CHAPTER 13

Surface Areas and Volumes

VERY SHORT ANSWER TYPE QUESTIONS

1. The curved surface area of a cylinder is 264 m2 and its volume is 924 m3. Find the ratio of its height to its diameter.

Ans : [Board Term-2, 2014]

Curved Surface area of cylinder $= 2\pi rh$

Volume of cylinder $= \pi r^2 h$

$$\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow \frac{r}{2} = \frac{7}{2}$$

Thus $r = 7$ m and substituting in $2\pi rh = 264$ we have

$$2 \times \frac{22}{7} \times 7 \times h = 264$$

$$h = 6 \text{ m}$$

Now $\quad \frac{h}{2r} = \frac{6}{14} = \frac{3}{6}$

Hence, $\quad h : d = 3 : 7$

2. A rectangular sheet paper 40 cm $\times$ 22 cm is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder.

Ans : [Foreign Set I, II, III, 2014]

Here, $h = 40$ cm, circumference $= 22$ cm

$$2\pi r = 22$$

$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} = 3.5 \text{ cm}$$

3. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

Ans : [Delhi CBSE, 2014]

Volume of cylinder : Volume of cone : Volume of hemisphere

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$$

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^2 \times h \qquad (h = r)$$

$$= 1 : \frac{1}{3} : \frac{2}{3}$$

or, $\quad 3 : 1 : 2$

4. What is the ratio of the total surface area of the solid hemisphere to the square of its radius.

Ans : [Board Term-2, 2012 Set (21,22)]

$$\frac{\text{Total surface area of hemisphere}}{\text{Square of its radius}} = \frac{3\pi r^2}{r^2} = \frac{3\pi}{1}$$

Total surface area of hemisphere : Square of radius $= 3\pi : 1$

5. Two cubes each of volume 8 cm3 are joined end to end, then what is the surface area of resulting cuboid.

Ans : [Board Term II, 2012 Set (23)]

Side of the cube, $\quad a = \sqrt[3]{8} = \sqrt{2}$ cm

Now the length of cuboid

$$l = 4 \text{ cm}$$

Breadth, $\quad b = 2$ cm

Height, $\quad h = 2$ cm

Surface area of cuboid $= 2(l \times b + b \times h + h \times l)$

$$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$$

$$= 2 \times 20 = 40 \text{ cm}^2$$

6. The radius of sphere is r cm. It is divided into two equal parts. Find the whole surface of two parts.

Ans : [Board Term-2, 2012, Set (26)]

Whole surface of each part

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Total surface of two parts

$$= 2 \times 3\pi r^2 = 6\pi r^2$$

7. What is the volume of a right circular cylinder of base radius 7 cm and height 10 cm ? Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (59)]

Here $r = 7$ cm, $h = 10$ cm,

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 10$$

$$= 1540 \text{ cm}^3$$

8. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of original cylinder.

Ans : [Board Term-2, 2012 Set (40)]

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h}$$

$$= \frac{1}{4} = 1 : 4$$

9. If the area of three adjacent faces of a cuboid are X, Y, and Z respectively, then find the volume of cuboid.

Ans : [Board Term-2, 2012, Set (5)]

Let the length, breadth and height of the cuboid is l, b and h respectively.

$$X = l \times b$$

$$Y = b \times h$$

$$Z = l \times h$$
$$XYZ = l^2 \times b^2 \times h^2$$
$$\text{Volume of cuboid} = l \times b \times h$$
$$l^2b^2h^2 = XYZ$$
or, $$lbh = \sqrt{XYZ}$$

10. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, find the ratio of their volumes.

Ans : [Board Term-2, 2012, Set (44)]

$$\frac{Volume\ of\ 1^{st}\ cylinder}{Volume\ of\ 2^{nd}\ cylinder} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$
$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$
$$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$$
$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$
$$= 20 : 27$$

11. Volume of two spheres are in the ratio 64 : 27, find the ratio of their surface areas.

Ans : [KVS 2014][Board Term-2, 2012, Set (22)]

$$\frac{Volume\ of\ I^{st}\ sphere}{Volume\ of\ II^{nd}} = \frac{64}{27}$$
$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$
$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$
$$\frac{r_1}{r_2} = \frac{4}{3}$$

Ratio of their surface areas

$$\frac{2\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

12. A solid metallic object is shaped like a double cone as shown in figure. Radius of base of both cones is same but their heights are different. If this cone is immersed in water, find the quantity of water it will displace.

Ans : [Board Term-2, 2012]

Volume of the upper cone $= \frac{1}{3}\pi r^2 h$

Volume of the lower cone $= \frac{1}{3}\pi r^2 H$

Total volume of both the cones $= \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 H$

$$= \frac{1}{3}\pi r^2 (h + H)$$

The quantity of water displaced will $\frac{1}{3}\pi r^2 (h + H)$ cube units.

13. Find the volume (in cm3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

Ans : [Board Term-2, 2012, Set (22)]

Edge of the cube $= 4.2$ cm.

Height of the cone $= 4.2$ cm.

Radius of the cone $= \frac{4.2}{2} = 2.1$ cm.

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2$

$$= 19.4 \text{ cm}^3$$

14. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm3.

Ans : [Board Term-2, 2012]

Let r be the radius of bowl.

Circumference of bowl

$$2\pi r = 132$$
$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Capacity i.e volume of the bowl

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$
$$= 19404 \text{ cm}^3$$

15. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ?

Ans : [Delhi Set-I 2017]

Let radius of sphere be r.

Given, volume of sphere = S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$
$$r = \frac{9}{2} \text{ units}$$

Diameter $d = \frac{9}{2} \times 2 = 9$ units

16. Find the number of solid sphere of diameter 6 cm can be made by melting a solid metallic cylinder of height 45 cm and diameter 4 cm.

Ans : [Delhi CBSE Term-2, 2014]

Let the number of sphere $= n$

Radius of sphere $= 3$ cm, radius of cylinder $= 2$ cm

Volume of spheres = Volume of cylinder

$$n \times \frac{4}{3}\pi r^3 = \pi r_1^2 h$$
$$n \times \frac{4}{3} \times \frac{22}{7} \times (3)^3 = \frac{22}{7} \times (2)^2 \times 45$$
$$36n = 180$$
$$n = \frac{180}{36} = 5$$

Number of solid sphere $= 5$.

17. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius.

Ans : [Board Term-2, 2014]

Let the radius of spherical ball $= R$.

Volume of spherical ball = Volume of three balls

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left[(3)^2 + (4)^3 + (5)^3\right]$$
$$R^3 = 27 + 64 + 125 = 216$$
, $$R = 6 \text{ cm}$$

18. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere.

Ans : [Board Term-2, 2014]

No. of spheres $= 12$

Radius of cone, $r = 1$ cm

Height of the cone $= 48$

Volume of 12 spheres = Volume of cone

Let the radius of sphere be R.

$$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (1)^2 \times 48$$

$$R^3 = 1$$

$$R = 1 \text{ cm}$$

19. Three cubes of iron whose edges are 3 cm, 4 cm and 5 cm respectively are melted and formed into a single cube, what will be the edge of the new cube formed ?

Ans : [Delhi CBSE Term-2, 2012 (13)]

Let the edge of single cube be x cm.

Volume of single cube= Volume of three cubes

$$x^3 = (3)^3 + (4)^3 + (5)^3$$

$$= 27 + 64 + 125 = 216$$

$$x = 6 \text{ cm}$$

20. A solid sphere of radius r melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

Ans : [Delhi Board Term-2, 2012, Set (22)]

Volume of sphere = Volume of cone

Let the radius of cone be R cm.

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$$

$$4r^3 = R^3 r$$

$$R^2 = 4r^2$$

$$R = 2r$$

21. If a cone is cut into two parts by a horizontal plane passing through the mid-points of its axis, find the ratio of the volume of the upper part and the cone.

Ans : [Board Term-2, 2011, Set A1]

As per question the figure is shown below.

$$\text{Volume of upper cone} = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times \frac{h}{2}$$

$$= \frac{1}{3}\pi\frac{r^2}{4} \times \frac{h}{2}$$

$$= \frac{1}{3}\pi\frac{r^2 h}{8}$$

$$\text{Volume of full cone} = \frac{1}{3}\pi r^2 h$$

$$\frac{\text{Volume of upper of cone}}{\text{Volume of cone}} = \frac{\frac{1}{3}\pi \times \frac{1}{8}r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{1}{8}$$

$$= 1 : 8$$

22. What is the frustum of a right circular cone of height 16 cm with radii of its circular ends as 8 cm and 29 cm has slant height equal to ?

Ans : [Board Term-2, 2014 A1]

As per question the figure is shown below.

Slant height of the frustum,

$$l = \sqrt{h^2 + d^2}$$

$$= \sqrt{(16)^2 + (12)^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400}$$

$$= 20 \text{ cm.}$$

23. The slant height of a bucket is 26 cm. The diameter of upper and lower circular ends are 36 cm and 16 cm. Find the height of the bucket.

Ans : [Board Term-2, 2012 31]

Here, $l = 26$ cm, upper radius $= 18$ cm,

lower radius $= 8$ cm

d =difference in radius $= 18 - 8 = 10$ cm.

Let h be the height of bucket

$$h = \sqrt{l^2 - d^2} = \sqrt{(26)^2 - (10)^2}$$

$$= \sqrt{676 - 100} = \sqrt{576} = 24 \text{ cm.}$$

24. A cylinder and a cone have base radii 5 cm and 3 cm respectively and their respective heights are 4 cm and 8 cm. Find the ratio of their volumes.

Ans : [Board Term-2, 2012 Set (59)]

$$\text{Volume of cylinder} = \pi(5)^2 \times 4 \text{ cm}^3$$

$$= 100\pi \text{ cm}^3$$

$$\text{Volume of cone} = \frac{1}{3}\pi \times 3^2 \times 8$$

$$= 24\pi$$

$$\text{Required ratio} = 100\pi : 24\pi$$

$$= 25 : 6.$$

SHORT ANSWER TYPE QUESTIONS - I

1. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere.

Ans : [Board Term-2, 2012 Set (5)]

Diameter of sphere = Radius of hemisphere

$= 6$ cm

Radius of sphere $= 3$ cm

Volume $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3^3 \text{ cm}^3.$

$= 113.14 \text{ cm}^3.$

2. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid.

Ans : [Board Term-2, 2012 Set (17)]

Diameter of hemisphere = Side of cubical block

$$2r = 7$$

or, $$r = \frac{7}{2}$$

Surface area of solid

= Surface area of the cube

− Area of base of hemisphere

+ curved surface area of hemisphere

$$= 6l^2 - \pi r^2 + 2\pi r^2$$

$$= 6 \times 49 - 11 \times \frac{7}{2} + 77 = 332.5 \text{ cm}^2$$

3. A glass cylinder with diameter 20 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. Use $\pi = \frac{22}{7}$

OR

A cylinder glass tube with radius 10 cm has water upto a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. By how much the water will rise in the glass tube. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set(34)]

Let h be the height of water raised measured.

Volume of water displaced in cylinder $= \pi(10)^2 h$

Volume of cube, $\pi(10)^2 h = 8 \times 8 \times 8$

$$h = \frac{8 \times 8 \times 8 \times 7}{22 \times 10 \times 10}$$

$$= 1.629 \text{ cm.}$$

4. Two cubes of 5 cm each are kept together joining edge to edge to form a cuboid. Find the surface area of the cuboid so formed.

Ans : [Board Term-2, 2015]

Let l be the length of the cuboid so formed.

Thus $l = 5 + 5 = 10$ cm, $b = 5$ cm; $h = 5$ cm.

$$\text{Surface area} = 2(l \times b + b \times h + h \times l)$$

$$= 2(10 \times 5 + 5 \times 5 + 5 \times 10)$$

$$= 2(50 + 25 + 50)$$

$$= 2 \times 125$$

$$= 250 \text{ cm}^2.$$

5. If the total surface area of a solid hemisphere is 462 cm2, find its volume. Use $\pi = \frac{22}{7}$

Ans : [CBSE O.D. 2014]

Total surface area of hemisphere,

$$3\pi r^2 = 462 \text{ cm}^2$$

$$\frac{22}{7} \times r^2 = \frac{462}{3}$$

$$r^2 = \frac{462 \times 7}{22 \times 3} = 49$$

$$r = 7 \text{ cm.}$$

Volume of hemisphere,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{2156}{3} = 718.67 \text{ cm}^3.$$

6. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

Ans : [Delhi CBSE, Term-2, 2014], [Foreign Set I, II, III, 2014]

Given, radius $r = 7$ m and height $h = 24$ m

Slant height of tent,

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$

$$= \sqrt{625} = 25 \text{ m.}$$

Curves surface area

$$\pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Curves surface area of tent will be required area of cloth. Let x meter of cloth is required

$$5x = 550 \text{ or, } x = \frac{550}{5} = 110 \text{ m.}$$

Thus 110 m of cloth is required.

$$\text{Cost of cloth} = 25 \times 110 = Rs.2750.$$

7. Find the number of plates, 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular of height 10 cm and diameter 4.5 cm.

Ans : [Board Term-2, 2014]

Each one of he circular plate is also a cylinder.

Volume of plate $V_p = \pi r^r h = \pi \times (.75)^2 (.2)$

$$= \frac{9\pi}{80} \text{ cm}^3$$

Volume of right circular cylinder

$$V_c = \pi(2.25)^2(10) = 405\frac{\pi}{8} \text{ cm}^3$$

$$\text{Number of plates} = \frac{\frac{405\pi}{8}}{\frac{9\pi}{80}} = \frac{405\pi}{9\pi} \times \frac{80}{8}$$

$$= 450 \text{ plates.}$$

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm3. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (44)]

As per question the figure is shown below.

Volume of remaining solid

= Volume of cylinder − Volume of cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 2.4$$

$$= 44 \times 0.1 \times 0.7 \times 0.8$$

$$= 4.4 \times .56 = 2.464 \text{ cm}^3.$$

9. A solid metallic of dimensions 9 m × 8m × 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

Ans : [Foreign Set-I, II 2017]

$$\text{Volume of cuboid} = 9 \times 8 \times 2 \text{ cm}^3$$

$$\text{Volume of cube} = 2 \times 2 \times 2 \text{ cm}^3$$

Let number of recast cubes be n.

Volume of n cubes = Volume of cuboid

$$n \times 2 \times 2 \times 2 = 9 \times 8 \times 2$$

$$n = \frac{9 \times 8 \times 2}{2 \times 2 \times 2} = 18$$

Hence, number of cubes recast = 18

10. A solid metallic cylinder of radius 3.5 cm and height 14 cm melted and recast into a number of small solid metallic ball, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

Ans : [CBSE S.A. 2 2016 Set-HODM4OL]

Let the number of recasted balls be N

Radius of cylinder $R = 3.5$ cm

Height of cylinder $h = 14$cm

Radius of recasted ball $r = \frac{7}{12}$

Volume of balls = Volume of cylinder

$$n\frac{4}{3}\pi r^2 = \pi R^2 h$$

$$n \times \frac{4}{3} \times \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12} = 3.5 \times 3.5 \times 14$$

$$n = \frac{3.5 \times 3.5 \times 14 \times 3 \times 12 \times 12 \times 12}{4 \times 7 \times 7 \times 7}$$

$$= 0.5 \times 0.5 \times 2 \times 3 \times 3 \times 12 \times 12$$

$$= 648$$

Hence, number of recasted balls = 648

11. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

Ans : [Board Sample Paper, 2016]

Radius of sphere $\frac{6}{2} = 3$ cm

Radius of cylinder vessel $\frac{12}{2} = 6$ cm

Let the level of water rise in cylinder be h.

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$= 36\pi \text{ cm}^3$$

Volume of sphere = Increase volume in cylinder

$$36\pi = \pi(6)^2 \times h$$

$$h = 1 \text{ cm}$$

Thus level of water rise in vessel is 1 cm.

12. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Ans : [Board Sample Paper 2016]

Volume of of any cylinder shape is $\pi r^2 h$.

Volume of coin $= \pi(0.75)^2 \times 0.2$ cm^3

Volume of cylinder $= \pi(2.25)^2 \times 10$ cm^3

$$\text{No. of coins} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$$

$$= \frac{\pi(2.25)^2 \times 10}{\pi(0.75)^2 \times 0.2} = \frac{(3)^2 \times 10}{0.2}$$

$$= 450$$

13. A cone of height 24 cm and radius of base 6 cm is made up of clay. If we reshape it into a sphere, find the radius of sphere.

Ans : [KVS 2014]

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\frac{4}{3} \times r_1^3 = (6)^2 \times \frac{24}{3}$$

$$4r_1^3 = 36 \times 24$$

$$r_1^3 = 6^3$$

$$r_1 = 6 \text{ cm}$$

Hence, radius of sphere is 6 cm.

14. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0.5 cm. What is the height of cylinder ?

Ans : [Board Term-2, 2012 (22)]

Volume of cylinder = Volume of sphere,

$$\pi r^2 h = \pi$$

where r and h are radius of base and height of cylinder

$$(0.5)^2 h = 1$$

$$\left(\frac{1}{2}\right)^2 h = 1$$

$$h = 4 \text{ cm.}$$

15. A metallic solid sphere of radius 4.2 cm is melted and recast into the shape of a solid cylinder of radius 6 cm. Find the height of the cylinder.

Ans : [Board Term-2, 2012 (1)]

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\frac{4}{3} \times (4.2)^3 = 6^2 \times h$$

$$h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$

Hence, height of cylinder $h = 2.744$ cm.

SHORT ANSWER TYPE QUESTIONS - II

1. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm2. Find the volume of the cone. (Use $\pi = 3.14$)

Ans : [Delhi Set II, 2016]

We have $r = 3, \pi rl = 47.1$

Thus $l = \frac{47.1}{3 \times 3.14} = 5$

$$h = \sqrt{5^2 - 3^3} = 4 \text{ cm}$$

Volume of cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4$$

$$= 37.68 \text{ cm}^3$$

2. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\pi = \frac{22}{7}$

Ans : [CBSE Delhi Set I, 2016]

Here, $r + h = 37$ (1)

and $2\pi r(r+h) = 1628$ (2)

Thus $2\pi r \times 37 = 1628$

$$2\pi r = \frac{1628}{37}$$

$$r = 7 \text{ cm}$$

Substituting $r = 7$ in (1) we have

$$h = 30 \text{ cm.}$$

Here volume of cylinder

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

3. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use $\pi = \frac{22}{7}$

Ans : [O.D. Set I, II, III, 2016]

Height of cylinder $= 2.1$ m

Radius of cylinder $=$ radius of cone $= \frac{3}{2}$ m

Slant height of cone $= 2.8$ m

Surface area of tent

$= C.S.A$ of cone $+ C.S.A$ of cylinder.

$= \pi rl + 2\pi rh = \pi r(l + 2h)$

Area of canvas required will be surface area of tent.

Thus $\pi r(l+2h) = \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1)$

$$= \frac{33}{7} \times 7 = 33 \text{ m}^2$$

Total Cost $= 33 \times 500 Rs$

$= 16,500 Rs$

4. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs.5 per 100 sq. cm.[Use $\pi = 3.14$]

Ans : [Outside Delhi CBSE Board 2015, Set I, II, III]

As per question the figure is shown below.

Side of given cube $a = 10$ cm

Area of cube(excluding base)

$A_1 =$ area of 4 walls + area of Top

$= 4a^2 + a^2 = 5a^2 = 5(10)^2 = 500 \text{ cm}^2$

Let r be the largest radius of hemisphere. From fig. (ii) we have

$\square ABCD$, in the square of side 10 cm.

In ΔABC, $\angle B = 90$

From Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$

$$(2r)^2 = (10)^2 + (10)^2$$

$$4r^2 = 200 \text{ cm}^2$$

$$r = \sqrt{\frac{200}{4}} = 5\sqrt{2} \text{ cm}$$

Hence, the required diameter of hemisphere

$$d = 2r = 2 \times 5\sqrt{2} = 10\sqrt{2} \text{ cm}$$

Now, area of unshaded part in fig (ii)

$A_2 =$ area of circle$-$ area of square $ABCD$

$= \pi r^2 - (a)^2 = \left[\pi \times 50 - (10)^2\right]$

$= (157 - 100) = 57 \text{ cm}^2$

Now, Total surface area of solid

$A = A_1 + A_2 + 2\pi r^2$

$= [500 + 57 + 2 \times 3.14 \times 50]$

$= 871 \text{ cm}^2$

The cost of painting of solid

$$= \left(871 \times \frac{5}{100}\right) = 43.55 \text{ Rs}$$

5. A hemispherical bowl of internal diameter 36 cm contains liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find. the height of the each bottle, if 10% liquid is wasted in this transfer.

Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

$$\text{Volume of bowl} = \frac{2}{3}\pi r^3$$

$$\text{Volume of liquid in bowl} = \frac{2}{3}\pi \times (18)^3 \text{ cm}^3$$

$$\text{Volume of one after wastage} = \frac{2}{3}\pi(18)^3 \times \frac{90}{100} \text{ cm}^3$$

$$\text{Volume of one bottle} = \pi r^2 h$$

Volume of liquid in 72 bottles

$$= \pi \times (3)^2 \times h \times 72 \text{ cm}^2$$

Volume of bottles $=$ volume in liquid after wastage

$$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}$$

$$h = \frac{\frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

Hence, the height of bottle $= 5.4$cm

6. A metallic has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

Ans : [Foreign Set I, II, III, 2015]

Volume of cylinder $= \pi r^2 h = \pi(3)^2 \times 5$

$= 45\pi \text{ cm}^3$

$$\text{Volume of conical hole} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9}$$

$$= \frac{2}{3}\pi \text{ cm}^3$$

$$\text{Metal left in cylinder} = 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3}$$

$$\frac{\text{Volume of metal left}}{\text{Volume of metal taken out}} = \frac{\frac{133}{3}\pi}{\frac{2}{3}\pi} = 133 : 2.$$

Hence, Volume of metal left : Volume of metal cut off $= 133 : 2$

7. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. Use $\pi = \frac{22}{7}$

Ans : [Foreign Set I, II, III, 2015]

Volume of water in cylinder = Volume of cylinder

$$\pi r^2 h = \pi \times (60)^2 \times 180$$

$$= 648000\pi \text{ cm}^3$$

Water displaced on dropping cone is equal to the volume of solid cone, which is

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (30)^2 \times 60$$

$$= 18000\pi \text{ cm}^3$$

Volume of water left in cylinder

= Volume of cylinder − Volume of cone

$= 648000\pi - 18000\pi = 630000\pi \text{ cm}^3$

$$= \frac{630000 \times 22}{1000000 \times 7} \text{ m}^3 = 1.98 \text{ m}^3$$

8. The rain water from 22 m × 20 m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collect from the roof the root fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

Ans : [Foreign Set I, II, III, 2015]

Volume of water collected in cylindrical vessel

$$= \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right) \text{ m}^3 = \frac{44}{5} \text{ m}^3$$

Let the rainfall be h m.

Rain water from roof $= 22 \times 20 \times h \text{ m}^3$

$$22 \times 20 \times h = \frac{44}{5}$$

$$h = \frac{44}{5} \times \frac{1}{22 \times 20} = \frac{1}{50} \text{ m}^3$$

$$= \frac{1}{50} \times 100 = 2 \text{ cm}$$

9. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe.

Ans : [Board Term-2, 2015]

Height of cylindrical pipe $h = 21$ dm

$= 210$ cm

External Radius $R = \frac{10}{2} = 5$ cm

Internal Radius $r = \frac{6}{2} = 3$

Volume of copper used in making the pipe

= (Volume of External Cylinder)

− (Volume of Internal Cylinder)

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - h^2)$

$$= \frac{22}{7} \times 210(5^2 - 3^2) = \frac{22}{7} \times 210 \times 8 \times 2$$

$= 10560 \text{ cm}^3$.

10. A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2015]

Radius of the glass $= 7$ cm

Height of the glass $= 10$ cm

Volume of 1 glass $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 10$$

$= 1540 \text{ cm}^3$

Volume of juice to fill 6 glasses

$= 6 \times 1540 = 9240 \text{ cm}^3$

$$\text{Volume in litre} = \frac{9240}{1000} = 9.240 \text{ litre.}$$

11. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. Use $\pi = \frac{22}{7}$

Ans : [CBSE O.D. 2014]

Side of cube $a = 7$ cm

The diameter of the largest possible sphere is the side of the cube.

Thus radius of sphere $= \frac{7}{2}$ cm.

Volume of the wood left

= volume of cube − volume of sphere

$= a^3 - \frac{4}{3}\pi r^3$

$$= 7 \times 7 \times 7 - \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 7 \times 7 \times 7\left(1 - \frac{11}{21}\right)$$

$$= 7 \times 7 \times 7 \times \frac{10}{21} = \frac{490}{3}$$

Hence, Volume of wood $= 163.3 \text{ cm}^3$.

12. In the given figure a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs. 500/ sq. metre. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi, Set-II 2016]

Radius of cylinder as well as conical part $= \frac{3}{2}$cm

Height of cylinder $h = 2.1$ m

Slant height of cone $r = 2.8$ cm

Total canvas required

$$2\pi rh + \pi rl = \frac{22}{7} \times \frac{3}{2}[4.2 + 2.8] \text{ m}^2$$

$$= \frac{22}{7} \times \frac{3}{2} \times 7.0 = 33 \text{ m}^2$$

$$\text{Total cost} = 33 \times 500 = 16{,}500$$

13. A girl empties a cylindrical bucket, full of sand, of radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct upto one place of decimal.

Ans : [Foreign Set I, II, III, 2014]

$$\text{Volume of cone} = \text{Volume of Cylinder}$$

$$\frac{1}{3}\pi r_2^2 h = \pi r_1 h^2$$

$$\frac{1}{3} \times r_2^2 \times 24 = 18 \times 18 \times 32$$

$$r_2^2 = 1296$$

Radius of cone $r_2 = 36$ cm

Now, slant height of cone

$$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 36^2}$$

$$= \sqrt{576 + 1296} = 43.2 \text{ cm.}$$

14. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. Use $\pi = \frac{22}{7}$

Ans : [Delhi 2013]

As per question the figure is shown below.

Radius of toy = radius of cylinder = 3.5 cm

Vol. of toy = Vol. of cylinder $- 2 \times$ Vol. of hemisphere

$$= \pi r^2 h - 2 \times \frac{22}{7}\pi r^3$$

$$= \pi r^2\left[h - \frac{4}{3}r\right]$$

$$= \frac{22}{7} \times (3.5)^2\left[10 - \frac{4}{3} \times 3.5\right]$$

$$= 22 \times 0.5 \times 3.5 \times 5.3$$

$$= 205.205 \text{ cm}^3.$$

15. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. Use $\pi = \frac{22}{7}$

Ans : [Delhi 2013]

As per question the figure is shown below.

$$\text{Radius of hemisphere } \frac{14}{2} = 7 \text{ cm}$$

$$\text{Height of cylinder} = 13 - 7 = 6 \text{ cm}$$

Total slanted area of cylinder

$$= \text{S.A of hemisphere} + \text{S.A. of hemisphere}$$

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi r(r + h)$$

$$= 2 \times \frac{22}{7} \times 7(7 + 6)$$

$$= 44 \times 13 = 572 \text{ cm}^2$$

16. The radii of two right circular cylinders are in the ratio of 2 : 3 and their height are in the ratio of 5 : 4. Calculate the ratio of their curved surface area and radio of their volumes.

Ans : [Board Term-2, 2012 Set (22)]

Let the radii of two cylinders be $2x$ and $3x$ and their heights be $5y$ and $4y$ respectively

Ratio of their curved surface areas

$$= \frac{2\pi \times 2x \times 5y}{2\pi \times 3x \times 4y} = \frac{5}{6}$$

Since their curved surface areas are in the ratio of 5 : 6.

Ratio of their volumes

$$= \frac{\pi \times (2x)^2 \times 5y}{2\pi \times 3x \times 4y} = \frac{5 \times 4}{4 \times 9} = \frac{5}{9}$$

Hence, their volumes are in the ratio of 5 : 9 and their $C.SA$ are in the ratio of 5 : 6.

17. A toy is in the form of a cone surmounted on a hemisphere of common base of diameter 7 cm. If the height of the toy is 15.5 cm, find the total surface area of the toy. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (21, 22)]

As per question the figure is shown below.

Radius $r = 3.5$ cm

and height $h = 12$ cm

Slant height of cone,

$$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 3.5^2} = 12.5$$

Total surface area of the toy

$$= \text{Surface area of hemisphere} + \text{Curved surface area of cone}$$

$$= 2\pi r^2 + \pi rl$$

$$= \pi r(2r_1 + l)$$

$$= \frac{22}{7} \times \frac{7}{2}\left(2 \times \frac{7}{2} + 12.5\right)$$

$$= 11 \times 19.5 = 214.5 \text{ cm}^2$$

18. Water is flowing at 7 m/s through a circular pipe of internal diameter of 4 cm into a cylindrical tank, the radius of whose base is 40 cm. Find the increase in water level in 30 minutes.

Ans : [Board Term-2, 2012 Set (40)]

Volume of water in 30 minutes

$$= \pi \times (2)^2 \times 700 \times 60 \times 30 \text{ cm}^3$$

Let height of water in tank $= h$ cm

and radius $= 40$ cm

Volume of water in the tank

$$\pi 40^2 \times h = 700 \times 60 \times 30 \times 4 \times \pi$$

$$h = \frac{700 \times 60 \times 30 \times 4}{40 \times 40}$$

$$= \frac{6300}{2} \text{ cm} = \frac{63}{2} \text{ m}$$

Hence, water level increased is 31.5 m.

19. The slant height of a frustum of a cone is 4 m and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Ans : [Outside Delhi Set-I 2017]

Let the radii of frustum be r_1 and r_2.

$$2\pi r_1 = 18 \text{ cm and } 2\pi r_2 = 6 \text{ cm}$$

$$\pi r_1 = \frac{18}{2} = 9 \text{ cm } \pi r_2 = \frac{6}{2} = 3 \text{ cm}$$

and slant height $= 4$ cm

Curved surface area of frustum

$$= \pi(r_1 + r_2) \times l$$
$$= (\pi r_1 + \pi r_2) \times l$$
$$= (9 + 3) \times 4$$
$$= 12 \times 4 = 48 \text{ cm}^2$$

Hence, curved surface area $= 48$ cm^2

20. A metallic solid sphere of radius 10.5 cm melted and recasted into smaller solid cones each of radius 3.5 cn and height 3 cm. How may cones will be made ?

Ans : [Delhi Set-II 2017]

Radius of given sphere $= 10.5$ cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5$$

$$= 4\pi \times 3.5 \times 10.5 \times 10.5 \text{ cm}^3$$

Radius of one recasted cone $= 3.5$cm

$$\text{Height} = 3 \text{ cm}$$

$$\text{Volume} = \frac{1}{3}\pi \times 3.5 \times 3.5 \times 3$$

$$= \pi \times 3.5 \times 3.5 \text{ cm}^3$$

Let the number of recasted cones be n.

$$n \times \pi \times 3.5 = 4 \times \pi \times 3.5 \times 10.5 \times 10.5$$

$$n = \frac{4 \times 3.5 \times 10.5 \times 10.5}{3.5 \times 3.5} = 126$$

Hence, number of recasted cones $= 126$.

21. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

Ans : [Delhi Set-III 2017]

Diameter of sphere $= 16$ cm

$$\text{radius} = \frac{16}{2} = 8 \text{ cm}$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times 8 \times 8 \times 8 \text{cm}^3$$

Radius and height of recasted cones $= 4$ cm and 8 cm respectively.

Volume of each cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{4} \times \pi \times 4 \times 4 \times 8 \text{ cm}^3$$

Let number of cones recasted be n

$$n = \frac{\text{Volume of Sphere}}{\text{Volume of One Cone}}$$

$$= \frac{\frac{4}{3} \times \pi \times 8 \times 8 \times 8}{\frac{1}{3} \times \pi \times 4 \times 4 \times 8} = 16$$

Hence number of recasted cones $= 16$ cm

22. A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged into water, by how much will the level of water rise in the cylindrical vessel ?

Ans :

Let the rise in level of water be h cm.

Radius of sphere $= 3$ cm. radius of cylinder

$$= \frac{12}{2} = 6 \text{ cm}$$

Volume of water displaced in cylinder will be equal to the volume of sphere.

Thus $\pi r^2 h = \frac{4}{3}\pi r^3$

$$\pi \times 6 \times 6 \times h = \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$h = \frac{4 \times 3 \times 3 \times 3}{3 \times 6 \times 6} = 1 \text{ cm}$$

Hence the water level rises $= 1$ cm.

23. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Ans :

As per question the figure is shown below.

Height of hemisphere $r = 3.5$ cm

Height of cone $h = 15.5 - 3.5 = 12$

Slant height of cone

$$\sqrt{r^2 + h^2} = \sqrt{12.5 + 144}$$
$$= \sqrt{156.25} = 12.5 \text{ cm}$$

TSA of toy= CSA of cone + CSA of hemisphere

$$\pi rl + 2\pi r^2 = \frac{22}{7} \times 12.5 \times 3.5 + 2 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 22 \times 12.5 \times 0.5 + 22 \times 3.5$$
$$= 22\left(12.5 \times \frac{5}{10} + 3.5\right)$$
$$= 22\left(12.5 \times \frac{1}{2} + 3.5\right)$$
$$= 22(6.25 + 3.5)$$
$$= 22(9.75) = 214.5 \text{ cm}^2$$

Thus total surface area of toy is 214.5 cm^2

24. A conical vessel, with base radius 5 cm height 24 cm, is full of water. This water emptied into a cylindrical vessel, of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi, Set-II 2016]

Here radius and height of conical vessel are 5 cm and 24 cm.

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 2.5 \times 24 \text{ cm}^3$$

When water is emptied into cylindrical vessel, water will rise in cylindrical vessel. Let rise in height be h.

Volume of water raised $= \pi r^2 h$

$$\frac{1}{3}\pi \times 25 \times 24 = \pi \times (10)^2 \times h$$

$$25 \times 8 = 100h$$

$$h = 2 \text{ cm}$$

25. Water is flowing at the rate of 0.7 m/sec through a circular pipe whose internal diameter is 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the level of water in half hour.

Ans : [Board Sample Paper, 2016]

Length of water that flows in 1 sec. $= 0.7$ m

Length of water that flows out in 30 minutes

$$= (0.7 \times 100 \times 60 \times 30) \text{ cm}$$

$$= 126000 \text{ cm}$$

Volume of water that flows out in 30 minutes

$$= \pi(1)^2 \times 126000 \text{ cm}^3$$

$$= 126000\pi \text{ cm}^3$$

Let the depth of water in the tank be x cm.

Volume of water tank $= \pi(40)^2 \times x \text{ cm}^3$

$$\text{Volume of tank} = \text{Volume of water flows}$$

$$\pi(40)^2 \times x = 126000\pi$$

$$x = 78.75 \text{ cm}$$

26. A well of diameter 4 m dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Ans : [Delhi Set I, II, III, 2016]

Diameter of earth dug out $= 4$m

radius of earth dug out $= 2$ m

Depth of the earth $= 21$,

Volume of earth $= \pi r^2 d$

$$= \frac{22}{7} \times 2 \times 2 \times 21$$

$$= 264 \text{ m}^2$$

Width of embankment $= 3$ m

Outer radius of ring $= 2 + 3 = 5$ m

Let the height of embankment be h.

Volume of embankment

$$\pi(R - r)^2 = 264$$

$$\frac{22}{7} \times (25 - 4) \times h = 264$$

$$h = \frac{264 \times 7}{22 \times 21} = 4$$

Height of embankment is 4 m.

27. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cones.

Ans : [Foreign Set I, II, III, 2016]

For cylindrical tub,

Diameter $D = 12$ cm

Radius $R = 6$ cm

Height $H = 15$ cm.

Volume $\pi R^2 H = \pi(6)^2 \times 15$

$$= 540\pi \text{ cm}^3$$

Each child will get the ice-cream $\frac{540\pi}{10}$ cm^3

$$= 54\pi \text{ cm}^3$$

For cone, height $h = 2 \times d = 2(2r) = 4r$

Volume of cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times 4r = \frac{4}{3}\pi r^3$$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

Total volume of cone and hemisphere

$$= \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{6}{3}\pi r^3 = 2\pi r^3$$

According to question,

$$2\pi r^3 = 54\pi$$

$$r^3 = 27$$

$$r = 3$$

Hence, Diameter $= 2r = 2 \times 3 = 6$ cm.

28. A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty ? Use $\pi = \frac{22}{7}$

Ans : [Foreign Set I, II, III, 2016]

Diameter of tank $= 3$m

Radius $r = \frac{3}{2}$ m

Volume of hemispherical tank,

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left(\frac{3}{2}\right)^3 \text{ m}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{27}{8} \text{ m}^3$$

$$= \frac{11}{7} \times \frac{9}{2} = \frac{99}{14} \text{ m}^3$$

Since 1 m^3 = 1000 litre, we have

$$V = \frac{99}{14} \times 1000 \text{ litre}$$

Volume of hemisphere

$$\frac{V}{2} = \frac{1}{2} \times \frac{99}{14} \times 1000 \text{ Litres}$$

Let time taken for this volume to flow out be t sec. Then according to question,

$$t \times 3\frac{4}{7} = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$t \times \frac{25}{7} = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$t = \frac{7}{25} \times \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$= 990 \text{ sec}$$

$$= 16 \text{ minutes } 30 \text{ sec.}$$

29. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

Volume of metal in 504 cones

$$= 504 \times \frac{\pi}{3} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3$$

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times r^3$$

$$= \text{Volume of } 504 \text{ cones}$$

$$\frac{4\pi}{3} \times r^3 = 504 \times \frac{\pi}{3} \times \frac{35}{20} \times \frac{35}{20} \times 3$$

$$r^3 = 126 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 7 \times 9 \times 2 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 3 \times 3 \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \times 3$$

$$r = 3 \times \frac{7}{2} = 10.5 \text{ cm}$$

Thus diameter is 21 cm.

$$\text{Surface area} \quad 4\pi r^2 = 4 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= 1386 \text{ cm}^2$$

30. A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. Find the radius of sphere.

Ans : [Board Term-2, 2014]

Let the radius of sphere be R.

$$\text{Volume of sphere} = \text{Volume of cone}$$

$$\frac{4}{3}\pi r^2 h = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times 2 \times 2 \times 8$$

$$R^3 = \frac{2 \times 2 \times 8}{4}$$

$$R^3 = 8$$

$$R = 2 \text{ cm}$$

31. A sphere of diameter 12 cm, is dropped in a right circular cylindrical verse, partly filled with water. If the sphere is completely submerged in water, the water level into the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Ans : [Outside Delhi Set-II, 2016]

$$\text{Diameter of sphere} = 12 \text{ cm}$$

$$\text{Its radius} = 6 \text{ cm}$$

$$\text{Volume} = \frac{4}{3}\pi \times 6^3 \text{ cm}^3$$

It is submerged into water, in cylindrical vessel, then water turn rise by $3\frac{5}{9} = \frac{32}{9}$ cm

$$\text{Volume submerged} = \text{Volume rise}$$

Let radius of cylinder be r cm

$$\frac{4}{3}\pi \times 6^3 = \pi \times r^2 \times \frac{32}{9} \text{ cm}$$

$$\frac{216 \times 3 \times 4}{32} = r^2$$

$$\frac{4 \times 27 \times 3}{4} = r^2 \Rightarrow 4 \times \frac{81}{4} \text{ cm}^3 = r^2$$

$$r = 9 \text{ cm}$$

$$\text{Diameter} \quad 2r = 2r = 2 \times 9 = 18 \text{ cm.}$$

32. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Ans : [Delhi Set-I 2017]

Radius of conical vessel $= 5$ cm

and its height $= 24$ cm

Volume of this vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 24$$

$$= 200\pi \text{ cm}^3$$

Internal radius of cylindrical vessel= 10

Let the height of emptied water be h.

Volume of water in cylinder,

$$\pi r^2 h = \frac{3}{4} \times \text{Volume of cone}$$

$$\pi \times 10 \times 10 \times h = \frac{3}{4} \times 200\pi$$

$$100\pi h = 150\pi$$

$$h = 1.5 \text{ cm}$$

Hence the height of water= 1.5 cm

33. Rampal decided to donate convas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is Rs. 40 per meter, find the amount by which Rampal helped the money.

Ans : [Outside Delhi Compt. Set-I, II, III 2017]

Diameter of tent $= 14$m and height $= 24$ m

radius of tent $= 7\text{m}$

Slant height $= \sqrt{h^2+r^2} = \sqrt{24^2+7^2}$

$= \sqrt{576+49} = 25$ m

Surface area of the tent

$\pi rl = \frac{22}{7} \times 7 \times 25 = 550$ m^2

Surface area of 10 tents

$= 550 \times 10 = 5500$

Total cost $= 5500 \times \frac{40}{2} = 110000$

Hence, Rampal helped the centre

$= Rs.110000$

34. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

Ans : [Sample Question Paper 2017]

Side of cube $= 14$ cm.

Cone of maximum size is curved out

Diameter of cone $= 14$ cm

Radius of cone $= 7$ cm

Slant height $l = \sqrt{h^2+r^2} = \sqrt{14^2+7^2}$

$= \sqrt{196+49} = \sqrt{245}$

$= 15.65$ cm.

Total surface area = Surface area cube + curved

Surface area of cone − Circular area of base of cone

$= 6a^2 + \pi rl - \pi r^2$

$= 6 \times 14 \times 14 + \frac{22}{7} \times 7 \times 15.65 - \frac{22}{7} \times 7 \times 7$

$= 1176 + [22(15.65-7)]$

$= 1176 \times 22 \times 8.65$

$= 223792.8$ cm^3

35. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?

Ans :

Water flow in 1 hr

= Area of cross-section× Speed of water

$= 5.4 \times 1.8 \times 25000$ m^3

$= 54 \times 18 \times 250$ m^3

Water floe in 4 minutes

$= 54 \times 18 \times 250 \times \frac{40}{60}$ m^3

$= 54 \times 6 \times 500$ m^3

Irrigated area $\times \frac{10}{100} = 54 \times 6 \times 500$

Irrigated area $= 54 \times 6 \times 500 \times 10$

$= 1620000$ m^3

36. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

Ans : [Outside Delhi Comp. Set-I, II III 2017]

Height of cylinder = height of cone $= 8$ cm

radius of cylinder = radius of cone $= 6$ cm

Slant height of cone $= \sqrt{8^2+6^2} = \sqrt{64+36}$

$= 10$ cm

Total surface area of remaining solid

= Surface area of cylinder+

+ Surface area of cone + area of top

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r(2h + l + r)$

$= \frac{22}{7} \times 6(2 \times 8 + 10 + 6)$

$= \frac{22}{7} \times 6 \times 32$

$= 603.43$

Hence total surface area $= 603.43$ cm^2

37. From a solid cylinder of height 24 cm and diameter 14 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

Ans : [Delhi Comp Set-I, II III 2017]

Height of the cylinder = height of the cone

$= 24$ cm.

and radius of cylinder = radius of cone

$= \frac{14}{2} = 7$ cm

Slant height of cone $= \sqrt{h^2+r^2} = \sqrt{24^2+7^2}$

$= \sqrt{576+49} = 25$ cm

Total surface area of remaining part

= Surface area of cylinder+

+ Surface area of cone + area of top

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r(2h + l + r)$

$= \frac{22}{7} \times 7(2 \times 24 + 25 + 7)$

$= 22 \times 80$

$= 1760$ cm^2

38. The perimeters of the ends of the frustum of a cone are 207.24 cm and 169.56 cm. If the height of the frustum be 8 cm, find the whole surface area of the frustum. (Use $\pi = 3.14$)

Ans : [Board Sample Paper, 2016]

Let R and r be the radii of the circular ends of the frustum where $R > r$

Now $2\pi R = 207.24$

$R = \frac{207.24}{2 \times 3.14} = 33 = 33$ cm

and $2\pi r = 169.56$ cm

$$r = \frac{169.56}{2 \times 3.14} = 27 = 27 \text{ cm}$$

Now $\quad l^2 = h^2 + (R - r)^2 = 8^2 + (33 - 27)^2 = 100$

$$l = 10 \text{ cm}$$

Whole surface area of the frustum

$$= \pi\left(R^2 + r^2 + (R + r)l\right)$$
$$= 3.14\left[(33)^2 + (27)^2 + (33 + 27)10\right]$$
$$= 3.14(1089 + 729 + 600)$$
$$= 3.14 \times 2418 \text{ cm}^2$$
$$= 7592.52 \text{ cm}^2.$$

39. A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends as 8 cm and 20 cm repetitively. Find the cost of milk which can completely fill the container at the rate of Rs. 35 per litre. Use $\pi = \frac{22}{7}$

Ans : [Foreign Set I, II, III, 2016]

As per question the figure is shown below.

If r_1 and r_2 be the radii of two circular ends and h be the height of frustum, then volume

$$= \frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right]$$

We have $\quad r_1 = 8 \text{ cm}$

$$r_2 = 20 \text{ cm}$$

and $\quad h = 21 \text{ cm}$

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times 21\left[(8)^2 + (20)^2 + 8 \times 21\right]$$
$$= 22[64 + 400 + 160]$$
$$= 22 \times 624$$
$$= 13728 \text{ cm}^3$$
$$= \frac{13728}{1000} \text{ lit} (\because 1000 \text{ cm}^3 = 1 \text{ lit.})$$

$$V = 13.728 \text{ litres}$$

$$\text{Total Cost} = Rs.13.728 \times 35$$
$$= Rs.\, 480.48$$

40. A cone is cut by a plane parallel to the base and upper part is removed. If the curved surface area of upper cone is $\frac{1}{9}$ times the curved surface of original cone. Find the ratio of line segment to which the con's height is divided by the plane.

Ans : [Board Term-2, 2014]

As per question the figure is shown below.

$$\frac{\text{Curved surface of upper cone}}{\text{Curved surface of original cone}} = \frac{1}{9}$$

$$\frac{\pi rl}{\pi RL} = \frac{1}{9}$$

$$\frac{rl}{RL} = \frac{1}{9} \quad \ldots(1)$$

Since by AA similarity $\Delta AOB - \Delta ACD$, thus

$$\frac{r}{R} = \frac{h}{H} = \frac{l}{L} \quad (2)$$

Substituting (2) in (1) we have

$$\frac{h}{H} \times \frac{h}{H} = \frac{1}{9}$$

$$\frac{h^2}{H^2} = \frac{1}{9}$$

or, $\quad \frac{h}{H} = \frac{1}{3}$

Hence $\quad \frac{\text{Height of upper cone}}{\text{Height of lower frustum}} = \frac{1}{3-1} = \frac{1}{2}$

Ratio of the line segments $OA : OC = 1 : 2$

41. The slant height of a frustum of a cone is 4 cm and the perimeter (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (12)]

As per question the figure is shown below.

We have $\quad l = 4 \text{ cm}$

$$2\pi R = 18 \text{cm}$$
$$R = \frac{18}{2\pi} = \frac{9}{\pi}$$

and $\quad 2\pi r = 6$

$$r = \frac{6}{2\pi} = \frac{3}{\pi} \text{ cm}$$

Curved surface area of frustum

$$\pi l(R + r) = \pi \times 4\left(\frac{9}{\pi} + \frac{3}{\pi}\right)$$
$$= 4\pi \times \frac{12}{\pi} = 48 \text{ cm}^2.$$

LONG ANSWER TYPE QUESTIONS

1. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 m high embankment. Find the width of the embankment.

Ans : [Delhi CBSE Board, 2012 Set I, II]

Depth of well = 14 m, radius = 12 m.

Volume of earth taken out

$$\pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 14$$
$$= 176 \text{ m}^3$$

Let r be the width of embankment. The radius of outer circle of embankment

$$= 2 + r$$

Area of upper surface of embankment

$$= \pi\left[(2 + r)^2 - (2)^2\right]$$

Volume of embankment= Volume of earth taken out

$$\pi\left[(2 + r)^2 - (2)^2\right] \times 0.4 = 176$$
$$\pi\left[4 + r^2 + 4r - 4\right] \times 0.4 = 176$$
$$r^2 + 4r = \frac{176 \times 7}{0.4 \times 22}$$
$$r^2 + 4r = 140$$
$$r^2 + 4r - 140 = 0$$
$$(r + 14)(r - 10) = 0$$
$$r = 10$$

Hence width of embankment = 10 m.

2. A hemispherical depression is cut from one face of a cubical block, such that diameter 'l' of hemisphere is equal to the edge of cube. find the surface area of the remaining solid.

Ans : [CBSE Set I, II, III, 2014]

Let the radius of hemisphere be r.

Therefore, $r = \frac{l}{2}$

Now, the required surface area

= Surface area of cubical block +

− Area of base of hemisphere +

+ Curved surface area of hemisphere.

$$= 6(l)^2 - \pi r^2 + 2\pi r^2$$

$$= 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 - \frac{\pi l^2}{4} + \frac{\pi}{2}l^2$$

$$= 6l^2 + \frac{\pi l^2}{4}$$

$$\text{Surface area} = \frac{1}{4}(24+\pi)l^2 \text{ units.}$$

$$= \frac{1}{4}\left(24 + \frac{22}{7}\right)^2 l^2$$

$$= \frac{1}{4} \times \frac{190}{7} \times \frac{190}{7} l^2$$

$$= 184.18l^2 \text{ unit}^2$$

3. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area in hectare will it irrigate in 30 minutes if 8 cm of standing water is needed ?

Ans : [KVS 2014][Delhi Set, 2014] [Board Term-2, 2012 (13)]

As per question the figure is shown below.

Water flows in 1 hr = 10 km

Water flows in $\frac{1}{2}$hr $= \frac{10}{2}$

$$= 5 \text{ km}$$

$$= 5000 \text{ m}$$

Now volume of water flows in $\frac{1}{2}$ hr

$$lbh = 5000 \times 6 \times 1.5 \text{m}^3$$

$$= 45000 \text{ m}^3.$$

According to the question,

Volume of water $\frac{1}{2}$ hr =area of irrigated field $\times \frac{8}{100}$

$$45000 = \text{Area} \times \frac{8}{100}$$

$$\text{Area} = \frac{45000 \times 100}{8} = 562500 \text{ m}^2$$

$$= 56.25 \text{ hectare.}$$

4. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled ?

Ans : [Delhi Set, 2014]

[Board Term-2, 2012 (13)]

Diameter of pipe = 20cm.

$$\text{Radius of pipe} = \frac{20}{2} = 10 \text{ cm} = 0.10\text{m}$$

Diameter of tank = 10 cm

$$\text{radius of the tank} = \frac{20}{2} = 5 \text{ cm}$$

Depth of tank = 2 m

$$\text{Volume of tank} = \pi r^2 h = \pi \times 5 \times 5 \times 2 = 50\pi$$

Speed of the water 3 km/hr.

$$= \frac{300}{60} = 50 \text{ m/min}$$

Volume of water supplied in one minute

$$\pi r^2 h = \pi \times 0.10 \times 0.10 \times 50$$

Time taken $t = \frac{50\pi}{\pi \times 0.10 \times 0.10 \times 50} = 100$

Hence time taken to fill the tank is 100 minutes.

5. The internal and external diameters of a hollow hemispherical vessel are 16 cm and 12 cm respectively. If the cost of painting 1 cm^2 of the surface area is Rs. 5.00, find the total cost of painting the vessel all over. (Use $\pi = 3.14$)

Ans :

Here $R = 8$ cm, $r = 6$ cm

$$\text{Surface area} = 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$

$$= \pi\left[2 \times 8^2 + 2 \times 6^2 + (8^2 - 6^2)\right]$$

$$= \pi\left[64 \times 2 + 36 \times 2 + (64 - 36)\right]$$

$$= \pi[128 + 72 + 28]$$

$$= 228 \times 3.14 = 715.92 \text{ cm}^2$$

Total cost $= 715.92 \times 5 = 3579.60$ Rs

6. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

Ans : [Delhi 2013]

Volume of water flowing through pipe in 1 sec.

$$\pi R^2 H = \pi \times (1)^2 \times 0.4 \times 100 \text{ cm}^3$$

Volume of water flowing in 30 min $(30 \times 60 \text{ sec})$

$$= \pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60$$

Volume of water in cylindrical talk in 30 min

$$\pi r^2 h = \pi \times (40)^2 \times h$$

Now

$$\pi \times (40)^2 \times h = \pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60$$

Rise in water level

$$h = \frac{\pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60}{\pi \times 40 \times 40}$$

$$= 45 \text{ cm.}$$

Thus level of water in the tank is 45 cm.

7. A toy is in the form of a cylinder of diameter $2\sqrt{2}$ m and height 3.5 m surmounted by a cone whose vertical angle is 90°. Find total surface area of the toy.

Ans : [Board Term-2, 2012 (44)]

As per question the figure is shown below.

Here $\angle C = 90^\circ$ and $AC = BC = l$

Thus $AB^2 = AC^2 + BC^2 = l^2 + l^2 = 2l^2$

Now $\left(2\sqrt{2}\right)^2 = 2l^2$

Thus $l = 2$ and $r = \sqrt{2}$ m

Slant height of conical portion

$l = 2$ m

Total surface area of toy

$$2\pi rh + \pi r^2 + \pi rl = \pi r\left[7 + \sqrt{2} + 2\right] \text{ m}^2$$
$$= \pi\sqrt{2}\left[9 + \sqrt{2}\right] \text{ m}^2$$
$$= \pi\left[2 + 9\sqrt{2}\right] \text{ m}^2$$

8. Find the volume of the largest solid right circular cone that can be cut out off a solid cube of side 14 cm.

Ans : [Board Term-2, 2012 (1)]

The base of cone is the largest circle that can be inscribed in the face of the cube and the height will be equal to edge of the cube.

$$\text{Radius of cone} = \frac{14}{2} = 7 \text{ cm}$$

$$\text{Height of cone} = 14 \text{ cm}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 14$$
$$= \frac{2156}{3} = 718.67.$$

9. Water is flowing at the rate of 15 km/hr through a cylindrical pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time the level of water in pond rise by 21 cm ?

Ans : [Board Term-2, 2012 Set (5)]

Speed of water flowing through the pipe

$= 15$ km/hr $= 15000$ m/hr

Volume of water flowing in 1 hr

$$\pi R^2 H = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000 \text{ m}^3$$
$$= 231 \text{ m}^3$$

Volume of water in the tank when the depth is 21 cm

$$lbh = 50 \times 44 \times \frac{21}{100} \text{ m}^3 = 462 \text{ m}^3$$

Time taken to fill 462 m^3

$$= \frac{462}{231} = 2 \text{ hrs.}$$

10. A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends, the length of the entire capsule is 15 mm and the diameter of the capsule is 5 mm. Find the Volume of the capsule.

Ans : [Board Term-2, 2012 Set (12)]

As per question the figure is shown below.

Total height $= 14$ mm

Height of cylinder $= 14 - 2 \times 2.5$

$= 14 - 5 = 9$ mm

Radius of cylinder $= 2.5$mm

Radius of hemisphere $= 2.5$ mm

Volume of capsule = Volume of two hemispheres + Volume of cylinder

$$= 2 \times \frac{2}{3}\pi r^3 + \pi r^2 h$$
$$= \frac{4}{3}\pi\left(\frac{5}{2}\right)^3 + \pi\left(\frac{5}{2}\right)^2 \times 9$$
$$= \left(\frac{5}{2}\right)^2 \times \pi\left[\frac{4}{3} \times \frac{5}{2} + 9\right]$$
$$= \frac{25}{4}\pi\left[\frac{10}{3} + 9\right] = \frac{25}{4}\pi\left[\frac{10 + 27}{3}\right]$$
$$= \frac{25}{4}\pi\left[\frac{37}{3}\right] = \frac{25}{4} \times \frac{22}{7} \times \frac{37}{3}$$
$$= \frac{10175}{42} \text{ mm}^3$$
$$= 242.26 \text{ mm}^3.$$

11. A milk tanker cylindrical in shape having diameter 2 m and length 4.2 m supplies mild to the two booths in the ratio of 3 : 2. One of the milk booths has cuboidal vessel having base area 3.96 sq. m. and the other has a cylindrical vessel having radius 1 m. Find the level of milk in each of the vessels. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 (28)]

Volume of milk $= \frac{22}{7} \times 1 \times 1 \times 4 \cdot 2 = 13 \cdot 2 \text{ m}^3$

Supply of milk to booth I

$$= 13 \cdot 2 \times \frac{3}{5} = 2 \cdot 64 \times 3 = 7.92 \text{ m}^3$$

Supply of milk to booth II

$$= 13 \cdot 2 \times \frac{2}{5} = 2 \cdot 64 \times 2 = 5.28 \text{ m}^3$$

Height in 1st vessel $= \frac{7 \cdot 92}{3 \cdot 96} = 2$ m

Height in 2nd vessel $= \frac{5 \cdot 28}{\frac{22}{7} \times 1} = \frac{5 \cdot 28 \times 7}{22} = 1.68$ m

12. In fig from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. Use $\pi = \frac{22}{7}$ and $\sqrt{5} = 2.236$.

Ans : [Delhi CBSE Board, 2015 Set I, II, III]

Let r be the radius of the top after cutting

$h = 12 - 4 = 8$ cm

Now $\frac{4}{r} = \frac{12}{6} \Rightarrow r = 2$ cm

Now slant length of frustum

$$l = \sqrt{h^2 + (R - r)^2}$$
$$= \sqrt{(8)^2 + (6 - 2)^2}$$
$$= \sqrt{64 + 16} = \sqrt{80}$$

$= 4\sqrt{5} = 4 \times 2.236$

$= 8.944$ cm

Total surface area of frustum

$= \pi\left[R^2 + r^2 + l(R + r)\right]$

$= \frac{22}{7}\left[(6)^2 + (2)^2 + 8.944(6 + 2)\right]$

$= \frac{22}{7}[36 + 4 + 71.552]$

$= \frac{22}{7} \times 111.552$

$= 350.59$ cm^2.

13. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi Set I, II, III, 2015]

As per question the figure is shown below.

Volume of cylinder,

$\pi r^2 h = \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times 10$ cm^3

$= 554.40$ cm^3

Volume of metal scooped out

$= 2 \times$ Volume of hemisphere

$= 2 \times \frac{2}{3} \times \pi r^3 = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{42}{10}\right)^3$

$= 310.46$ cm^3

Volume of rest of cylinder

$= 554.40 - 310.46 = 243.94$ cm^3

Now from rest volume a wire of thickness 1.4 cm i.e radius 0.7 cm is formed. Let length of wire be l. Thus volume of wire and rest cylinder will be equal.

Volume of wire, $\pi r^2 l = 243.94$ cm^3

$\frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times l = 243.94$ cm^3

$l = \frac{243.94 \times 10 \times 10}{22 \times 7}$

$h = 158.4$ cm

14. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

Ans : [CBSE O.D. 2014]

Diameter of spherical marble = 14 cm

Radius $r_1 = \frac{1.4}{2} = 0.2 = \frac{7}{10}$ cm

Diameter of cylindrical vessel = 7 cm

Radius $R = \frac{7}{2} = 3.5$ cm

Let h be the rise in water level then,

Volume of 150 spherical marbles = volume of water rise

$150 \times \frac{4}{3} \times \pi \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \pi \times \frac{7}{2} \times \frac{7}{2} \times h$

$h = \frac{4 \times 7}{5}$

$\frac{28}{5} = h$

$h = 5.6$ cm

Thus 5.6 cm will be rise in the level of water.

15. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys formed so.

Ans : [Outside Delhi Compt. 2017]

Height of cylinder = 15 cm

Diameter = 12 cm

Radius = 6 cm

Radius of cone = 3 cm

and height = 9 cm

Let the number of toys recast be n.

Volume of n conical toys = Volume of cylinder

$n \times \frac{1}{3}\pi \times 3 \times 3 \times 9 = \pi \times 6 \times 6 \times 15$

$n = \frac{6 \times 6 \times 15}{3 \times 9}$

$= 20$

Hence the number of toys = 20.

16. A well diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment.

Ans : [CBSE Foreign 2017]

The volume of soil taken out from the well

$\pi^2 rh = \frac{3}{2} \times \frac{3}{2} \times 14\pi$ m^3

The radius of embankment with well

$= \frac{3}{5} + 5 = \frac{13}{2}$ m

Let the height of embankment be x. Then the volume of soil used in embankment,

$\pi(R^2 - r^2)x = \pi r^2 h$

$\pi\left[\left(\frac{13}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]x = \frac{3}{2} \times \frac{3}{2} \times 14\pi$

$\frac{160}{4}x = \frac{3}{2} \times \frac{3}{2} \times 14$

$x = \frac{3 \times 3 \times 14}{160} = 0.7875$ m

Hence the height of embankment = 78.75cm

17. Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank of dimensions 50 m $\times$ 44 m. Find the time in which the

level of water in the tank will rise by 7 cm.

Ans : [Delhi Compt 2017]

Speed of water in pipe $= 5$ km/hour

In an hour length of water $= 5000$ m

Let time taken to fill the tank be t.

Total length of water $= t \times 5000$ m

Volume of water flown= Volume of water in tank

$$\pi r^2 h = l \times b \times h$$

$$\frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 500t = 50 \times 44 \times \frac{7}{100}$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 5000t = 50 \times 44 \times \frac{7}{100}$$

$$t = \frac{50 \times 44}{22 \times 50} = 2$$

Hence, Time taken to fill the tank $= 2$ hours.

18. A bucket open at the top is in form of a frustum of a cone with a capacity of 12308.8 cm3. The radii of he top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use $\pi = 3.14$)

Ans : [Delhi Set I, II, III, 2016]

Here $R = 20, r = 12, V = 12308.8$

$$V = \frac{1}{3}\pi(R^2 + r^2 + Rr)h$$

$$12308.8 = \frac{1}{3} \times 3.14(400 + 240 + 144)h$$

$$12308.8 = \frac{1}{3} \times 3.14 \times 784$$

$$h = 15 \text{ cm}$$

Now $l = \sqrt{(20-12)^2 + 15^2} = 17$ cm

Total area of metal sheet used,

$= CSA +$ Base area

$= \pi[(20+12) \times 17 + 12 \times 12]$

$= 2160.32$ cm^2

19. The radii of the circular ends of a frustum of cone of height 6 cm are 14 cm and 6 cm respectively. Find the lateral area and total surface area of the frustum.

Ans : [Board Term-2, 2012 Set (59)]

We have $r_1 = 14$ cm, $r_2 = 6$ cm, $h = 6$ cm

$$l = \sqrt{h^2 + (r_1 + r_2)^2}$$

$$= \sqrt{6^2 + (14-6)^2} = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64} = 10\text{cm}$$

Lateral surface area,

$$\pi(r_1 + r_2)l = \frac{22}{7} \times (14+6) \times 10 \text{ cm}^2$$

$$= 628.57 \text{ cm}^2$$

Total surface area

$$\pi\left[r_1^2 + r_2^2 + l(r_1 + r_2)\right] = \frac{22}{7} \times [(196+36) + 20 \times 10]$$

$$= \frac{22}{7} \times 432 = 1357.71 \text{ cm}^2$$

20. A cone of radius 10 cm is divided into two parts by a plane parallel to its base through the mid-point of its height. Compose the Volume of the two parts.

Ans : [Delhi Set-III 2017]

As per question the figure is shown below.

Since $\Delta ABC \sim \Delta APQ$ we have

$$\frac{h}{2h} = \frac{r_1}{10} \Rightarrow r_1 = 5 \text{ cm}$$

Volume of smaller cone

$$= \frac{1}{3}\pi(5)^2 \times h$$

Volume of frustum $= \frac{1}{2}\pi \times h(5^2 + 10 + 5 \times 10)$

$$= \frac{1}{3}\pi \times h \times 175$$

Required ratio $= \frac{\frac{1}{3} \times \pi \times 25 \times h}{\frac{1}{3} \times \pi \times h \times 175} = \frac{1}{7}$

21. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the two parts.

Ans : [Delhi Set-I, II, III 2017]

As per question the figure is shown below.

Let the radius of cone be r_2 and cut off cone be r_1

Height of the cone $= 10$ cm

And the height the cone cut off $= 5$cm

Since $\Delta AOC \sim \Delta AO'D$, we have

$$\frac{AO}{AO'} = \frac{r_2}{r_1} = \frac{10}{5}$$

$$r_2 = 2r_1$$

Volume of cut off cone $= \frac{1}{3}\pi r_1^2 \times 5$

$$= \frac{1}{3}\pi r_1^2 \text{ sq. units}$$

Volume of original cone $= \frac{1}{3}\pi(2r_1)^2 \times 10$

$$= \frac{40}{3}\pi r_1^2 \text{ sq. units}$$

Volume of frustum

= Volume of original cone − Volume of cut of cone

$$= \frac{40}{3}\pi r_1^2 - \frac{5}{3}\pi r_1^2 = \frac{35}{3}\pi r_1^2 \text{ sq.units}$$

Ratio of two parts $= \frac{35\pi r_1^2}{5\pi r_1^2} = \frac{7}{1}$

Hence the ratio of two parts $= 7 : 1$

22. A metallic right circular cone 20 cm high and whose vertical angel is 60° is cut into two parts at the middle of its height by a plane parallel to its base if the frustum so obtained be drawn into a wire of uniform diameter $\frac{1}{16}$ cm, find the length of the wire.

Ans : [Foreign Set-I 2017]

As per question the figure is shown below.

Total height of cone $= 20$ cm

and Vertex angle $= 30^\circ$

Let the radius of cone be r_2. Then we have

$$\frac{r_2}{20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$r_2 = \frac{20}{\sqrt{3}} \text{ cm}$$

The height of the cone cut off $= 10$ cm

Let its radius be r_1. Then

$$\frac{r_1}{10} = \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ cm}$$

$$r_1 = \frac{10}{\sqrt{30}} \text{ cm}$$

Let the length of wire be l. Its radius is $\frac{1}{32}$ cm.

Now Volume of frustum = Volume of wire

$$\frac{1}{3}\pi \times h\left[(r_1)^2 + (r_2)^2 + (r_1 r_2)\right] = \pi r^2 l$$

$$\frac{1}{3} \times 10 \times \pi\left[\left(\frac{10}{3}\right)^2 + \left(\frac{20}{3}\right)^2 + \frac{10}{3} \times \frac{20}{3}\right] = \pi\left(\frac{1}{32}\right)^2 \times l$$

$$\frac{1}{3} \times 10\left[\frac{100}{9} + \frac{400}{9} + \frac{200}{9}\right] = \frac{1}{32 \times 32} \times l$$

$$\frac{1}{3} \times 10 \times \frac{700}{9} = \frac{1}{32} \times \frac{1}{32} \times l$$

$$l = \frac{32 \times 32 \times 700 \times 10}{3 \times 9}$$

$$= 796444.4 \text{ cm.}$$

Hence, the length of wire is 7964.44 m.

23. A right circular cone is divided into three parts trisecting its height by two planes drawn parallel to the base. Show that volumes of the three portions starting from the top are in the ratio $1 : 7 : 19$.

Ans : [Foreign Set-III 2017]

As per question the figure is shown below.

Let the radii of three cones from top be r_1, r_2 and r_3 respectively.

Let the height of given cone be $3h$. So, the height of cone ADE is $2h$ and height of cone ABC is h.

Since $\Delta ABC \sim ADE$,

$$\frac{r_1}{r_2} = \frac{h}{2h} \Rightarrow 2r_1 = r_2$$

Since $\Delta ADE \sim AFG$

$$\frac{r_1}{r_2} = \frac{h}{3h} \Rightarrow 3r_1 = r_3$$

Volume of cone ABC $= \frac{1}{3}\pi r_1^2 h$

Volume of cone ADE $= \frac{1}{3}\pi (r_2)^2 2h$

$$= \frac{1}{3}\pi (2r_1)^2 .2h$$

Volume of frustum $BCED$ $= \frac{1}{3}\pi 4r_1^2 2h - \frac{1}{3}\pi r_1^2 h$

$$= \frac{7}{3}\pi r_1^2 h$$

Volume of frustum $DEGF$

$$= \frac{1}{3}\pi r_3^2 .3h - \frac{1}{3}\pi r_1^2 .2h$$

$$= \frac{1}{3}\pi (3r_1)^2 3h - \frac{1}{3}(2r_1)^2 .2h$$

$$= \frac{1}{3}\pi r_1^2 h(27 - 8) = \frac{19}{3}\pi r_1^2 h$$

$$\text{Ratio} = \frac{1}{3}\pi r_1^2 h : \frac{7}{3}\pi r_1^2 h : \frac{19}{3}\pi r_1^2 h$$

Hence, required ratio $= 1 : 7 : 19$.

24. From a rectangular block of wood, having dimensions 15 cm $\times$ 10 cm $\times$ 3.5 cm, a pen stand is made by making four conical depressions. The radius of each one of the depression is 0.5 cm and the depth 2.1 cm. Find the volume of wood left in the pen stand.

Ans : [Delhi Compt. Set-I, II, III 2017]

Volume of cuboidal block

$$l \times b \times h = 15 \times 10 \times 3.5 = 525 \text{ cm}^3$$

Volume of one cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1 \text{ cm}^3$$

$$= 0.55 \text{ cm}^3$$

Volume of 4 cones $= 0.55 \times 4 = 2.2$ cm^3

Volume of wood remaining in pen stand

$$= 525 - 2.2 = 522.80 \text{ cm}^3$$

25. The height of a cone is 30 cm. From its topside a small cone is cut by a plane parallel to its base. If volume of smaller cone is $\frac{1}{27}$ of the cone then at what height it is cut from the base ?

Ans : [Delhi Set-II, 2017]

As per question the figure is shown below.

Let the radii of smaller cone and original cone be r_1 and r_2 respectively and the height of smaller cone be h.

Since $\Delta ABC \sim \Delta APQ$ we have

$$\frac{h}{30} = \frac{r_1}{r_2} \quad (1)$$

Volume smaller cone $= \frac{1}{27} \times$ Volume of original cone

$$\frac{1}{3}\pi r_1^2 \times h = \frac{1}{27} \times \frac{1}{3}\pi r_2^2 \times 30$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \frac{h}{30} = \frac{1}{27}$$

From (1) using $\frac{h}{30} = \frac{r_1}{r_2}$ we have

$$\left(\frac{h}{30}\right)^2 \times \frac{h}{30} = \frac{1}{27}$$

$$\left(\frac{h}{30}\right)^3 = \frac{1}{27}$$

$$h^3 = \frac{30 \times 30 \times 30}{27}$$

$$h = 10 \text{ cm}$$

Hence, required height $= (30 - 10) = 20$ cm

HOTS QUESTIONS

1. The ratio of the volumes of two spheres is $8 : 27$. If r and R are the radii of sphere respectively, then find the $(R-r) : r$.

Ans : [Board Term-2, 2012, Set (22)]

Ratio of volumes

$$\frac{\text{Volume of } 1^{st}\text{sphere}}{\text{Volume of } 2^{nd}\text{sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27}$$

or, $$\frac{r}{R} = \frac{2}{3}$$

$$R = \frac{3}{2}r$$

$$(R-r) : r = \left(\frac{3}{2}r - r\right) : r$$

$$= \frac{r}{2} : r = 1 : 2$$

2. A decorative block, made up of two solids - a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. Use $\pi = \frac{22}{7}$.

Ans : [Delhi Set I, II, III, 2016]

Surface area of block

$$= 216 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} + 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

$$= 225.625 \text{ cm}^2.$$

3. In fig., from a cuboidal solid metallic block of dimensions 15 cm $\times$ 10 cm $\times$ 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. Use $\pi = \frac{22}{7}$

Ans : [Delhi CBSE Board, 2015 Set-I, II, III]

Total Surface area $= 2(lb + bh + hl) + 2\pi rh$

Here, $l = 15$ cm, $b = 10$ cm, $h = 5$cm, $r = \frac{7}{2}$ cm

TSA of Cuboidal block

$$= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$$

$$= 550 \text{ cm}^2.$$

Area of C.S. of Cylinder

$$2\pi rh = 2 \times \frac{22}{7} \times \frac{7}{2} \times 5$$

$$= 110 \text{ cm}^2$$

Area of two Circular bases $= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$$= 77 \text{ cm}^2$$

Required area $= 550 + 110 - 77 = 583 \text{ cm}^3.$

4. A right triangle whose sides are 15 cm is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. (Use $\pi = 3.14$)

Ans : [Board Term-2, 2012 Set (28)]

As per question the figure is shown below.

We have $$AC^2 = 20^2 + 15^2 = 625$$

$$AC = 25 \text{ cm}$$

$$ar(\Delta ABC) = ar(\Delta ABC)$$

$$\frac{1}{3} \times BC \times AB = \frac{1}{2} \times AC \times BD$$

$$15 \times 20 = 25 \times BD$$

$$BD = 12 \text{ cm}$$

Volume of double cone,

= Volume of upper cone + Volume of lower cone

$$= \frac{1}{3}\pi(BD)^2 \times AD + \frac{1}{3}\pi(BD)^2 \times CD$$

$$= \frac{1}{3}\pi(BD)^2\{AD + CD\} = \frac{1}{3}\pi(BD)^2(AC)$$

$$= \frac{1}{3} \times 3.14 \times 144 \times 25 = 3768 \text{ cm}^2$$

Surface area= C.S.A. of upper cone + C.S.A. of lower cone

$$= \pi(12)(20) + \pi(12)(15)$$

$$= 12\pi\{20 + 15\}$$

$$= 12 \times 3.14 \times 35$$

$$= 1318.8 \text{ cm}^2$$

5. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pipe, given that 1 cm3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Ans : [Board Term-2, 2012 Set (31)]

As per question the figure is shown below.

Radius of lower cylinder $R = 12$ cm

Radius of upper cylinder $r = 8$ cm

Height of upper cylinder $h = 60$ cm

Height of lower cylinder $H = 220$ cm

Volume of solid iron pole,

$$\pi R^2 H + \pi r^2 h = 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$$

$$= 111532.8 \text{ cm}^3$$

Mass of pole $= 111532.8 \times 8$ g

$$= 892.2624 \text{ kg}.$$

6. A heap of wheat is in the form of cone of diameter 6 m and height 3.5 m. Find its volume . How much canvas cloth is required to just cover the heap ? Use $\pi = \frac{22}{7}$

Ans :

Volume of wheat in the form of cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 3.5$$

$$= 11 \times 3 = 33 \text{ m}^3$$

$$l = \sqrt{3^2 + 3.5} = 4.609 \text{ m}$$

Canvas required to cover the heap

$$\pi rl = \frac{22}{7} \times 3 \times 4.609$$

$$= 43.45 \text{ m}^2.$$

7. A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball.

Ans : [Foreign Set I, II, III, 2015]

Volume of water in cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (5)^3 \times 8 = \frac{200}{3}\pi \text{ cm}^3$$

Volume of water flows out

$$= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi \text{ cm}^3$$

Let the radius of one spherical ball be r cm

$$\frac{4}{3}\pi r^3 \times 100 = \frac{50}{3}\pi$$

$$r^3 = \frac{50}{4 \times 100} = \frac{1}{8}$$

or, $$r = \frac{1}{2} = 0.5 \text{ cm}$$

8. A cone is cut by a plane parallel to the base and upper part is removed. It the C.S.A. of the remainder is $\frac{15}{16}$ of the C.S.A. of whole cone, find the ratio of the line segments to which cone's height is divided by the plane.

Ans : [Board Term-2, 2014]

As per question the figure is shown below.
Let the height of larger cone be H and height of smaller cone be h. Let radius of larger and smaller cones be R and r

Since $\Delta ONC \sim \Delta OMA$, we have

$$\frac{h}{H} = \frac{r}{R} = \frac{l}{L}$$

C.S.A. of the frustum $= \frac{15}{16}$(C.S.A. of cone OAB)

C.S.A. of cone OCD

$$= 1 - \frac{15}{16} = \frac{1}{16} \text{ (C.S.A. of cone } OAB)$$

$$\frac{\text{C.S.A. of cone OCD}}{\text{C.S.A. of cone OAB}} = \frac{1}{16}$$

$$\frac{\pi rl}{\pi RL} = \frac{1}{16}$$

or, $$\left(\frac{r}{R}\right)\left(\frac{l}{L}\right) = \frac{1}{16}$$

$$\left(\frac{h}{H}\right)\left(\frac{h}{H}\right) = \frac{1}{16} \qquad \left(\frac{l}{L} = \frac{h}{H}\right)$$

$$\frac{h}{H} = \frac{1}{4}$$

$$h = \frac{1}{4}H$$

$$ON = \frac{1}{4}H$$

$$MN = \frac{3}{4}H$$

$$ON : MN = 1 : 3$$

9. A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the longest side. Find the surface area of figure obtained. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 (44)]

As per question the figure is shown below.
By revolving right triangle about longest side double cone is generated. Let radius of double cone $= x$ cm.
In ΔADE and ΔADC,

$$\angle AED = \angle DAC = 90^\circ$$

$$\angle ADE = \angle ADC \text{ (common angle)}$$

Thus due to AA symmetry we have

or, $$\Delta ADC \sim \Delta ADC$$

$$\frac{x}{AC} = \frac{AD}{DC} = \frac{DE}{AD}$$

$$\frac{x}{4} = \frac{3}{5} = \frac{DE}{3}$$

$$x = \frac{12}{5} = 2.4 \text{ cm}$$

$$DE = \frac{9}{5} = 1.8 \text{ cm}$$

Surface area of double cone

$$\pi rl_1 + \pi rl_2 = \pi r(l_1 + l_2)$$

$$= \frac{22}{7} \times 2.4 \times (3+4)$$

$$= 22 \times 2.4 = 52.8 \text{ cm}^2.$$

10. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and toy. (Use $\pi = 3.14$)

Ans : [Board Term-2, 2012 Set (34)]

As per question the figure is shown below.
Let BPC is a hemisphere and ABC is a cone.

Radius of hemisphere = Radius of cone

$$= \frac{4}{2} = 2 \text{ cm}$$

h = Height of cone = 2 cm

Volume of toy $= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$\frac{1}{3}\pi r^2(2r+h) = \frac{1}{3} \times 3.14 \times 2 \times 2(2 \times 2 + 2)$$

$$= \frac{1}{3} \times 3.14 \times 4 \times 6$$

$$= 25.12 \text{ cm}^3$$

Let right circular cylinder $EFGH$ circumscribe the given solid toy.

Radius of cylinder = 2 cm

Height of cylinder = 4 cm

Volume of right circular cylinder

$$\pi r^2 h = 3.14 \times (2)^2 \times 4 \text{ cm}^3 \qquad \text{...(ii)}$$

$$= 50.24 \text{ cm}^3$$

Difference of two volume

= Volume of cylinder − Volume of toy

$$= 50.24 - 25.12 = 25.12 \text{ cm}^3.$$

11. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm3. Find the height of the toy. Also find the cost of painting the hemisphere part of the toy at the rate of Rs. 10 per cm2. Use $\pi = \frac{22}{7}$

Ans : [Delhi CBSE Board 2015 set I, II, III]

As per question the figure is shown below.

Radius of cone = Radius of hemisphere

$r = 3.5$ cm

Total volume, $V = 166\frac{5}{6}$ cm^3 $= \frac{1001}{6}$ cm^3

Let the height of cone be h.

Total volume

= Volume of cone + Volume of hemisphere

$$\frac{1001}{6} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$\frac{1001}{6} = \frac{1}{3}\pi(3.5)^2 h + \frac{2}{3}\pi(3.5)^2$$

$$\frac{1001}{6} = \frac{1}{3}\pi[12.25h + 2 \times 42.875]$$

$$\frac{1001 \times 3 \times 7}{6 \times 22} = 12.25h + 85.75$$

$$\frac{21021}{132} = 12.25h + 85.75$$

$$12.25h = 159.25 - 85.75$$

$$h = \frac{73.5}{12.25} = 6$$

Height of the toy $= 6 + 3.5 = 9.5$ cm.

Surface area of hemisphere

$$2\pi r^2 = 2 \times \frac{22}{7} \times 3.5 \times 3.5 = 77 \text{ cm}^2$$

Cost of painting $= 10 \times 77 = 770$ Rs

12. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, if the In crease in the level of the water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

Ans : [Delhi CBSE Board 2015 Set I, II, III]

Let the internal diameter of the pipe be r m.

Water flows in 1 hour = 2.52 km.

Water flows in $\frac{1}{2}$ hour $= \frac{2.52}{2} = 1.26$ km

$= 1260$ m

Volume of water flows in $\frac{1}{2}$hour $= \pi r^2 h$

$= \pi r^2 \times 1260$

Volume of the in cylindrical tank

$$= \pi \times \left(\frac{40}{100}\right)^2 \times 3.15$$

Volume of water flow = Volume of increase water

$$\pi r^2 \times 1260 = \pi\left(\frac{2}{5}\right)^2 \times 3.15$$

or, $$1260r^2 = \frac{2}{5} \times \frac{2}{5} \times 3.15$$

or, $$r^2 = \frac{4}{25} \times \frac{315}{100} \times \frac{1}{1260} = \frac{1}{2500}$$

or, $$r = \frac{1}{50} \text{ m} = 2 \text{ cm}$$

Internal diameter of pipe = 4 cm.

13. A solid is consisting of a right circular cone of height 120 cm and radius 60 cm standing on hemisphere of radius 60 cm. It is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm ant its height is 180 cm.

Ans : [Board Term-2, 2015]

As per question the figure is shown below.

Height of cone, $h = 120$ cm,

Radius of cone $r = 60$ cm

Radius of hemisphere $= 60$ cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 60 \times 60 \times 120$

$= 3.14 \times 60 \times 60 \times 40$

$= 452160$ cm^3

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times 60 \times 60 \times 60$

$= 452160$ cm^3

Total volume = Volume of cone + Volume of hemisphere

$= 452160 + 452160$

$= 904320$ cm^3

Height of cylinder = 180 cm,

radius = 60 cm.

Volume of water in the cylinder

= Volume of cylinder

$= \pi r^2 h$

$= 3.14 \times 60 \times 60 \times 180$

$= 2034720$ cm^3

Water left in the cylinder = Volume of water − Volume of (cone + sphere)

$= 2034720 - 904320$

$= 1130400$ cm^3

14. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter. If there common diameter is 56 m, the height of cylindrical part is 6 m and the total height of the tent above the ground is 27 m, find the area of canvas used in the tent.

Ans : [Delhi Compt. Set-I, II, III 2017]

Total height of tent = 27 m

Height of cylindrical part = 6 m

Height of conical part $= 27 - 6 = 21$ m

Slant height of cone $= \frac{56}{2} = 28$ m

Slant height of cone $= \sqrt{r^2 + h^2}$

$= \sqrt{28^2 + 21^2}$

$= \sqrt{784 + 441} = \sqrt{1225}$

$= 35$ m

Area of canvas used $= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$= \frac{22}{7} \times 28(2 \times 6 + 35)$

$= 22 \times 4 \times 47$

$= 4136$ m^2

15. From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cut-out. Find the total surface area of the remaining solid.

Ans : [Outside Delhi Set-II, III 2017]

Radius $r = 0.7$ cm

and height $h = 2.4$ cm

Slant height $l = \sqrt{h^2 + r^2} = \sqrt{(2.4)^2 + (0.7)^2}$

$= 2.5$ m

Total surface area of remaining solid

= C.S.A. of cylinder + C.S.A. of cone + area of top.

$= 2\pi rh + \pi rl + \pi r^2$

$= \frac{22}{7} \times 0.7(2 \times 2.4 + 2.5 + 0.7)$

$= \frac{22}{7} \times 0.7 \times 8 = \frac{176}{10}$

Hence total surface area $= 17.6$ cm^2