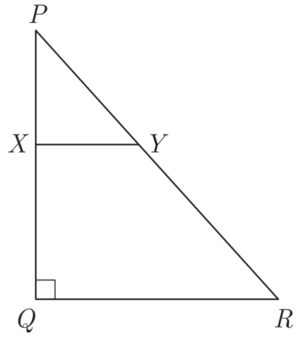
$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

and $\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$

Thus $\frac{c_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, system is consistent and dependent.

22. In the given figure, PQR is a triangle right angled at Q and $XY || QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1 : 2$. Calculate the length of PR and QR. [2]



Ans :

Since $XY \;||\; OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$
$$\frac{1}{2} = \frac{PY}{PR - PY}$$
$$= \frac{4}{PR - 4}$$
$$PR - 4 = 8$$
$$PR = 12 \text{ cm}$$

In right ΔPQR we have

$$QR^2 = PR^2 - PQ^2$$
$$= 12^2 - 6^2$$
$$= 144 - 36 = 108$$

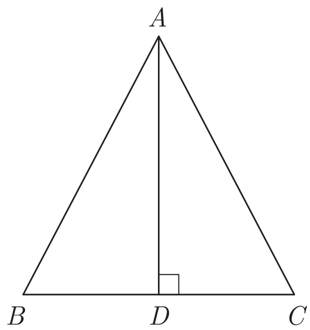
Thus $QR = 6\sqrt{3}$ cm

or

In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.

Ans :

In ΔABD, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

Since $AB = BC = CA$, we get

$$BC^2 = AD^2 + BD^2,$$

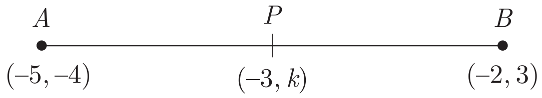
Here, $BC = 2BD$

$$(2BD)^2 = AD^2 + BD^2$$
$$4BD^2 - BD^2 = AD^2$$
$$3BD^2 = AD^2$$

23. Find the ratio in which the point $(-3, k)$ divides the line segment joining the points $(-5, -4)$ and $(-2, 3)$. Also find the value of k. [2]

Ans :

As per question, line diagram is shown below.



Let AB be divides by P in ratio n:1.
x co-ordinate for section formula

$$-3 = \frac{(-2)n+1(-5)}{n+1}$$

$$-3(n+1) = -2n-5$$

$$-3n-3 = -2n-5$$

$$5-3 = 3n-2n$$

$$2 = n$$

Ratio $\frac{n}{1} = \frac{2}{1}$ or 2:1

Now, y co-ordinate,

$$k = \frac{2(3)+1(-4)}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

24. Find the number of plates, 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular of height 10 cm and diameter 4.5 cm. [2]

Ans :

Each one of he circular plate is also a cylinder.

Volume of plate $V_p = \pi r^r h = \pi \times (.75)^2(.2)$

$$= \frac{9\pi}{80} \text{ cm}^3$$

Volume of right circular cylinder

$$V_c = \pi(2.25)^2(10) = 405\frac{\pi}{8} \text{ cm}^3$$

$$\text{Number of plates} = \frac{\frac{405\pi}{8}}{\frac{9\pi}{80}} = \frac{405\pi}{9\pi} \times \frac{80}{8}$$

$$= 450 \text{ plates.}$$

or

A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ? [2]

Ans :

Given, Radius of sphere, $r_1 = \frac{d}{2} = \frac{6}{2} = 3$ cm

Radius of cylinder vessel, $r_2 = \frac{12}{2} = 6$ cm

Let the level of water rise in cylinder be h.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$= 36\pi \text{ cm}^3$$

Volume of sphere = Increase volume in cylinder

$$36\pi = \pi(6)^2 \times h$$

$$h = 1 \text{ cm}$$

25. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ? [2]

Ans :

Radius of sphere $\frac{6}{2} = 3$ cm

Radius of cylinder vessel $\frac{12}{2} = 6$ cm

Let the level of water rise in cylinder be h.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$= 36\pi \text{ cm}^3$$

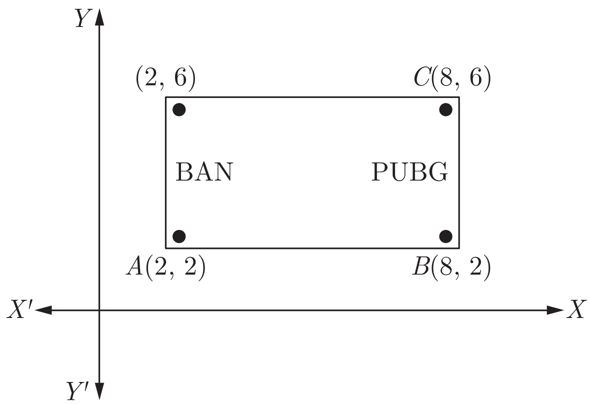
Volume of sphere = Increase volume in cylinder

$$36\pi = \pi(6)^2 \times h$$

$$h = 1 \text{ cm}$$

Thus level of water rise in vessel is 1 cm.

26. Read the following passage and the question that follows:
One tends to become lazy. Also, starting at your mobile screen for long hours can affect your eyesight and give you headaches . Those who are addicted to playing PUBG can get easily stressed out or face anxiety issues in public due to lack of social interaction.
To raise social awareness about ill effects of playing PUBG, a school decided to start "BAN PUBG" campaign, students are asked to prepare campaign students are asked to prepare campaign board in the shape of rectangle (as shown in the figure).



(i) Find the area of the board.
(ii) If cost of 1 cm² of board is ₹8, then find the cost of board. [2]

Ans :

(i) From the figure, we have

$$AB = \sqrt{(8-2)^2+(2-2)^2}$$

$$= \sqrt{(6^2)+(0)^2} = 6 \text{ cm}$$

$$BC = \sqrt{(8-8)^2+(6-2)^2}$$

$$= \sqrt{(0)^2+4^2} = 4 \text{ cm}$$

Area of board = Area of rectangle $ABCD$

$$= AB \times BC$$

$$= 6 \times 4 = 24 \text{ cm}^2 \quad ...(1)$$

(ii)Total cost of board

$$= \text{Area of board} \times \text{Rate}$$
$$= 24 \times 8 = ₹192$$

Section C

27. The tenth term of an A.P., is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms. [3]

Ans :

Let the first term be a, common difference be d, n^{th} term be a_n and sum of n term be S_n.

$$a_n = a+(n-1)d$$
$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$a+9d = -37 \quad ...(1)$$
$$3(2a+5d) = -27$$
$$2a+5d = -9 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$(2a+18d)-(2a+5d) = -74+9$$
$$13d = -65$$
$$d = -5$$

Substituting the value of d in equation (1), we get

$$a+9\times-5 = -37$$
$$a = -37+45$$
$$a = 8$$

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$$= \frac{8}{2}[2\times 8+(8-1)(-5)]$$
$$= 4[16-35]$$
$$= 4\times -19 = -76$$

Hence, $S_n = -76$

28. If one the zero of a polynomial $3x^2-8x+2k+1$ is seven times the other, find the value of k. [3]

Ans :

We have $f(x) = 3x^2-8x+2k+1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha+\beta = -\left(-\frac{8}{3}\right)$

$$\alpha+7\alpha = 8\alpha = \frac{8}{3}$$

So $\alpha = \frac{1}{3}$

Product of zeroes, $\alpha\times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$
$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$
$$7\times\frac{1}{9} = \frac{2k+1}{1}$$
$$\frac{7}{3}-1 = 2k$$
$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

or

Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2+4x-3$ and verify relationship between zeroes and coefficients of the polynomial.

Ans :

We have $p(x) = 4x^2+4x-3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x)=0$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right)+4\left(\frac{1}{2}\right)-3$$
$$= 1+2-3 = 0$$

and $p\left(-\frac{3}{2}\right) = 4\left(\frac{9}{2}\right)+4\left(-\frac{3}{2}\right)-3$

$$= 9-6-3 = 0$$

Thus $\frac{1}{2}, -\frac{3}{2}$ are zeroes of polynomial $4x^2+4x-3$.

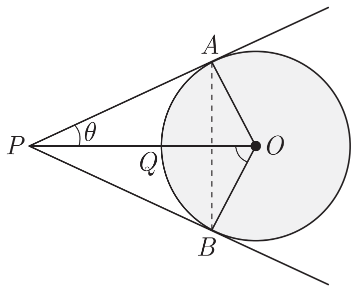
Sum of zeroes $= \frac{1}{2}-\frac{3}{2} = -1 = \frac{-4}{4}$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes $= \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$

$$= \frac{\text{Constan term}}{\text{Coefficient of } x^2} \qquad \text{Verified}$$

29. In the given figure, OP is equal to the diameter of a circle with center O and PA and PB are tangents. Prove that ABP is an equilateral triangle. [3]



Ans :

We redraw the given figure by joining A to B as shown below.

Since OA is radius and PA is tangent at A, $OA \perp AP$. Now in right angle triangle ΔOAP, OP is equal to diameter of circle, thus

$$OP = 2OA$$
$$\frac{OA}{OP} = \frac{1}{2}$$
$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Since PO bisect the angle $\angle APB$,

Hence, $\angle APB = 2\times 30^\circ = 60^o$

Now, in ΔAPB,

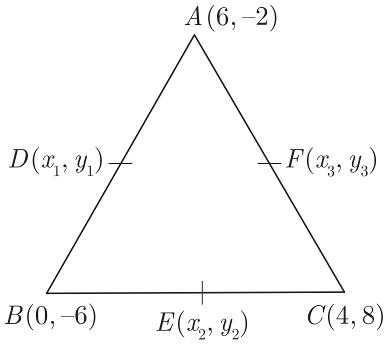
$$AP = AB$$
$$\angle PAB = \angle PBA$$
$$= \frac{180^\circ - 60^\circ}{2} = 60^o$$

Thus ΔAPB is an equilateral triangle.

30. The vertices of ΔABC are $A(6,-2), B(0,-6)$ and $C(4,8)$. Find the co-ordinates of mid-points of AB, BC and AC. [3]

Ans :

Let mid-point of AB, BC and AC be $D(x_1, y_1)$, $E(x_2, y_2)$ and $F(x_2, y_3)$. As per question, triangle is shown below.



Using section formula, the co-ordinates of the points D, E, F are

For D, $x_1 = \frac{6+0}{2} = 3$ $y_1 = \frac{-2-6}{2} = -4$

For E, $x_2 = \frac{0+4}{2} = 2$ $y_2 = \frac{-6+8}{2} = 1$

For F, $x_3 = \frac{4+6}{2} = 5$ $y_3 = \frac{-2+8}{2} = 3$

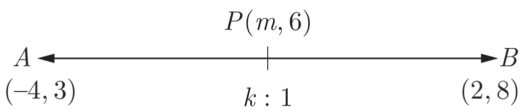
The co-ordinates of the mid-points of AB, BC and AC are $D(3,-4)$, $E(2,1)$ and $F(5,3)$ respectively.

or

Find the ratio in which the point $p(m,6)$ divides the line segment joining the points $A(-4,3)$ and $B(2,8)$. Also find the value of m.

Ans :

As per question, line diagram is shown below.



Let the ratio be $k:1$

Using section formula, we have

$$m = \frac{2k+(-4)}{k+1} \quad (1)$$

$$6 = \frac{8k+3}{k+1} \quad (2)$$

$$8k+3 = 6k+6$$

$$2k = 3$$

$$k = \frac{3}{2}$$

Thus ratio is $\frac{3}{2}:1$ or 3:2.

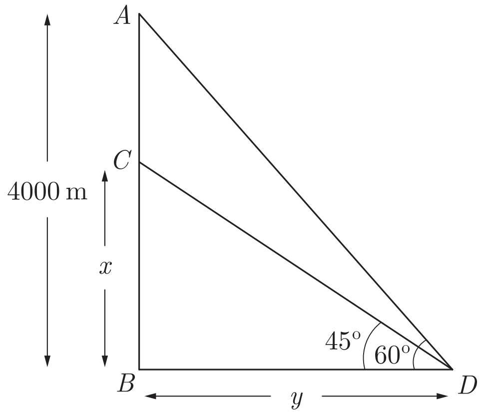
Substituting value of k in (1) we have

$$m = \frac{2(\frac{3}{2})+(-4)}{\frac{3}{2}+1} = \frac{3-4}{\frac{5}{2}} = \frac{-1}{\frac{5}{2}} = \frac{-2}{5}$$

31. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$) [3]

Ans :

Let the height of first plane be $AB = 4000$ m and the height of second plane be $BC = x$ m. As per given in question we have drawn figure below.



Here $\angle BDC = 45°$ and $\angle ADB = 60°$

In ΔCBD, $\frac{x}{y} = \tan 45° = 1 \Rightarrow x = y$

and in ΔABD, $\frac{4000}{y} = \tan 60° = \sqrt{3}$

$$y = \frac{4000\sqrt{3}}{3}$$

$$= 2309.40 \text{ m}$$

Thus vertical distance between two,

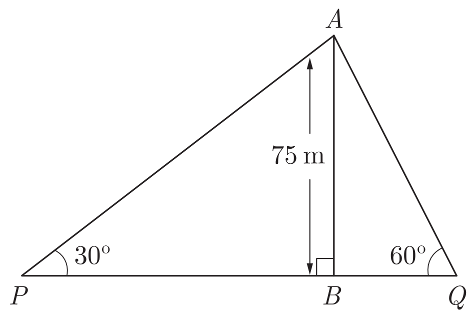
$$4000 - y = 4000 - 2309.40 = 1690.59 \text{ cm}$$

or

Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. find the distance between the two men. (Use $\sqrt{3} = 1.73$)

Ans :

Let AB be the building and the two men are at P and Q. As per given in question we have drawn figure below.



In ΔABP, $\tan 30° = \frac{AB}{BP}$

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$

$$BP = 75\sqrt{3} \text{ m}$$

In ΔABQ, $\tan 60° = \frac{AB}{BQ}$

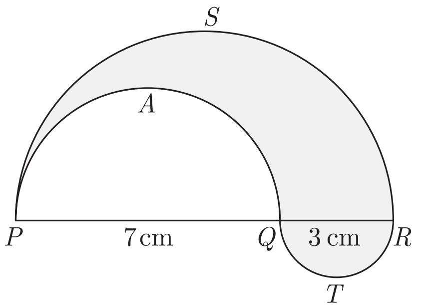
$$\sqrt{3} = \frac{75}{BQ}$$

$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

Distance between the two men,

$$PQ = BP + BQ = 75\sqrt{3} + 25\sqrt{3}$$
$$= 100\sqrt{3} = 100 \times 1.73 = 173$$

32. In the fig., PSR, RTQ and PAQ are three semi-circles of diameters 10 cm, 3 cm and 7 cm region. Use $\pi = \frac{22}{7}$. [3]



Ans :

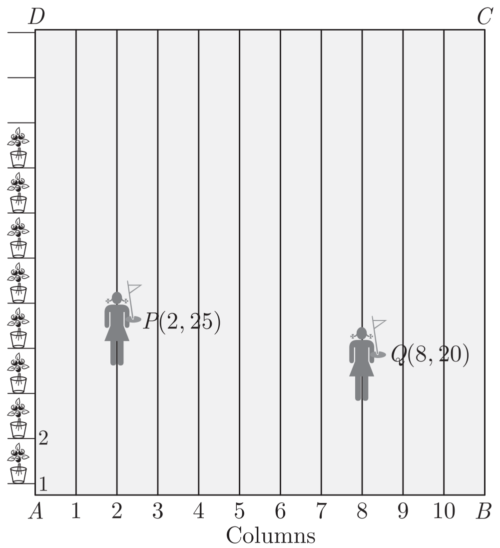
Perimeter of shaded region = Perimeter of semi-circles $PSR + RTQ + PAQ$

Perimeter of shaded region

$$= \pi(5) + \pi(1.5) + \pi(3.5)$$
$$= \pi(10)$$
$$= \frac{22}{7} \times 10 = \frac{22}{7}$$

Perimeter of shaded region = 31.4 cm. (approx.)

33. Read the following passage and the question that follows:
To conduct Sport Day activities, in your rectangular shaped school ground $ABCD$, lines have been draw with chalk power at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in figure. Niharika runs $\frac{1}{4}^{th}$ the distance AD in the 2^{nd} line and posts a green flag. Preet runs $\frac{1}{5}^{th}$ the distance AD on the eights line and posts a red flag. [3]
(i) What is the distance between both the flags?
(ii) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?
(iii) Which mathematical concept is used in the above problem?



Ans :

(i) Considering A as origin (0,0), AB as X-axis and AD as Y-axis.
Niharika runs in the 2^{nd} line with green flag and distance covered (parallel to AD)

$$= \frac{1}{4} \times 100 = 25 \text{ m}$$

Co-ordinates of green flag are (2, 25) and label it as P i.e., $P(2,25)$.
Similarly, Preet runs in the eights line with red flag and distance covered (parallel to AD)

$$= \frac{1}{5} \times 100 = 20 \text{ m}$$

Co-ordinates of red flag are (8,20) and label it as Q, i.e., $Q(8,20)$
Now, using distance formula, distance between green flag and blue flag.

$$PQ = \sqrt{(8-2)^2 + (20-25)^2}$$
$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25}$$
$$= \sqrt{61} \text{ m}$$

(ii) Also, Rashmi has to post a blue flag at the mid-point of PQ, therefore by using mid-point formula, we have $\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$ i.e., $\left(5, \frac{45}{2}\right)$
Hence, the blue flag is in the fifth line, at a distance of $\frac{45}{2}$ i.e., 22.5 m along the direction parallel to AD.
(iii) Co-ordinate Geometry.

34. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. Use $\pi = \frac{22}{7}$ [3]

Ans :

Volume of water in cylinder = Volume of cylinder

$$\pi r^2 h = \pi \times (60)^2 \times 180$$
$$= 648000\pi \text{ cm}^3$$

Water displaced on dropping cone is equal to the volume of solid cone, which is

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (30)^2 \times 60$$
$$= 18000\pi \text{ cm}^3$$

Volume of water left in cylinder

$$= \text{Volume of cylinder} - \text{Volume of cone}$$
$$= 648000\pi - 18000\pi = 630000\pi \text{ cm}^3$$
$$= \frac{630000 \times 22}{1000000 \times 7} \text{ m}^3 = 1.98 \text{ m}^3$$

Section D

35. For Uttarakhand flood victims two sections A and B of class contributed Rs. 1,500. If the contribution of X-A was Rs. 100 less than that of X-B, find graphically the amounts contributed by both the sections. [4]

Ans :

Let amount contributed by two sections X-A and X-B be Rs. x and Rs.y.

$$x + y = 1,500 \qquad ...(1)$$

$$y - x = 100 \quad ...(2)$$

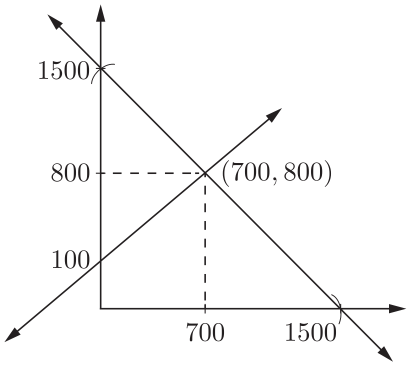
From (1) $y = 15000 - x$

x	0	700	1,500
y	1,500	800	0

From (2) $y = 100 + x$

x	0	700
y	100	800

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point (700, 800)
Hence X-A contributes 700 Rs and X-B contributes 800 Rs.

36. Prove that $n^2 - n$ is divisible by 2 for every positive integer n. [4]

Ans :

We have $n^2 - n = n(n-1)$
Thus $n^2 - n$ is product of two consecutive positive integers.
Any positive integer is of the form $2q$ or $2q+1$, for some integer q.

Case 1 : $n = 2q$

If $n = 2q$ we have

$$n(n-1) = 2q(2q-1)$$
$$= 2m,$$

where $m = q(2q-1)$ which is divisible by 2.

Case 2 : $n = 2q+1$

If $n = 2q+1$, we have

$$n(n-1) = (2q+1)(2q+1-1)$$
$$= 2q(2q+1) = 2m$$

where $m = q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

or

If d is the HCF of 30 and 72, find the value of x and y satisfying $d = 30x + 72y$.

Ans :

Using Euclid's Division Lemma, we have

$$72 = 30 \times 2 + 12 \quad ...(1)$$
$$30 = 12 \times 2 + 6 \quad ...(2)$$
$$12 = 6 \times 2 + 0 \quad ...(3)$$

Thus HCF(30, 72) = 6

Now $6 = 30 - 12 \times 2$ From (2)

$6 = 30 - (72 - 30 \times 2) \times 2$ From (1)

$$6 = 30 - 72 \times 2 + 30 \times 4$$
$$6 = 30(1-4) - 72 \times 2$$
$$6 = 30 \times 5 + 72 \times (-2)$$
$$6 = 30x + 72y$$

Thus $x = 5$ and $y = -2$ .Here x and y are not unique.

37. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, prove that $q(p^2 - 1) = 2p$ [4]

Ans :

We have $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

$$q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)[(\cos\theta + \sin\theta)^2 - 1]$$
$$= (\sec\theta + \operatorname{cosec}\theta)(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$$
$$= (\sec\theta + \operatorname{cosec}\theta)[1 + 2\sin\theta\cos\theta - 1]$$
$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$
$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\sin\theta\cos\theta$$
$$= 2(\sin\theta + \cos\theta) = 2p$$

Hence Proved.

38. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately. [4]

Ans :

Let time taken by pipe A be x minutes and time taken by pipe B be $x+5$ minutes.

In one minute pipe A will fill $\frac{1}{x}$ tank.

In one minute pipe B will fill $\frac{1}{x+5}$ tank.

Thus pipes $A+B$ will fill $\frac{1}{x} + \frac{1}{x+5}$ tank in one minute.

As per question, two pipes running together can fill a tank in $11\frac{1}{9} = \frac{100}{9}$ minutes, in one minute $\frac{9}{100}$ tank will be filled.

Now according to the question we have

$$\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$$
$$\frac{x+5+x}{x(x+5)} = \frac{9}{100}$$
$$100(2x+5) = 9x(x+5)$$
$$200x + 500 = 9x^2 + 45x$$
$$9x^2 - 155x - 500 = 0$$
$$9x^2 - 180x + 25x - 500 = 0$$
$$9x(x-20) + 25(x-20) = 0$$
$$(x-20)(9x+25) = 0$$
$$x = 20, \frac{-25}{9}$$

As time can't be negative we take $x = 20$ minutes

and $x + 5 = 25$ minutes

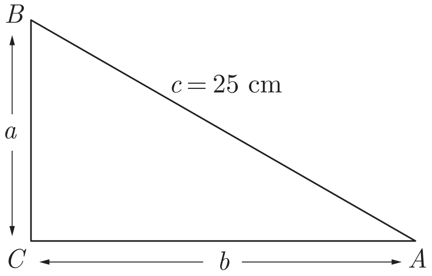
Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

or

The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

Ans :

As per question statement figure is given below.



Here $a+b+c = 60, c = 25$

$$a+b = 60-c = 60-25 = 35$$

Using Pythagoras theorem

$$a^2+b^2 = 25^2 = 625$$

Substituting the values in $(a+b)^2 = a^2+b^2+2ab$,

$$35^2 = 625+2ab$$

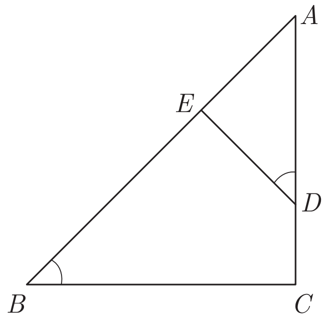
$$1225-625 = 2ab$$

or, $ab = 300$

Hence, Area of ΔABC

$$\frac{1}{2}ab = 150\,\text{cm}^2.$$

39. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$. Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE. [4]



Ans :

In ΔADE and ΔABC, $\angle A$ is common

and $\angle ADE = \angle ABC$ (Given)

Due to AA similarity

$$\Delta ADE \sim \Delta ABC$$

$$\frac{AD}{AB} = \frac{DE}{BC}$$

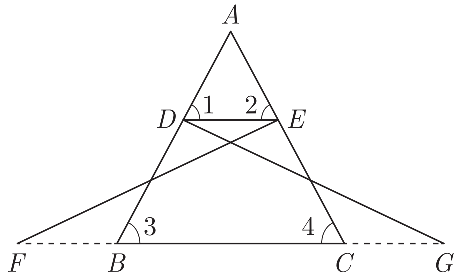
$$\frac{AD}{AE+BE} = \frac{DE}{BC}$$

$$\frac{7.6}{7.2+4.2} = \frac{DE}{8.4}$$

$$DE = \frac{7.6\times 8.4}{11.4} = 5.6 \text{ cm}$$

or

In the following figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \cong \Delta ABC$.



Ans :

Since, $\Delta FEC \cong \Delta GBD$

$$EC = BD \quad ...(1)$$

Since, $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \quad ...(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$DE \parallel BC, \quad \text{(Converse of BPT)}$$

Due to corresponding angles, we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 = \angle 4$$

Thus in ΔADE and ΔABC,

$$\angle A = \angle A$$

$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4$$

By AAA criterion of similarity,

$$\Delta ADE \sim \Delta ABC$$ Hence proved

40. Find the value of x and y, if the median for the following data is 31. [4]

Classes	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	6	y	6	5	40

Ans :

C.I.	f	$c.f.$
0-10	5	5
10-20	x	5+x
20-30	6	11+ x
30-40	y	11+ x+y
40-50	6	17+ x+y
50-60	5	22+ x+y

Here from table, $N = 22+x+y = 40$

$\Rightarrow$ $x+y = 18$ ...(1)

Since, Median = 31, which lies between 30-40

$\therefore$ Median class $= 30-40$

$$\text{Median} = l+\left(\frac{\frac{N}{2}-c.f.}{f}\right)\times h$$

$$\Rightarrow \quad 31 = 30+\left[\frac{20-(11+x)}{y}\right]\times 10$$

$$\Rightarrow \quad 1 = \frac{(9-x)\times 10}{y}$$

$$\Rightarrow \quad y = 90-10x$$

$$10x + y = 90 \qquad ...(2)$$

From equation (1),

$$y = 18 - x$$

From equation (2)

$$10x + 18 - x = 90$$

$$9x = 72$$

$$x = \frac{72}{9} = 8$$

Similarly, from equation (1), we get

$$8 + y = 18$$

$$y = 10$$