CHAPTER 12

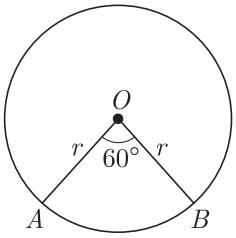
Areas Related to Circles

VERY SHORT ANSWER TYPE QUESTIONS

1. What is the perimeter of the sector with radius 10.5 cm and sector angle 60°.

Ans : [Board Term-2, 2012 Set (40)]

As per question the digram is shown below.



Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$$

$$= 21 + 11 = 32 \text{ cm}$$

2. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring.

Ans : [Delhi 2013]

Circumference of the outer circle $2\pi r_1 = 88$ cm

$$r_1 = \frac{88 \times 7}{22 \times 22} = 14 \text{ cm}$$

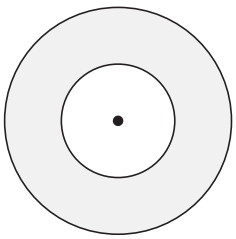
Circumference of the outer circle $2\pi r_2 = 66$ cm

$$r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm} == 10.5 \text{ cm}$$

Width of the rirg,

$$r_1 - r_2 = 14 - 10.5 \text{ cm} = 35 \text{ cm}$$

3. Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, find the area of the shaded ring shaped region in square cm.



Ans : [CBSE Board Term-2, 2012]

$$\text{Area of circle} = \pi r^2$$

$$\text{Area of the shaded region} = \pi(2)^2 - \pi(1)^2$$

$$= 4\pi - \pi = 3\pi \text{ sq cm}$$

4. The diameter of two circle with centre A and B are 16 cm and 30 cm respectively. If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C.

Ans : [Board Term-2, 2012 Set (22)]

Area of circle $= \pi r^2$, Let the radius of circle with centre $C = R$

According to question we have,

$$\pi(8)^2 + \pi(15)^2 = \pi R^2$$

$$64\pi + 225\pi = \pi R^2$$

$$289\pi = \pi R^2$$

$$R^2 = 289 \text{ or } R = 17 \text{ cm}$$

Circumference of circle

$$2\pi r = 2\pi \times 17$$

$$= 34\pi \text{ cm}$$

5. The diameter of a wheel is 1.26 m. What the distance covered in 500 revolutions.

Ans : [Board Term-2, 2012 Set (50)]

Distance covered in 1 revolution is equal to circumference of wheel and that is πd.

Distance covered in 500 revolutions

$$= 500 \times \pi \times 1.26$$

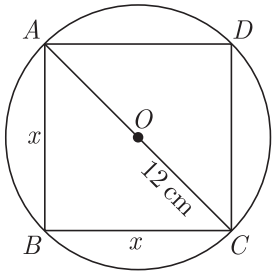
$$= 500 \times \frac{22}{7} \times 1.26$$

$$= 1980 \text{ m.} = 1.98 \text{ km}$$

6. What is the area of the largest square that can be inscribed in a circle of radius 12 cm.?

Ans : [Board Term-2, 2012 Set (31)]

As per question the digram is shown below.



Radius of the circle =12 cm

Diameter of circle = 24 cm

Diagonal of square = 24 cm

Let the side of square= x cm

From Pythagoras theorem we have

$$x^2 + x^2 = (24)^2$$
$$2x^2 = 24 \times 24$$
$$x^2 = \frac{24 \times 24}{2} = 288$$

Thus area of square,

$$x^2 = 288\ \text{cm}^2$$

7. What is the name of a line which intersects a circle at two distinct points?

Ans : [Board Term-2, 2012 (40)]

A line intersecting the circle at two distinct points is called a secant.

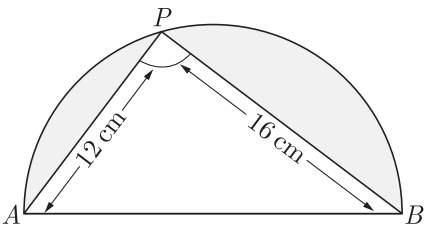
8. What is the perimeter of a sector of a circle whose central angle is 90^o and radius is 7 cm?

Ans : [Board Term-2, 2012 Set(59)]

As per question the digram is shown below.
Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^o}$$
$$= 2r + 7 + 2 \times \frac{22}{7} \times 7 \times \frac{90}{360}$$
$$14 + 11 = 25\ \text{cm}$$

9. In the given figure, AB is the diameter where $AP = 12$ cm and $PB = 16$ cm. Taking the value of π as 3, find the perimeter of the shaded region.



Ans : [Board Term-2, 2012 Set (21)]

From Pythagoras theorem we have

$$AB = \sqrt{(16)^2 + (12)^2}$$
$$= \sqrt{256 + 144}$$
$$= \sqrt{400} = 20cm$$

Radius of circle $= 10$ cm.

Perimeter of shaded region

$$\pi r + AP + PB = 3 \times 10 + 12 + 16$$
$$= 30 + 12 + 16 = 58\ \text{cm}$$

10. Find the area of circle that can be inscribed in a square of side 10 cm.

Ans : [Board Term-2, 2012 Set (44)]

Radius of the circle $= \frac{10}{2} = 5$ cm

Area of the circle,

$$\pi r^2 = \pi \times (5)^2 = 25\pi\ cm^2$$

11. A thin wire is in the shape of a circle of radius 77 cm. It is bent into a square. Find the side of the square (Taking, $\pi = \frac{22}{7}$).

Ans : [Board Term-2, 2012 Set (5)]

Let side of square be x cm.

Perimeter of the circle = Perimeter of square

$$2\pi r = 4x$$
$$2 \times \frac{22}{7} \times 77 = 4x$$
$$x = \frac{2 \times 22 \times 11}{4} = 121$$

Thus side of the square is 121 cm.

12. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm?

Ans : [Board Term-2, 2012 Set (34)]

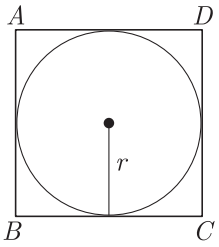
Area of the circle = sum of areas of two circles

$$\pi R^2 = \pi \times (40)^2 + \pi (9)^2$$
$$R^2 = 1600 + 81$$
$$R = \sqrt{1681} = 41\ \text{cm}$$

Thus diameter of given circle $= 41 \times 2 = 82$ cm

13. Find the area (in cm^2) of the circle that can be inscribed in a square of side 8 cm.

Ans : [board Term-2, 2012 Set (28, 32, 33)]



Side of square = diameter of circle = 8 cm

Radius of circle, $r = \frac{8}{2} = 4$ cm

Area of circle, $\pi r^2 = \pi \times 4 \times 4 = 16\pi\ \text{cm}^2$

14. If the radius of a circle is doubled, what about its area?

Ans : [Board Term-2, 2012 Set (23)]

Let the radius of the circle be r. Then are will be πr^2

Now the radius is doubled

Area $= \pi (2r)^2 = 4\pi r^2 = 4 \times \pi r^2$

The area will be 4 times the area of the first circle.

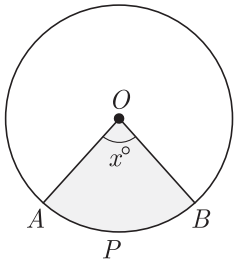
15. If the perimeter and the area of the circle are numerically equal, the find the radius of the circle.

Ans : [Board Term-2, 2012 Set(13)]

Perimeter of the circle = area of the circle.

$$2\pi r = \pi r^2$$
$$r = 2\ \text{units}$$

16. In given fig., O is the centre of a circle. If the area of the sector $OAPB$ is $\frac{5}{36}$ times the area of the circle, then find the value of x.



Ans : [Board Term-2, 2012, Set (12)]

Area of sector $OAPB = \frac{5}{36}$ times the area of circle

Thus $\pi r^2 \times \frac{x}{360} = \frac{5}{36}\pi r^2$

$\frac{x}{360} = \frac{5}{36}$

$x = 50^o$

17. If circumference of a circle is 44 cm, then what will be the area of the circle?

Ans : [Board Term-2, 2012 (25)]

Circumference of a circle = 44 cm

Radius of the circle $= \frac{22}{2 \times \frac{22}{7}} = 7$ cm

Area of the circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\, cm^2$

18. A steel wire when bent in the form of a square encloses an area or 121 cm2. If the same wire is bent in the form of a circle, then find the circumference of the circle.

Ans : [Board Term-2, 2012 (26)]

Area of square = $(\text{side})^2 = 121$ cm^2

Side of square = $= \sqrt{121} = 11$ cm

Parameter of square$= 4 \times 11 = 44$ cm

Circumference of the circle = Perimeter of the square

$= 44$ cm

19. Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of diameter 36 cm and 20 cm

Ans : [Board term-2, 2012, A1]

Circumference of the circle $= 2\pi r$

$2\pi r = 2\pi \times 18 + 2\pi \times 10$

$r = 18 + 10$

$r = 28$ cm

Hence radius of given circle is 28 cm.

20. Find the diameter of a circle whose area is equal to the sum of areas of two circles os diameter 16 cm and 12 cm.

Ans : [Board Term-2, 2012, (22)]

Let r be the radius of the circle

Area of the circle = Sum of areas of two circles

$\pi r^2 = \pi \times (8)^2 + \pi(6)^2$

$\pi r^2 = \pi(64 + 36)$

$r^2 = 100$ or, $r = 10$ cm

Diameter of the circle $= 2 \times 10 = 20$ cm.

21. If the circumference of a circle increases from 4π to 8π, then what about its area ?

Ans : [Delhi 2013]

Circumference of the circle $= 4\pi$ cm or, $r = 2$ cm.

Increased circumference $= 8\pi$ cm or, $r = 4$ cm.

Area of the 1^{st} circle $= \pi \times (2)^2 = 4\pi$ cm

Area of the new circle $= \pi(4)^2 = 16\pi = 4 \times 4\pi$

Area of the new circle= 4 times the area of first circle.

22. If the radius of the circle is 6 cm and the length of an arc 12 cm. Find the area of the sector.

Ans : [Board Term-2, 2014]

Area of the sector$= \frac{1}{2} \times$(length of the corresponding arc)$\times$ radius

$= \frac{1}{2} \times l \times r = \frac{1}{2} \times 12 \times 6$

$= 36$ cm^2

23. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. $(Use\ \pi = 3.14)$

Ans : [Board Term-2, 2012 Set (5)]

Radius of circle $r = 10$ cm, central angle $= 90^\circ$

Area of minor segment

$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90^\circ\right]$

$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5$ cm^2

24. If the perimeter of a semi-circular protractor is 36 cm, find its diameter. $\left(Use\ \pi = \frac{22}{7}\right)$

Ans : [Board Term-2, 2012 Set (59)]

Perimeter $\pi r + 2r = (\pi + 2)r = 36$

or, $\left(\frac{22}{7} + 2\right)r = 36$ or, $r = 7$

Diameter $= 14$ cm.

SHORT ANSWER TYPE QUESTIONS - I

1. Find the area of the square that can be inscribed in a circle of radius 8 cm.

Ans : [Board Term-2, 2015]

As per question the digram is shown below.

Radius of the circle $= 8$ cm

Diameter of circle $= 16$ cm

Diagonal of square $= 16$ cm

Let the side of square $= x$ cm

From Pythagoras theorem we have

$$x^2 + x^2 = (16)^2$$
$$2x^2 = 16 \times 16$$
$$x^2 = \frac{16 \times 16}{2} = 128$$

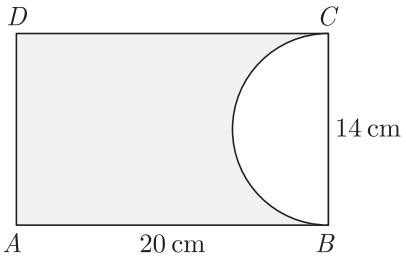
Thus area of square,

$$x^2 = 128 \text{ cm}^2$$

2. A paper is in the from of a rectangle $ABCD$ in which $AB = 20$ cm, $BC = 14$ cm. A semi-circular portion with BC as diameter is cut off. Find the area of the part. Use $\pi = \frac{22}{7}$.

Ans : [Foreign Set I, II, III, 2014] [Board Term-2 2012 Set (40)]

As per question the digram is shown below.



Area of remaining part

= Area of rectangle − Area of semi-circle

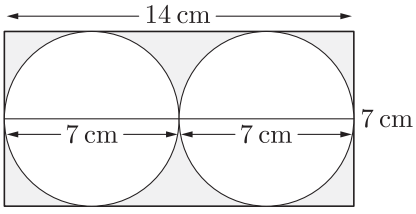
$$= 20 \times 14 - \frac{22 \times 7 \times 7}{7 \times 2}$$
$$= 280 - 77]$$

Hence, area of remaining part = 203 cm

3. Two circular pieces of equal radii and maximum areas, touching each other are cut out from a rectangular cardboard of dimensions 14 cm × 7 cm. find the area of the remaining cardboard. $\left(Use\ \pi = \frac{22}{7}\right)$

Ans : [Delhi 2013]

As per question the digram is shown below.



Area of the remaining cardboard

= Area of rectangular cardborad $-2 \times$ Area of circle

$$= 14 \times 7 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$
$$= 98 - \frac{44}{7} \times \frac{49}{4} = 98 - 77 = 21$$

Hence, area of remaining card board = 21 cm^2

4. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, find the circumference (in cm) of the circle.

Ans : [Delhi 2012]

Let r be the radius of the circle

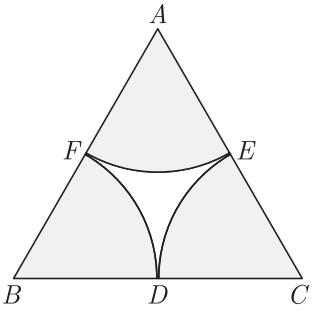
Now, circumference − radius = 37

$$2\pi r - r = 37$$
$$2 \times \frac{22}{7} r - r = 37$$
$$r\left(\frac{22-7}{7}\right) = 37$$
$$r \times \frac{37}{7} = 37$$
$$r = \frac{37 \times 7}{37} = 7 \text{ cm}$$

Circumference of the circle,

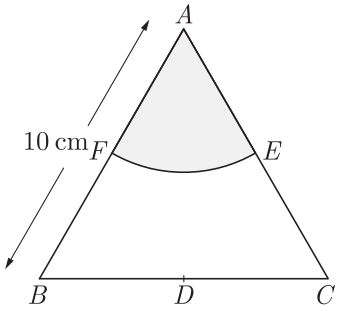
$$2\pi r = 2 \times \frac{22}{7} \times 7$$
$$= 44 \text{ cm}.$$

5. In fig., arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, to intersect the side BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. (Use $\pi = 3.14$).



Ans : [Board Term-2, 2011 Set (34)]

We have redrawn figure as shown below.



Since ΔABC is an equilateral triangle

$$\angle A = \angle B = \angle C = 60^\circ$$

Area of sector, $AFEA = \frac{\theta}{360} \times \pi r^2$

$$= \frac{60}{360} \times \pi (5)^2 = \frac{25}{6}\pi \text{ cm}^2$$

Here areas of all three sectors are equal.

Thus total area of shaded region

$$= 3\left(\frac{25}{6}\pi\right) = \frac{25 \times 3.14}{2}$$

$= 39.25 \text{ cm}^2$

6. If the perimeter of a protractor is 72 cm, calculate its are. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2, 2012 Set (22)]

Perimeter of semi-circle

$$\pi r + 2r = 72 \text{ cm}$$
$$(\pi + 2)r = 72 \text{ cm}$$
$$\left[\frac{22}{7} + 2\right] = 72 \text{ cm}$$
$$r\left[\frac{22+14}{7}\right] = 72 \text{ cm}$$
$$\frac{36}{7}r = 72$$
$$r = 14 \text{ cm}$$

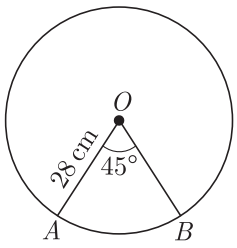
Area of protractor $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$

$= 308 \text{ cm}^2$

7. Find the area of the corresponding major sector of a circle of radius 28 cm and the central angle 45°.

Ans : [Board Term-2, 2015]

As per question statement figure is shown below;



Area of major sector,

= area of circle − area of minor sector

$= \pi r^2\left(1 - \frac{\theta}{360}\right)$

$= \frac{22}{7} \times 28 \times 28\left(1 - \frac{45}{360}\right)$

$= \frac{22}{7} \times 28 \times 28 \times \frac{7}{8}$

$= 2156 \text{ cm}^2$

8. The diameters of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. Find the number of revolutions of rear wheel to cover the distance which the front wheel covers in 800 revolutions.

Ans : [Delhi 2013]

Circumference of front wheel

$$\pi d = \frac{22}{7} \times 80 = \frac{1760}{7} \text{ cm}$$

Distance covered by front wheel in 800 revolutions

$$= \frac{1760}{7} \times 800$$

Circumference of rear wheel

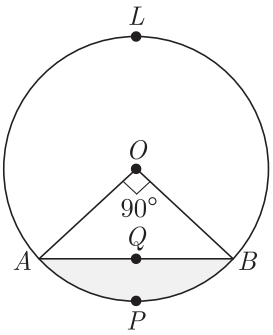
$$= \frac{22}{7} \times 200 = \frac{4400}{7} \text{ cm}$$

No. of revolutions made by rear wheel

$$= \frac{\frac{1760}{7} \times 800}{\frac{4400}{7}} = \frac{1760 \times 800}{4400} = 320$$

SHORT ANSWER TYPE QUESTIONS - II

1. In the given figure, a chord AB of the circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $A\angle LBQA$. (Use $\pi = 3.14$)



Ans : [Foreign Set I, II, III, 2016]

Area of sector $OAPB$,

$$= \frac{90}{360}\pi(10)^2 = 25\pi \text{ cm}^2$$

Area of ΔAOB,

$$= \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

Area of minor segment $AQBP$,

$= (25\pi - 50) \text{ cm}^2$

$= 25 \times 3.14 - 50$

$= 78.5 - 50$

$= 28.5 cm^2 \text{ cm}^2$

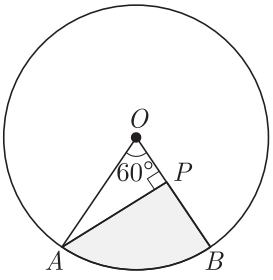
Also area of circle $= \pi(10)^2$

$= 3.14 \times 100 = 314 \text{ cm}^2$

Area of major segment $ALBQA = 314 - 28.5$

$= 285.5 \text{ cm}^2$

2. In the given figure, AOB is a sector of angle 60° of a circle with centre O and radius 17 cm. If $AP \perp OB$ and $AP = 15$ cm, find the area of the shaded region.



Ans : [CBSE S.A.2 2016 Set-HODM4OL]

Here $OA = 17$ cm $AP = 15$ cm and ΔOPA is right triangle

Using Pythagoras theorem, we have

$$OP = \sqrt{17^2 - 15^2} = 8 \text{ cm}$$

Area of the shaded region

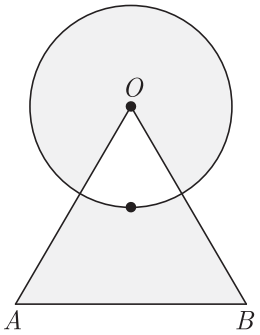
= Area of the sector ΔOAB – Area of ΔOPA

$$= \frac{60}{360} \times \pi r^2 - \frac{1}{2} \times b \times h$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15$$

$$= 151.38 - 60 = 91.38\, cm^2$$

3. Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Ans : [Board Sample Paper 2016], [Foreign Set I, II, III, 2016]

Since OAB is an an equilateral triangle,we have

$\angle AOB = 60^\circ$

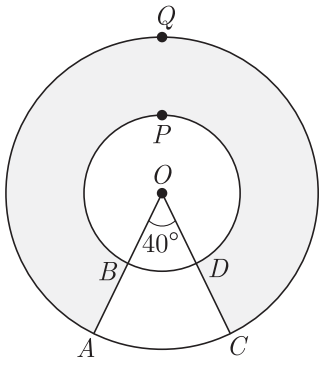
Area of shaded region = Area of major sector + (Area of ΔAOB – Area of minor sector)

$$= \frac{300}{360} \times \frac{22}{7} \times (6)^2 + \left(\frac{\sqrt{3}}{4}(12)^2 - \frac{60}{360} \times \frac{22}{7} \times 6^2\right)$$

$$= \frac{660}{7} + 36\sqrt{3} - \frac{132}{7}$$

$$= 36\sqrt{3} + \frac{528}{7} \text{ cm}^2$$

4. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40^\circ$. Use $\pi = \frac{22}{7}$.



Ans : [O.D. Set I, II, III, 2016]

Radii of two concentric circle = 7 cm and 14 cm

Angle $\angle AOC = 40^\circ$,

Angle $\angle AOC = 360^\circ - 40^\circ = 320^\circ$

Area of shaded region

$$= \frac{\theta}{360}\pi[R^2 - r^2]$$

$$= \frac{320}{360} \times \frac{22}{7}[14^2 - 7^2]$$

$$= \frac{8}{9} \times 22 \times (14 \times 2 - 7)$$

$$= \frac{8}{9} \times 22 \times 21 = \frac{8}{3} \times 22 \times 7$$

$$= \frac{8 \times 154}{3} \text{ cm}^2$$

Required area $= \frac{1232}{3} \text{ cm}^2$

$$= 410.67 \text{ cm}^2$$

5. Find the area of minor segment of a circle of radius 14 cm, when its centre angle is 60°. Also find the area of corresponding major segment. Use $\pi = \frac{22}{7}$.

Ans : [Outside Delhi Set I, II, III, 2015]

Here, $r = 14$ cm, $\theta = 60^\circ$

Area of minor segment $= \pi r^2 \frac{\theta}{360} - \frac{1}{2} r^2 \sin\theta$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60}{360^\circ} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

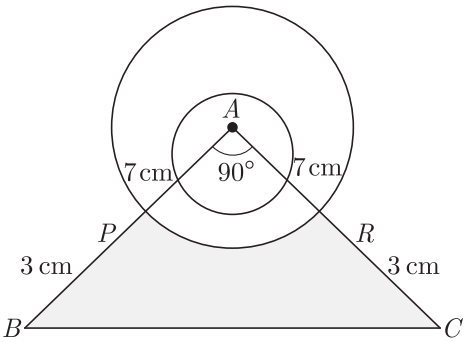
$$= \left(\frac{308}{3} - 49\sqrt{3}\right) = 17.9 \text{ cm}^2 \text{ approx.}$$

Area of major segment $= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$

$$= \frac{1540}{3} + 49\sqrt{3} = 598.10$$

$$= 598 \text{ cm}^2 \text{ approx.}$$

6. A momento is made as shown in the figure. Its base $PBCR$ is silver plate from the Front side. Find the area which is silver plated. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2,2015]

From the given figure

Area of right-angled ΔABC

$$= \frac{1}{2} \times 10 \times 10 = 50$$

Area of quadrant APR of the circle of radii 7 cm

$$= \frac{1}{4} \times \pi \times (7)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 49 = 38.5 \text{ cm}^2$$

Area of base $PBCR$

$$= \text{Area of } \Delta ABC - \text{Area of quadrant } APR$$
$$= 50 - 38.5 = 11.5 \text{ cm}^2$$

7. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle. Use $\pi = \frac{22}{7}$.

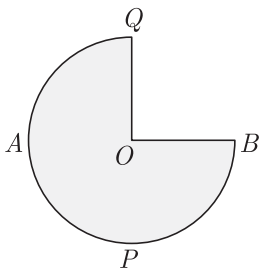
Ans : [Board Term-2, 2015]

Let radius of the circle be r cm

$$\text{Diameter} = 2r \text{ cm}$$
$$\text{Circumference} = 2\pi r$$
$$\text{Circumference} = \text{Diameter} + 16.8$$
$$2\pi r = 2r + 16.8$$
$$2\left(\frac{22}{7}\right)r = 2r + 16.8$$
$$\frac{44}{7}r = 2r + 16.8$$
$$44r = 14r + 16.8 \times 7$$
$$30r = 177.6$$
$$r = \frac{117.6}{30} = 3.92$$

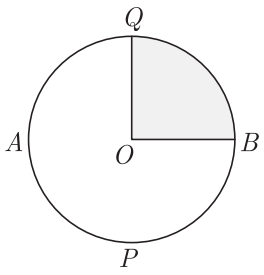
Thus $r = 3.92$ cm

8. In fig., APB and AQP are semi-circle, and $AO = OB$. If the perimeter of the figure is 47 cm, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Delhi CBSE Board, 2015, Set I, II, III]

We have redrawn the given figure as shown below;



Let r be the radius of given circle

Perimeter of given figure = 47 cm

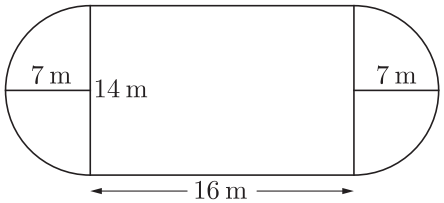
Perimeter of full circle− perimeter of $\left(\frac{1}{4}\right)^{th}$ circle

$$= 47 + 2r$$
$$2\pi r - \frac{1}{4}(2\pi r) + 2r = 47$$
$$\frac{3\pi r}{2} + 2r = 47$$
$$r\left(\frac{3}{2} \times \frac{22}{7} + 2\right) = 47$$
$$r\left(\frac{33}{7} + 2\right) = 47$$
$$r = \frac{47 \times 7}{47} = 7 \text{ cm}$$

Now, area of shaded region

$$A = \text{area of circle} - \frac{1}{4} \text{ area of circle}$$
$$= \frac{3}{4} \text{ area of circle}$$
$$= \frac{3}{4} \times \pi r^2 = \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{3}{2} \times 77 = 115.5 \text{ cm}^2$$

9. Find the area of the adjoining diagram.



Ans : [Board Term-2, 2014]

Required area,

= area of two semi-circles of same radii + area of rectangle

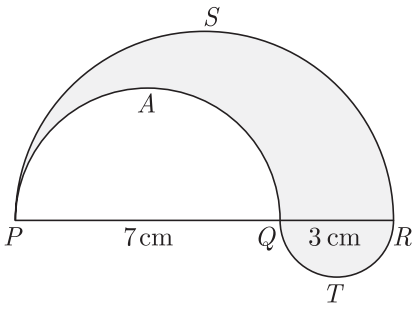
= area of one circle + area of rectangle

$= \pi r^2 + (l \times b)$

(where r is radius of circle and l and b are length and breadth of rectangle)

$$= \frac{22}{7} \times 7 \times 7 + (16 \times 14)$$
$$= 154 + 224 = 378\, m^2$$

10. In the fig., PSR, RTQ and PAQ are three semi-circles of diameters 10 cm, 3 cm and 7 cm region. Use $\pi = \frac{22}{7}$.



Ans : [Delhi CBSE, Term II 2014]

Perimeter of shaded region = Perimeter of semi-circles $PSR + RTQ + PAQ$

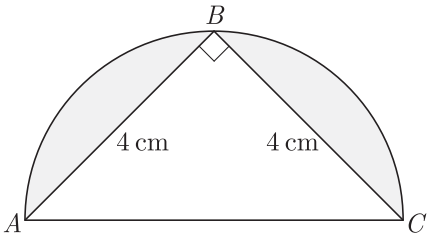
Perimeter of shaded region

$$= \pi(5) + \pi(1.5) + \pi(3.5)$$
$$= \pi(10)$$

$$= \frac{22}{7} \times 10 = \frac{22}{7}$$

Perimeter of shaded region $= 31.4$ cm. (approx)

11. In the figure, ΔABC is in the semi-circle, find the area of the shaded region given that $AB = BC = 4$ cm.(Use $\pi = 3.14$)



Ans : [Board Term-2, 2014]

As ΔABC is a triangle in semi-circle

$AC = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ cm

Radius of circle $\frac{4\sqrt{2}}{2} = 2\sqrt{2}$ cm

Area of shaded portion,

$=$ Area of the semi-circle$-$(Area of ΔABC)

$= \left\{\frac{1}{2}\pi \times (2\sqrt{2})^2\right\} - \left\{\frac{1}{2} \times 4 \times 4\right\}$

$= \left\{\frac{1}{2} \times 3.14 \times 8\right\} - 8$

$= 12.56 - 8 = 4.56$ cm^2

12. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of sector formed by the arc.

Ans : [Delhi Set Compt. Set-I, II, III 2017]

We have $r = 21$ cm and $\theta = 60^\circ$

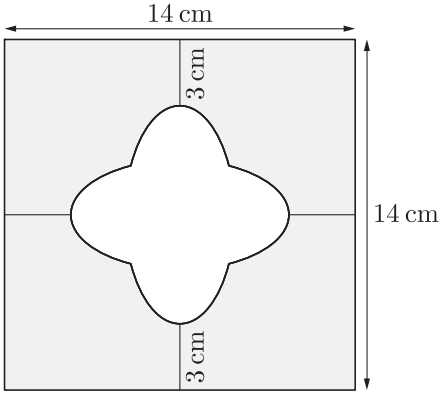
Area formed the sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1}{6} \times 22 \times 3 \times 21$

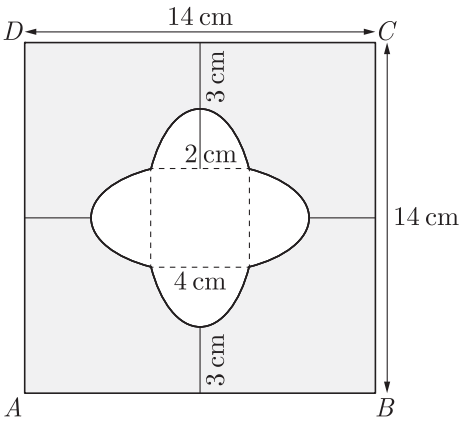
$= 11 \times 21 = 231$ cm^2

13. In fig., find the area of the shaded region [$\pi = 3.14$]



Ans : [Delhi Set I, II, III 2015][Board Term-2, 2011 Set-B1]

We have redrawn the given figure as shown below;



Area of square $ABCD$

$= 14 \times 14 = 196$ cm^2

Radius of the semi-circle formed inside $= 2$ cm

Area of 4 semi circle $= 4 \times \frac{1}{2}\pi r^2$

$= 2 \times 3.14 \times 2 \times 2 = 25.12$ cm^2

Length of the side of square formed inside the semi-circle $= 4$ cm.

Area of the square $= 4 \times 4 = 16$ cm^2

Area of the shaded region,

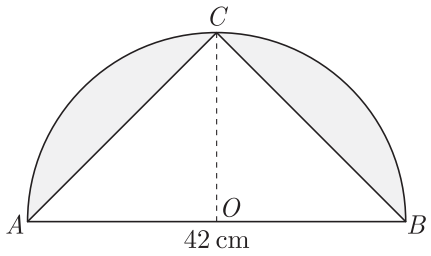
$=$ area of square $ABCD$

$-$ (Area of 4 semi-circle +Area of square)

$= 196 - (25.12 + 16)$

$= 196 - 41.12 = 154.88$ cm^2

14. In the figure, ΔACB is in the semi-circle. Find the area of shaded region given that $AB = 42$ cm.



Ans : [Board Term-2, 2014]

Base of triangle $=$ diameter of semicircle

$= 42$ cm

and its heigh $=$ radius of semicircle

$= \frac{42}{2} = 21$ cm

Area of shaded portion,

$=$ Area of semicircle$-$area of ΔABC

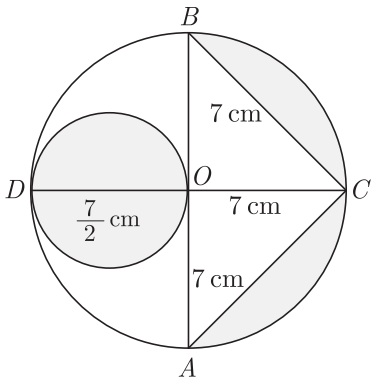
$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times$base$\times$height

$= \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{2} \times 42 \times 21$

$= 693 - 441 = 252$

Hence, the area of shaded portion $= 252$ cm^2

15. AB and CD are two diameters of a circle perpendicular to each other and OD is the diameter of the smaller circle. If $OA = 7$ cm, find the area of the shaded region.



Ans : [Board Term-2, 2012 Set (13)]

Area of a circle with DO as diameter

$$\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ sq.cm}$$

Area of semi-circle with AB as diameter

$$\frac{\pi r^2}{2} = \frac{22 \times 7 \times 7}{7 \times 2} = 77 \text{ sq.cm}$$

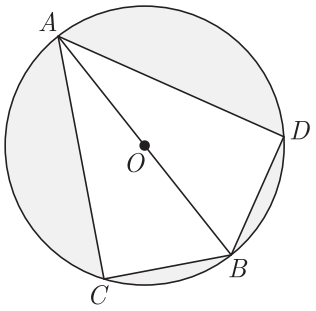
Area of $\Delta ABC = \frac{1}{2} \times 14 \times 7 = 49$ sq.cm

Area of shaded region

$=$ Area of circle $+$ Area of semi-circle $-$ Area of ΔABC

$= \frac{77}{2} + 77 - 49 = 66.5$ cm^2

16. Find the area of the shaded region in figure, if $BC = BD = 8$ cm, $AC = AD = 15$ cm and O is the centre of the circle. (Take $\pi = 3.14$)



Ans : [Board Term-2, 2012 Set (34)]

Since $\angle ADB$ and $\angle ACB$ angle in a semicircle,

$\angle ADB = \angle ACB = 90^\circ$

Since $\Delta ADB \cong \Delta ACB$

Thus $\text{ar}\Delta ADB = \text{ar}\Delta ACB$

$= \frac{1}{2} \times 15 \times 8 = 60$ cm^2

and ar $\Delta ADB + ar\Delta ACB$

$= 2 \times 60 = 120$ cm^2

Now in ΔABC $AB = \sqrt{AC^2 + BC^2}$

$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$

$= 17$ cm

Area of circle $\pi r^2 = \frac{22}{7} \times \frac{17}{2} \times \frac{17}{2}$

$= 226.87$ cm^2

Area of shaded portion,

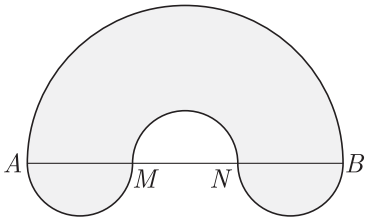
$=$ area of circle$-$area of sum of ΔACB and ΔADB.

$= 226.87 - 120 = 106.87$ cm^2

Hence, area of shaded region

$= 106.87$ cm^2

17. In the given figure, AB is the diameter of the largest semi-circle. $AB = 21$ cm, $AM = MN = NB$. Semi-circle are drawn with AM, MN and NB as shown. Using $\pi = \frac{22}{7}$, calculate the area of the shaded region.



Ans : [Board Term-2, 2012 Set (21)]

Area of semi-circle with diameter 21 cm,

$A = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{21}{7} \times \frac{21}{7} = \frac{693}{4}$ cm^2

Here $AM = MN = NB = \frac{21}{3} = 7$ cm

Thus radii of smaller semi circle $= \frac{7}{2}$ cm

Area of semi-circle with diameter AM,

$=$ Area of semi-circle with diameter MN

$=$ Area of semi- circle with diameter NB

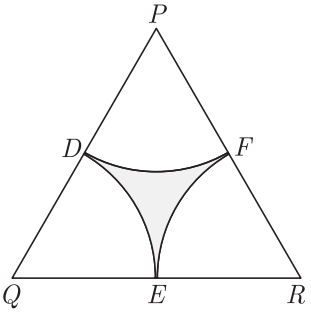
$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4}$ cm^2

Area of shaded region

$=$ Area largest semicircle $+$ smallest semicircle

$= \frac{693}{4} + \frac{77}{4} = \frac{770}{4} = 192.5\, cm^2$

18. In the given figure, ΔPQR is an equilateral triangle of side 8 cm and D, E, F are centres of circular arcs, each of radius 4 cm. Find the area of shaded region. (Use $\pi = 3.14$) and $\sqrt{3} = 1.732$



Ans : [Board Term-2, 2012, Set (28)]

Area of shaded region

$$= \text{Area of } \Delta PQR - 3(\text{area of sector})$$

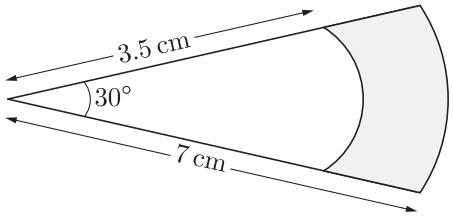
$$= \frac{\sqrt{3}}{4}(side)^2 - 3\left[\frac{\theta}{360^\circ} \times \pi r^2\right]$$

$$= \frac{\sqrt{3}}{4} \times 8 \times 8 - 3\left[\frac{60}{360^\circ} \times 3.14 \times 4 \times 4\right]$$

$$= 16\sqrt{3} - 3.14 \times 8 = 16 \times 1.732 - 25.12$$

$$= 27.712 - 25.12 = 2.59\, cm^2 \text{ cm}^2$$

19. In fig., sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2, 2012, Set B1]

Area of shaded region,

$$= \pi[R^2 - r^2]\frac{\theta}{360^\circ}$$

$$= \frac{22}{7}[7^2 - (3.5)^2]\frac{30^\circ}{360^\circ}$$

$$= \frac{22}{7}(7 + 3.5)(7 - 3.5) \times \frac{1}{12}$$

$$= \frac{22}{7} \times 10.5 \times 3.5 \times \frac{1}{12}$$

$$= 22 \times \frac{5}{10} \times \frac{35}{10} \times \frac{1}{4} = \frac{77}{8} = 9.62 \text{ cm}^2$$

20. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm2. If the wire is bent in the form of a circle, find the area enclosed by the circle. Use $\pi = \frac{22}{7}$.

Ans : [Outside Delhi Set-I, II, III 2017]

Let l be length of wire. If it is bent in the form of an equilateral triangle, side of triangle will be $\frac{l}{3}$
Area enclosed by the triangle,

$$\frac{\sqrt{3}}{4} \times \left(\frac{l}{3}\right)^2 = 121\sqrt{3}$$

$$\frac{1}{4} \times \left(\frac{l}{3}\right)^2 = 121$$

$$\frac{1}{2} \times \frac{l}{3} = 11$$

$$l = 66 \text{ cm}$$

Same wire is bent in the form of circle. Thus circumference of circle will be 66.

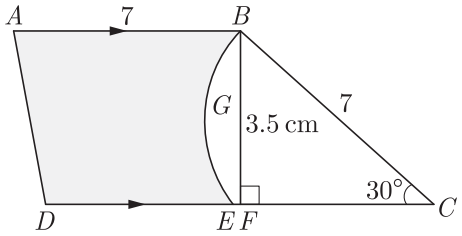
$$2\pi r = 66$$

$$r = \frac{66}{2\pi} = \frac{66}{2 \times \frac{22}{7}} = \frac{21}{2}$$

Area enclosed by the circle

$$\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} = 346.5 \text{ cm}^2$$

21. Adjoining fig, $ABCD$ is a trapezium with $AB \,||\, DC$ and $\angle BCD = 30^\circ$. Fig. $BGEC$ is a sector of a circle with centre C and $AB = BC = 7$ cm, $DE = 4$ cm and $BF = 3.5$ cm, then find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Outside Delhi Compt. Set-I, II, III 2017]

We have $AB = 7$ cm

$DE = 4$ cm, and

$BF = 3.5$ cm

Now $BC = DE + EC = 4 + 7 = 11$ cm

Area of Trapezium $ABCD$

$$= \frac{1}{2}(\text{Sum of } || \text{ lines}) \times \text{ distance between}$$

$$= \frac{1}{2}(11 + 7) \times 3.5 = \frac{1}{2} \times 18 \times 3.5$$

$$= 31.5 \text{ cm}^2$$

Area of shaded region

$$= \frac{30^o}{360^o} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{12} \times 22 \times 7$$

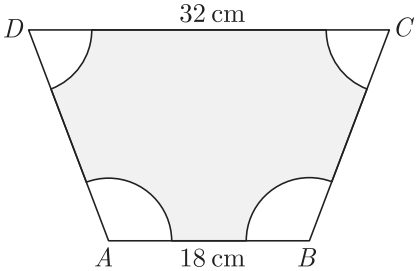
$$= 12.83 \text{ cm}^2$$

$$= \text{Area of trapezium} - \text{ar of sector}$$

$$= 31.5 - 12.83 = 18.67$$

$$= 18.67 \text{ cm}^2$$

22. In the given figure $ABCD$ is a trapezium with $AB \,||\, DC, AB = 18$ cm and $DC = 32$ cm and the distance between AB and AC is 14 cm. If arcs of equal radii 7 cm taking A, B, C and D have been drawn, then find the area of the shaded region.



Ans : [Foreign Set-I, II, III 2017]

In trapezium $ABCD, AB = 18$ we have
$AB = 18$ cm, $CD = 32$ cm $AB \,||\, CD$ and distance between || lines $= 14$ cm and the radius of each sector $= 7$cm.

Area of trapezium $ABCD$

$$= \frac{1}{2}(18 + 32) \times 14 = \frac{1}{2} \times 50 \times 14$$

$= 350 \text{ cm}^2$

Let, $\angle A = \theta, \angle B = \theta_2, \angle C = \theta_3$ and $\angle D = \theta_4$

Area of sector A,

$$\frac{\theta_1}{360}\pi r^2 = \frac{\theta_1}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{\theta_1}{360} \times 154 \text{ cm}^2$$

area of sector $B = \frac{\theta_2}{360} \times 154 \text{ cm}^2$

area of sector $C = \frac{\theta_3}{360} \times 154 \text{ cm}^2$

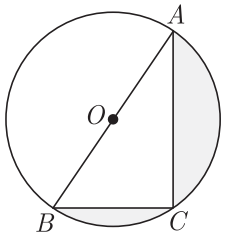
area of sector $D = \frac{\theta_4}{360} \times 154 \text{ cm}^2$

area of 4 sectors $= \frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{360} \times 154$

$$= \frac{360}{360} \times 154 = 154 \text{ cm}^2$$

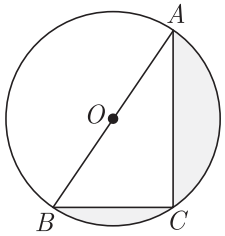
LONG ANSWER TYPE QUESTIONS

1. In the figure, O is the centre of circle such that diameter $AB = 13$ cm and $AC = 12$cm. BC is joined. Find the area of the shaded region. $(\pi = 3.14)$



Ans : [O.D. Set I, II, III, 2016]

We redraw the given figure as below.



Radius of semi circle $ACB = \frac{13}{2}$ cm

Area of semicircle $= \frac{\pi}{2}r^2 = \frac{3.14}{2} \times \frac{13}{2} \times \frac{13}{2}$

$$= \frac{3.14 \times 169}{8} = \frac{530.66}{8} \text{ cm}^2$$

Semicircle subtend 90° at circle, thus $\angle ACB = 90^\circ$

In ΔABC

$$AC^2 + BC^2 = AB^2$$

$$12^2 + BC^2 = 169$$

$$BC^2 = (160 - 144) = 25$$

$$BC = 5 \text{ cm}$$

Also area $\Delta = \frac{1}{2} \times \text{Base} \times \text{Hight}$

Area of ΔABC $\Delta = \frac{1}{2} \times AC \times BC$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

Area of shaded region $= \frac{530.66}{8} - 30$

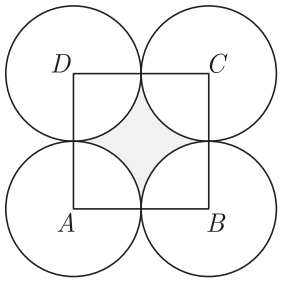
$$= (66.3325 - 30) \text{ cm}^2$$

$$= 36.3325 \text{ cm}^2$$

2. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circle is $\frac{24}{7}$ cm2. Find the radius of each circle.

Ans : [Board Sample paper, 2016]

As per question statement the figure is shown below.



Let r cm be the radius of each circle.

Area of square – Area of 4 sectors $= \frac{24}{7}$ cm^2

$$(2r)^2 - 4\left(\frac{90}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$$

$$4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$$

$$\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

$$6r^2 = 24$$

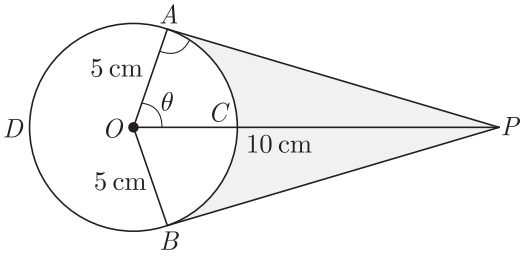
$$r^2 = 4$$

$$r = \pm 2$$

Thus radius of each circle is 2 cm.

3. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



Ans : [Delhi Set I, II, III, 2016]

Here AP is tangent at point A on circle.

Thus $\angle OAP = 90°$

Now $\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

or, $\theta = 60°$

Reflex $\angle AOB = 360° - 60° - 60° = 240°$

Now arc ADB $\frac{2 \times 3.14 \times 5 \times 120}{360} = 20.93$ cm

Hence length of elastic in contact $= 20.93$ cm

Now, $AP = 5\sqrt{3}$ dm

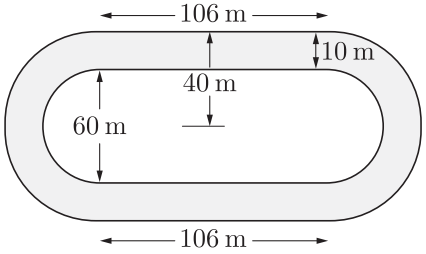
Area $(\Delta OAP + \Delta OBP) = 25\sqrt{3} = 43.25$ cm^2

Area of sector $OACB$

$$= \frac{25 \times 3.14 \times 120}{360} = 26.16 \text{ cm}^2.$$

Shaded Area $= 43.25 - 26.16 = 17.09$ cm^2

4. Fig. depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide everywhere, find the area of the track.



Ans : [Board Term-2, 2011, Set B1]

Width of the inner parallel lines $= 60$ m

And the width of the outer lines $= 40 \times 2 = 80$ m

Radius of the inner semicircles $= \frac{60}{2} = 30$ m

Radius of the other semicircles $= \frac{80}{2} = 40$ m

Area of inner rectangle $= 106 \times 60 = 3180$ m^2

Area of outer rectangle $= 106 \times 80 = 4240$ m^2.

Area of the inner semicircle

$$= 2\frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{19800}{7} \text{ m}^2$$

Area of outer semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{35200}{7} \text{ m}^2$$

Area of racing track

= (area of outer rectangle + area of outer semicircles)

– (area of inner rectangle + area of inner semicircles)

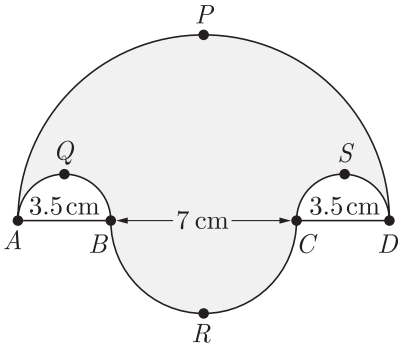
$$= 4240 + \frac{35200}{7} - \left(\frac{3180 + 19800}{7}\right)$$

$$= 1060 + \frac{15400}{7} = \frac{7420 + 15400}{7}$$

$$= \frac{22820}{7} = 3260 \text{ m}^2$$

Hence, area of track is 3260 m^2

5. Find the area of the shaded region in Figure, $\widehat{APD}, \widehat{AQB}, \widehat{BRC}$ and $\widehat{CSD}$, are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. Use $\pi = \frac{22}{7}$.



Ans : [Foreign Set I, II, III, 2016]

Diameter of the largest semi circle

$= 14$ cm

Radius $= \frac{14}{2} = 7$ cm

Diameter of two equal unshaded semicircle

$= 3.5$ cm

Radius of each circle $= \frac{3.5}{2}$ cm

Diameter of smaller shaded semi-circle $= 7$ cm

Radius $= 3.5$ cm

Area of shaded portion

= area of largest semi-circle+

+ area of smaller shades semicircle+

– area of two unshaded semicircles

$$= \frac{1}{2} \times \frac{88}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$- 2 \times \frac{22}{7} \times \frac{1}{2} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

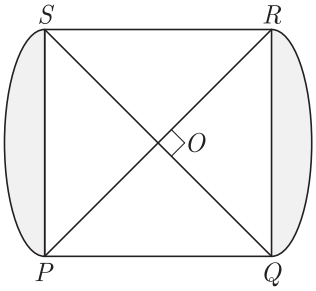
$$= \frac{1}{2} \times \frac{22}{7}\left[7^2 + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{4}\right)^2\right] \text{ cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7}\left[49 + \frac{49}{4} - \frac{49}{8}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times 49\left[\frac{9}{8}\right]$$

$$= \frac{693}{8} \text{ sq. cm or } 86.625 \text{ cm}^2$$

6. In figure, $PQRS$ is square lawn with side $PQ = 42$ metre. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



Ans : [Outside Delhi Set I, II, III, 2015]

Radius of circle with centre O is OR.

Let $OR = x$ then using Pythagoras theorem we have

$$x^2 + x^2 = (42)^2 \text{ or } x = 21\sqrt{2} \text{ m}$$

Area of segment of circle with centre angle 90°

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (21\sqrt{2})^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 \times 2$$

$$= 11 \times 3 \times 21 = 693$$

Area of triangle ΔROQ

$$= \frac{1}{2} \times (21\sqrt{2})^2 = 21 \times 21 = 441$$

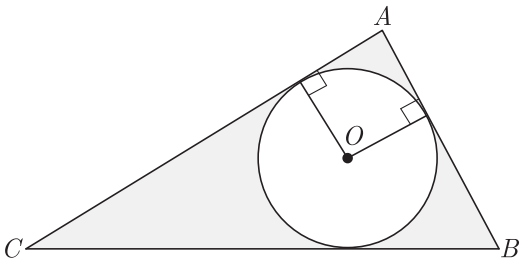
Area of the one side flower bed

$$= 693 - 441 = 252 \text{ m}^2$$

Area of flower bed of both

$$= 2 \times 252 = 504 \text{ m}^2$$

7. In the figure, ABC is a right angled triangle right angled at $\angle A$. Find the area of the shaded region, if $AB = 6$ cm, $BC = 10$ cm and O is the centre of the circle of the triangle ABC.



Ans : [Board Term-2, 2015]

Let r be the radius of in circle

Using the tangent properties we have

$$BC = 8 - r + 6 - r$$

$$10 = 14 - 2r$$

or, $2r = 4$ or, $r = 2$ cm

Area of circle $\pi r^2 = \frac{22}{7} \times 2 \times 2 = \frac{88}{7} = 12.57 \text{ cm}^2$

Now,area of ΔABC,

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Area of shaded region

$$= \text{Area of } \Delta ABC - \text{Area of the circle}$$

$$= 24. - 12.57 \text{ cm}^2 = 11.43 \text{ cm}^2$$

8. Two circular beads of different sizes are joined together such that the distance between their centres is 14cm. The sum of their areas is 130π cm2. Find the radius each bead.

Ans : [Board Term-2, 2015]

Let the radii of the circles are r_1 cm and r_2 cm

$$r_1 + r_2 = 14 \quad \ldots(1)$$

Sum, of their areas,

$$130\pi = \pi(r_1^2 + r_2^2)$$

$$130\pi = \pi(r_1^2 + r_2^2)$$

$$r_1^2 + r_2^2 = 130 \quad \ldots(2)$$

Now $(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1 r_2$

$$(14)^2 = 130 + 2r_1 r_2$$

$$2r_1 r_2 = 196 - 130 = 66$$

$$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1 r_2$$

$$= 130 - 66 = 64$$

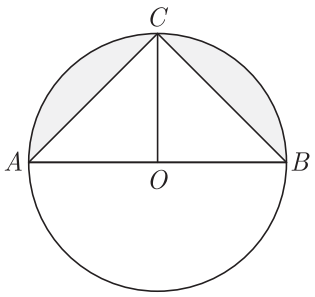
Thus $r_1 - r_2 = 8 \quad \ldots(3)$

From (1) and (3), $2r_1 = 22$

$$r_1 = 11 \text{ cm}$$

$$r_2 = 14 - 11 = 3 \text{ cm}.$$

9. A round thali has 2 inbuilt triangular for serving vegetables and a separate semi-circular area for keeping rice or chapati. If radius of thali is 21 cm, find the area of the thali that is shaded in the figure.



Ans : [Board Term-2, 2014]

Since AOB is the diameter of the circle. So Area of shaded region

$$= (\text{Area of semi-circle} - \text{Area of } \Delta ABC)$$

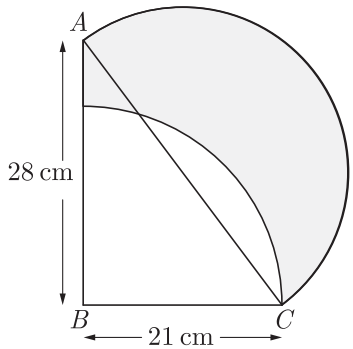
$$\text{Area of semi-circle} = \frac{\pi r}{2} \times \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= \frac{1386}{2} = 693 \text{ cm}^2$$

$$\text{Area of triangle} = \frac{1}{2} \times 42 \times 21 = 441 \text{ cm}^2$$

$$\text{Area of shaded region} = 693 - 441 = 252 \text{ cm}^2$$

10. In the fig., ABC is a right-angle triangle, $\angle B = 90^\circ, AB = 28$ cm and $BC = 21$ cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region.



Ans : [CBSE Foreign 2014][CBSE Board Term-2 2011]

In right angled triangle ΔABC using Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$
$$= 28^2 + 21^2 = 784 + 441$$

or $AC^2 = 1225$

Thus $AC = 35$ cm

Area of shaded region,

= area of ΔABC +

+ area of semi-circle with diameter AC +

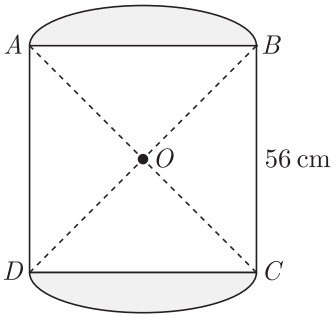
− area of quadrant with radius BC

$$= \frac{1}{2}(21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2 - \frac{1}{4} \times \frac{22}{7} \times (21)^2$$

$$= 294 + 481.25 - 346.5$$

$$= 775.25 - 346.5 = 428.75 \text{ cm}^2.$$

11. In fig., two circular flower beds have been shown on two sides of a square lawn $ABCD$ of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and flower beds.



Ans : [Board Term-2, 2011, Set A1]

Side of square = 56

Diagonal of square $= 56\sqrt{2}$

Radius of circle $= \frac{1}{2} \times 56\sqrt{2} = 28\sqrt{2}$

Total area = Area of sector OAB +

+ Area of sector ODC +

+ Area of ΔOAD +

+ Area of ΔOBC

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (28\sqrt{2})^2 + \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (28\sqrt{2})^2$$
$$+ \frac{1}{4} \times 56 \times 56 + \frac{1}{4} \times 56 \times 56$$

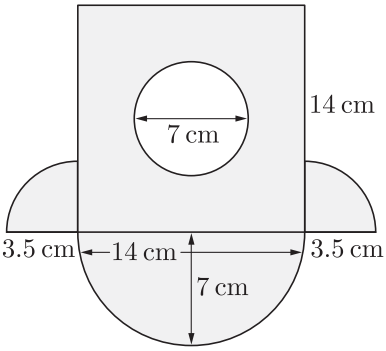
$$= \frac{1}{4} \times \frac{22}{7} \times (28\sqrt{2})^2 + \frac{1}{4} \times \frac{22}{7} \times (28\sqrt{2})^2$$
$$+ \frac{1}{4} \times 56 \times 56 + \frac{1}{4} \times 56 \times 56$$

$$= \frac{1}{4} \times 28 \times 56\left(\frac{22}{7} + \frac{22}{7} + 2 + 2\right) \text{ m}^2$$

$$= 7 \times 56\left(\frac{22 + 22 + 14 + 14}{7}\right) \text{ m}^2$$

$$= 56 \times 72 = 4032 \text{ m}^2.$$

12. In fig., find the area of the shaded region Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2, 2011, Set B1]

Area of square $= (14)^2 = 196 \text{ cm}^2$

Area of internal circle $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$$= \frac{77}{2} = 38.5 \text{ cm}^2$$

Area of semi-circle with 14 cm diameter

$$= \frac{1}{2} \times \frac{22}{7} \times 7^2 \text{ cm}^2$$

$$= 77 \text{ cm}^2$$

Area of two quarter circles of radius $\frac{7}{2}$ cm

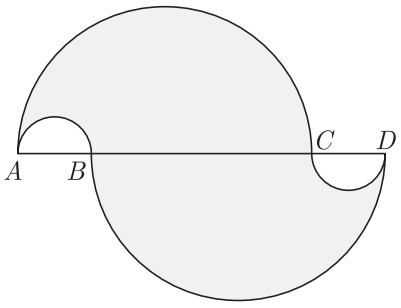
$$= 2 \times \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{4} = 19.25 \text{ cm}^2$$

Shaded area $= 196 - 38.5 + 77 + 19.25$

$$= 292.25 - 38.5$$

$$= 253.75 \text{ cm}^2.$$

13. In fig., $AC = BD = 7$ cm and $AB = CD = 1.75$ cm. Semi-circles are drawn as shown in the figure. Find the area of the shaded region. $\left(Use\ \pi = \frac{22}{7}\right)$



Ans : [Board Term-2, 2011, Set B1]

Area of shaded region

= 2(Area of semi-circle of radius $\frac{7}{2}$ cm)

− 2(Area of semi-circle of radius $\frac{7}{4}$ cm)

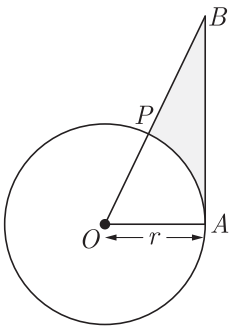
$$= 2\left[\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right] - 2\left[\frac{1}{2} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\right]$$

$$= \left(\frac{77}{2} - \frac{77}{8}\right) = \frac{77}{4}\left[1 - \frac{1}{4}\right] = \frac{77}{2} \times \frac{3}{4} = \frac{231}{8} \text{ cm}^2$$

$$= 28.87 \text{ cm}^2$$

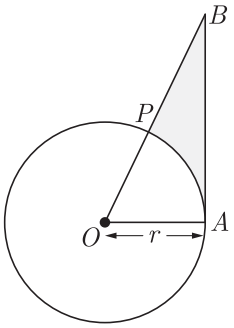
14. The given fig. is shown a sector OAP of a circle

with centre O, containing $\angle\theta$.AB is perpendicular to the radius OA and meets OP produced at B . Prove that the perimeter of shaded region is $r=\left[\tan\theta+\sec\theta+\frac{\pi\theta}{180}-1\right]$



Ans : [CBSE Outside 2015, 16]

As per question statement we have redrawn this figure as given below.



Here OAP is sectors of circle with centre O, $\angle POA=\theta$ and $OA\perp AB$

Perimeter of shaded region $=BP+AB+\overset{\frown}{AP}$ (1)

Now $\tan\theta=\frac{AB}{r}\Rightarrow r\tan\theta=AB$ ...(2)

$\sec\theta=\frac{OB}{r}\Rightarrow r\sec\theta=OB$

$OB-OP=BP\Rightarrow r\sec\theta-r=OP$ ...(3)

Length of arc AP

$$\overset{\frown}{AP}=\frac{\theta}{360}\times 2\pi r$$

$$=\frac{\theta}{360}\times 2\pi r=\frac{\theta\pi r}{180} \quad ...(4)$$

Putting value from eq(2), (3), (4) in eq (1) we have

Perimeter of shaded region

$$=r\tan\theta+r\sec\theta-r+\frac{\theta\pi r}{180}$$

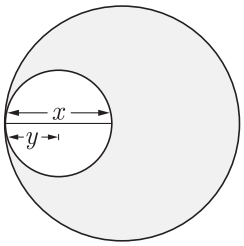
$$=r\left[\tan\theta+\sec\theta+\frac{\theta\pi}{180}-1\right]$$

Hence, Proved.

15. Two circles touch internally. The sum of their areas is 1.16π and the difference between their centres is 6 cm. Find the radii of the circles.

Ans : [Foreign Set-I, II, III 2017]

Let the radius of larger circle be x and the radius of smaller circle be y. As per question statement we have shown diagram shown below.



Now $x-y=6$ ...(1)

and $\pi x^2+\pi y^2=116\pi$

$\pi(x^2+y^2)=116\pi$

$x^2+y^2=116$ ...(2)

From (1) and (3)we have

$x^2+(x-6)^2=116$

$x^2+x^2-12x+36=116$

$x^2-6x-40=0$

$x^2-10x+4x-40=0$

$x(x-10)+4(x+10)=0$

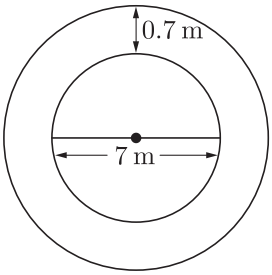
$x=10$, and $y=10-6=4$

Hence, radii of the circles are 10 cm and 4 cm.

16. A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path. If its cost is Rs.110 per sq. m.

Ans : [Foreign Set-I, II, III 2017]

As per question statement we have shown diagram shown below.



The diameter of park$=7$ m

$$\text{radius}=\frac{7}{2}=3.5 \text{ m}$$

Width of path $=0.7$ m

Radius of park with path

$=3.5+0.7=4.2$ m

Area of the path $=\pi(4.2)^2-\pi(3.5)^2$

$$=\frac{22}{7}(17.64-12.25)$$

$$=\frac{22}{7}\times 5.39=22\times 0.77$$

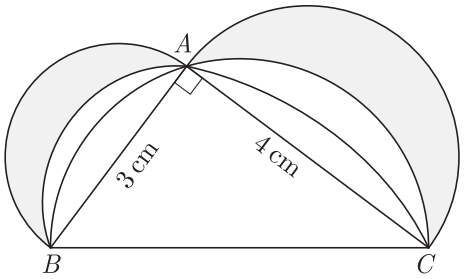
$=16.94$ m^2

Cost of the cementing the path

$= 16.94 \times 110$

$= Rs.1863.40$

17. In the given figure, ΔABC is a right angled triangle in which $\angle A = 90^\circ$. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



Ans : [Outside Delhi Set-II 2017]

In ΔABC we have

$\angle A = 90^\circ$, $AB = 3 = 3$ cm, and $AC = 4$ cm

Now $BC = \sqrt{AB^2 + AC^2} = \sqrt{3^2 + 4^2} = 5$ cm.

Area of shaded Area

$= \text{Area of semicircle with radius } \frac{3}{2} \text{ cm}$

$+ \text{ area of semi circle with radius } \frac{4}{2} \text{ cm}$

$+ \text{ Area of triangle } \Delta ABC)$

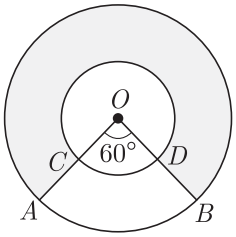
$- \text{ Area of semicircle with radius } \frac{5}{2} \text{ cm}$

$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}(2)^2 + \frac{1}{2} \times 3 \times 4 - \frac{\pi}{2}\left(\frac{5}{2}\right)^2$

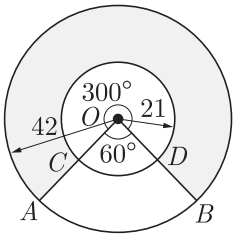
$= \frac{9\pi}{8} + 2\pi + 6 - \frac{25\pi}{8} = \frac{9\pi + 16\pi - 25}{8} + 6$

$= 6 \text{ cm}^2$

18. In the given figure, two concentric circle with centre O have radii 21 cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans :



Here $\angle AOB = 60^\circ$ and $\angle COD = 60^\circ$

$R = 42$ cm, $r = 21$ cm

Reflex of $\angle AOB$

$\theta = (360^\circ - 60^\circ) = 300^\circ$

Now, area of shaded region

$= \frac{\theta}{360^\circ} \times \pi R^2 - \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{\theta}{360^\circ} \times \pi \times (R^2 - r^2)$

$= \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (42 - 21)(42 + 21)$

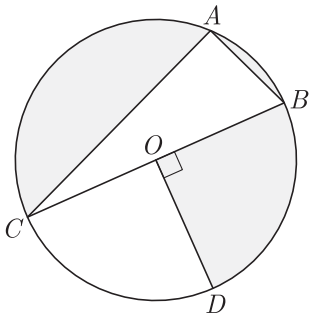
$= \frac{5}{6} \times \frac{22}{7} \times 21 \times 63$

$= 5 \times 11 \times 63$

$= 3465 \text{ cm}^2$

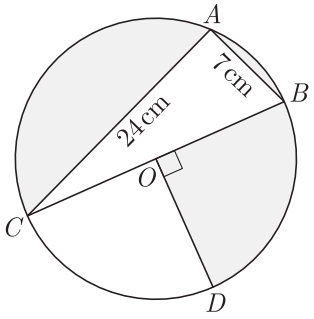
Thus area of shaded region is 3465 cm^2.

19. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90^\circ$. Find the area of the shaded region.



Ans :

We have redrawn the given figure as shown below.



Here ΔCAB is right angle triangle with $\angle CAB = 90^\circ$

In right ΔCAB, by Pythagoras theorem, we have

$BC^2 = AC^2 + AB^2 = 24^2 + 7^2 = 576 + 49 = 625$

Thus $BC = 25$ cm which is diameter.

Now radius is $\frac{25}{2}$ or 12.5 cm.

Area of shaded region,

$= \text{area of semicircle} + \text{area of quadrant} - \text{area of } \Delta ACB$

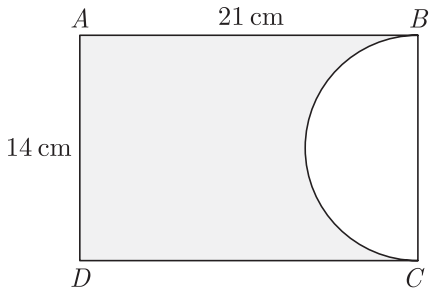
$= \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 - \frac{1}{2} \times AB \times BC$

$= \frac{3}{4}\pi r^2 - \frac{1}{2} \times 7 \times 24 = \frac{3}{4} \times \frac{22}{7} \times \frac{625}{4} - 7 \times 12$

$= 368.3035 - 84 = 284.3$ cm^2

Thus area of shaded region $= 284.3035$ cm^2

20. In the given figure, $ABCD$ is a rectangle of dimensions 21 cm $\times$ 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Ans : [Outside Delhi Set-I, 2017]

Area of shaded region

= Area of rectangle $ABCD$ − area of semicircle

$= 21 \times 14 - \frac{1}{2} \times \pi \times 7 \times 7$

$= 294 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \quad = 294 - 77 = 217$ cm^2

Perimeter of shaded are

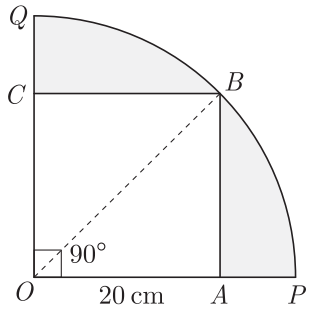
$= AB + AD + CD + \overset{\frown}{CB}$

$= 21 + 14 + 21 + \frac{22}{7} \times 7$

$= 21 + 14 + 21 + 22 = 78$ cm

Hence, area of shaded region $= 217$ cm^2 and perimeter $= 78$ cm.

21. A square $OABC$ is inscribed in a quadrant $OPBQ$ of a circle. If $OA = 20$ cm, find the area of the shaded region. [Use $\pi = 3.14$]



Ans : [Delhi CBSE, Term-2, 2014]

We have $OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{20^2 + 20^2}$

$= \sqrt{800}$

Thus $OB = 20\sqrt{2}$ cm or,

radius $r = 20\sqrt{2}$

Area of shaded region

= Area of sector $OQBPO$ − Area of square $OABC$

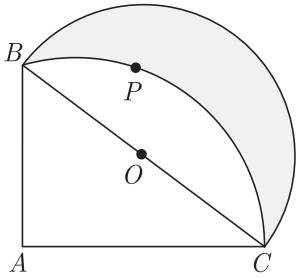
$= \frac{90^\circ}{360^\circ} \times 3.14 \times 20\sqrt{2} \cdot 20\sqrt{2} - (20)^2$

$= \frac{1}{4} \times 3.14 \times 800 - 400$

$= 2(314) - 400 = 628 - 400$

Required area is 228 cm^2.

22. In given figure $ABPC$ is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the are of the shaded region.



Ans : [Sample Question Paper 2017]

Radius of the quadrant $AB = AC = 14$ cm

$BC = \sqrt{14^2 + 14^2} = 14\sqrt{2}$ cm

Radius of semicircle $= \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Area of semicircle $= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}$

$= 154$ cm^2

Area of segment $BPCO$

$\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 = r^2\left(\frac{\pi\theta}{360} - \frac{1}{2}\right)$

$= 14 \times 14\left(\frac{22}{7} \times \frac{90}{360} - \frac{1}{2}\right)$

$= 14 \times 14\left(\frac{11}{14} - \frac{1}{2}\right)$

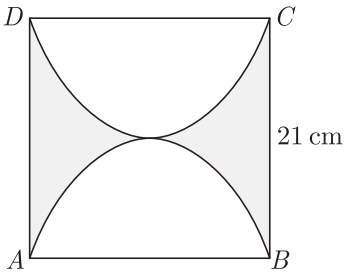
$= 14 \times 14 \times \frac{2}{7} = 56$ cm^2

Hence, area of shaded region $= 56$ cm^2

HOTS QUESTIONS

1. Find the perimeter of the shaded region if $ABCD$ is a square of side 21 cm and APB and CPD are

semicircle. Use $\pi = \frac{22}{7}$.



Ans : [Board Sample paper 2016]

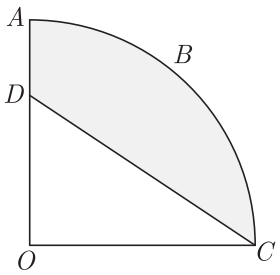
Perimeter of the shaded region

$= AD + BC +$

$+$lengths of the arcs of semi circles APB and CPD

$= 21 + 21 + 2\left(\frac{22}{7} \times \frac{21}{2}\right) = 42 + 66 = 108$ cm.

2. In the figure $OABC$ is a quadrant of a circle of radius 7 cm. If $OD = 4$ cm, find the area of shaded region.



Ans : [Foreign Set I, II, III, 2014]

Area of shaded region

$=$ Area of sector $OCBAD -$ Area of ΔODC

$= \frac{90^\circ}{360^\circ} \times \pi \times (7)^2 - \frac{1}{2} \times 7 \times 4$

$= \frac{49\pi}{4} - 14 = 24.5$ cm^2

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand from 9 a.m. to 9.35 a.m.

Ans : [Board Term-2, 2012 Set (13)]

Angle subtended in 1 minute

$\theta =$ angle subtended in 35 minutes

$= 35 \times 6 = 210^\circ$

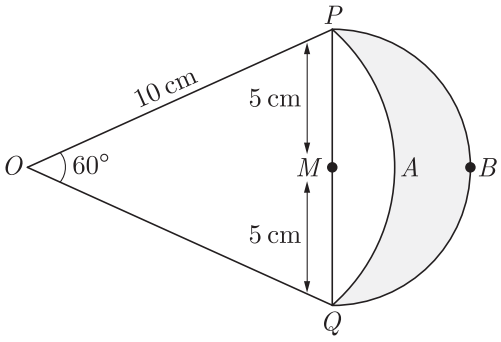
Area swept by the minute hand

$=$ Area of a sector

$= \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{14 \times 14 \times 210}{360}$

$= \frac{1078}{3} = 259.33$ cm^2

4. Figure shows two arcs PAQ and PQB. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M. If $OP = PQ = 10$ cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm2.



Ans : [Delhi Set I, II, III, 2016]

We have $OP = OQ = PQ = 10$

$\angle POQ = 60^\circ$

Area of segment $PAQM$

$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)$ cm^2

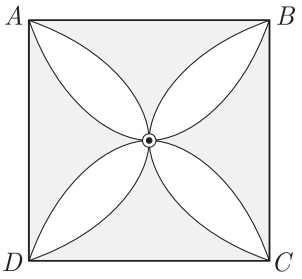
Area of semicircle $= \frac{25\pi}{2}$ cm^2

Area of shaded region

$= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$

$= 25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm^2.

5. In fig. $ABCD$ is a square of side 14 cm. Semi-circle are drawn with each side of square as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [CBSE Delhi, 2016]

Area of square $= 196$ cm^2

Area of semicircle $= AOB + DOC$

$= \frac{22}{7} \times 49 = 154$ cm^2

So, area of two shaded parts

$196 - 154 = 42$ cm^2

Hence, area of four shaded parts $= 84$ cm^2.

6. The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of distances travelled by their tips in 24 hours. (Use $\pi = 3.14$)

Ans : [Foreign Set I, II, III, 2015]

Long hand makes 24 rounds in 24 hours and short hand makes 2 round in 24 hours.

Radius of the circle formed by long hand = 6 cm. and radius of the circle formed by short hand = 4 cm.
Distance travelled by tips of hands in one round is equal to the circumference of circle.
Distance travelled by long hand in one round

$= \text{circumference of the circle } 2 \times 6 \times \pi$

Distance travelled by long hand in 24 rounds

$= 24 \times 12\pi = 288\pi$

Distance travelled by short hand in a round $= 2 \times 4\pi$

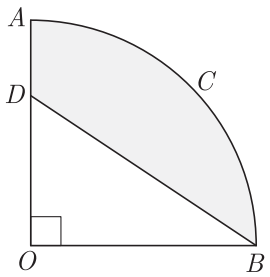
Distance travelled by short hand in 2 round

$= 2 \times 8\pi = 16\pi$

Sum of the distance $= 288\pi + 16\pi = 304\pi$

$= 304 \times 3.14 = 954.56$ cm

7. In the given figure $DACB$ is a quadrant of a circle with centre O and radius 3.5 cm. If $OD = 2$ find the area of the region.



Ans : [Delhi Set-I, II, 2017]

Area of shaded region,

$= \text{area of quadrant } OACB - \text{ar } \Delta DOB$

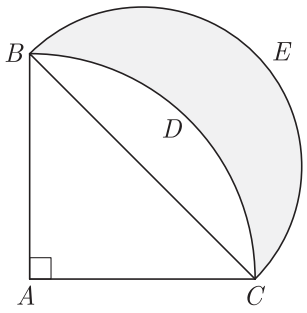
$= \frac{1}{4}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 - \frac{1}{2} \times 2 \times 3.5$

$= 19.625 - 3.5 = 6.125$

Hence the area of shaded region is 6.125 cm.

8. As $ABDC$ is a quadrant of a circle of radius 28 cm and a semi-circle BEC is drawn with BC as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Sample Question Paper 2017]

As ABC is a quadrant of the circle, $\angle BAC$ will be 90°.

In ΔABC, $BC^2 = AC^2 + AB^2$

$= (28)^2 + (28)^2 = 2(28)^2$

$BC = 28\sqrt{2}$ cm

Radius of semi-circle drawn on BC,

$= \frac{28\sqrt{2}}{2} = 14\sqrt{2}$

Area of semi-circle $= \frac{1}{2} \times \frac{22}{7} \times (14\sqrt{2})^2$

$= 616 \text{ cm}^2$

Area of $\Delta ABC = \frac{1}{2} \times 28 \times 28 = 392 \text{ cm}^2$

Area of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

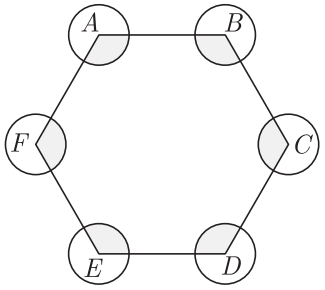
$= 616 \text{ cm}^2$

Area of the shaded region

= Area of semi-circle + area of Δ − Area of quadrant

$= 616 + 392 - 616 = 392 \text{ cm}^2$.

9. In fig., $ABCDEF$ is any regular hexagon with different vertices A, B, C, D, E, and F as the centres of circle with same radius 'r' are drawn. Find the area of the shaded portion.



Ans : [Board Term-2, 2011, B1]

Let number of sides is n.

$n \times \text{each angle} = (n-2) \times 180°$

$6 \times \text{each angle} = 4 \times 180°$

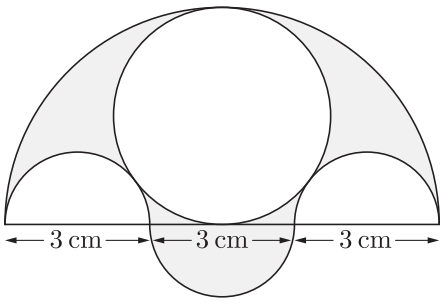
$\text{each angle} = 120°$

$\text{area of a sector} = \frac{120°}{360°} \times \pi r^2$

Area of 6 shaded regions $= 6 \times \frac{120°}{360°} \times \pi r^2$

$= 2\pi r^2$

10. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



Ans :

Area of shaded region

= Area of semicircle with $d = 9$ cm

+ Area of semicircle with $d = 3$ cm

$-2 \times$ area of semicircle with $d = 3$ cm

− area of circle with $d = 4.5$ cm

$$= \frac{1}{2} \times \pi \times \left(\frac{9}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - 2 \times \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - \pi \times \left(\frac{4.5}{2}\right)^2$$

$$= \frac{\pi}{8}[(9)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[4(4.5)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[2(4.5)^2 - (3)^2] = \frac{\pi}{8}[2(3 \times 1.5)^2 - (3)^2]$$

$$= \frac{\pi(3)^2}{8}[2(1.5)^2 - 1] = \frac{9\pi}{8}[4.5 - 1]$$

$$= \frac{9 \times 22}{8 \times 7} \times 3.5 = \frac{99}{8} = 12.375 \text{ cm}^2$$

Thus area of shaded region is 12.375 cm^2