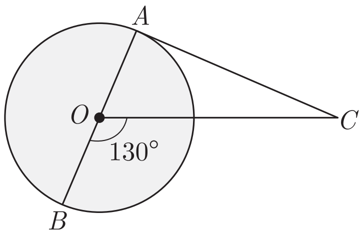
CHAPTER 10

Circle

VERY SHORT ANSWER TYPE QUESTIONS

1. In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A . If $\angle BOC = 130°$, the find $\angle ACO$.



Ans : [Foreign Set I, II, III, 2016]

Here OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have

$$\angle OAC = 90^o$$

From exterior angle property,

$$\angle BOC = OAC + \angle ACO$$

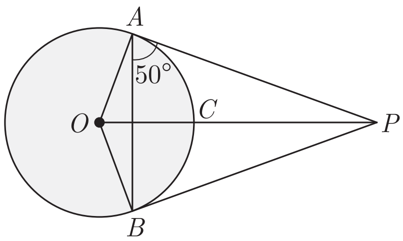
$$130^o = 90° + \angle ACO$$

$$\angle ACO \; 130^o - 90^o = 40^o$$

2. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^o$, then find $\angle AOB$.

Ans : [Delhi Set I, II, III, 2016]

As per the given question we draw the figure as below.



Since $PA \perp OA$, $\angle OAP = 90°$

$$\angle OAB = \angle OAP - \angle BAP$$

$$= 90^o - 50^o = 40^o$$

Since OA and OB are radii, we have

$$\angle OAB = \angle OBA = 40^o$$

Now

$$\angle AOB + \angle OAB + \angle OBA = 180^o$$

$$\angle AOB + 40^o + 40^o = 180^o$$

$$\angle AOB = 180^o - 80^o = 100^o$$

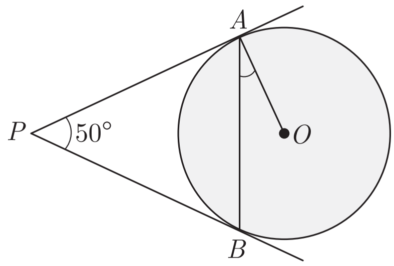
Hence $\angle AOB = 100^o$

3. What is the maximum number of parallel tangents a circle can have on a diameter?

Ans : [Board Term-2, 2012 Set (34)]

Tangent touches a circle on a distinct point. Thus on the diameter of a circle only two parallel tangents can be drawn. It has been shown in figure given below.

4. In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^o$. Write the measure of $\angle OAB$.



Ans : [Delhi CBSE Board, 2015, Set I, II, III]

We have $\angle APB = 50^o$

$$\angle PAB = \angle PBA = \frac{180^o - 50^o}{2} = 65^o$$

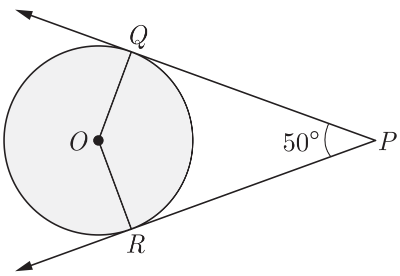
Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^o$$

Now $\angle OAB = \angle OAP - \angle PAB$

$$= 90^o - 65^o = 25^o$$

5. In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 50^o$, Then find $\angle OQR$.



Ans : [Board Term-2, 2012] ,Delhi CBSE Term II, 2015 Set I, II, III]

We have $\angle QPR = \angle 50^o$ (Given)

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

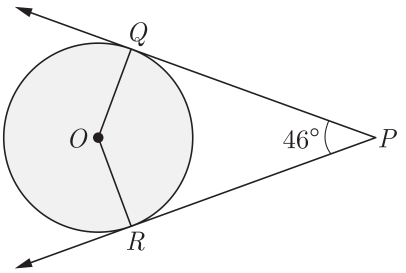
$$\angle QOR + \angle QPR = 180^o$$

$$\angle QOR = 180^\circ - \angle QPR$$
$$= 180^\circ - 50^\circ = 130^\circ$$

From $\Delta\, OQR$ we have

$$\angle OQR = \angle ORQ = \frac{180^\circ - 130^\circ}{2}$$
$$= \frac{50^\circ}{2} = 25^\circ$$

6. If PQ and PR are two tangents to a circle with center O. If $\angle QPR = 46^\circ$ then find $\angle QOR$.



Ans : [Delhi, CBSE, Term-2, 2014]

We have $\angle QPR = 46^\circ$

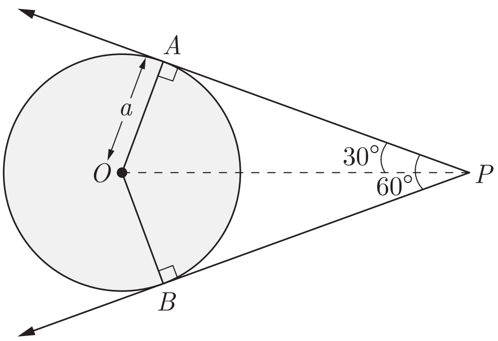
Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^\circ$$
$$\angle QOR + 46^\circ = 180^\circ$$
$$\angle QOR = 180^\circ - 46^\circ = 134^\circ$$

7. If the angle between two tangents drawn from an external point P to a circle of radius a and center O, is 60°, then find the length of OP.

Ans : [Outside Delhi Set II, 2018]

As per the given question we draw the figure as below.



Tangents are always equally inclined to line joining the external point P to center O.

$$\angle APO = \angle BPO = \frac{60}{2} = 30^\circ$$

Also radius is also perpendicular to tangent at point of contact.

In right $\Delta\, OAP$ we have

$$\angle APO = 30^\circ$$

Now, $$\sin 30^\circ = \frac{OA}{OP}$$

Here OA is radius whose length is a, thus

$$\frac{1}{2} = \frac{a}{OP}$$

or $$OP = 2a$$

8. If a circle can be inscribed in a parallelogram how will the parallelogram change?

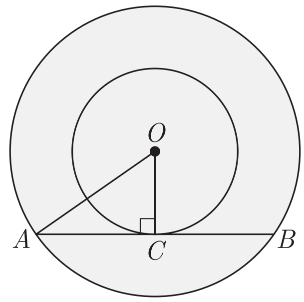
Ans : [Board Term II, 2014]

It changes into a rectangle or a square.

9. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (In cm) which touches the smaller circle.

Ans : [Foreign Set I, II, III, 2014]

As per the given question we draw the figure as below.



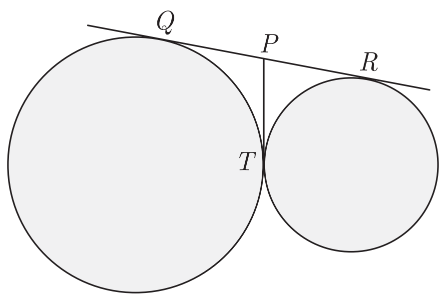
Here AB is the chord of large circle which touch the smaller circle at point C. We can see easliy that $\Delta\, AOC$ is right angled triangle.

Here, $AO = 5$ cm, $OC = 3$ cm

$$AC = \sqrt{AO^2 - OC^2}$$
$$= \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9} = \sqrt{16} = 4 \text{ cm}$$

Length of chord, $AB = 8$ cm.

10. In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If $QP = 3.8$ cm, then find length of QR.



Ans : [Delhi, Set 2014] [Board, Term-2, 2012 Set (44)]

Let us first consider large circle. Since length of tangents from external points are equal, we can write

$$QP = PT$$

Thus $$QP = PT = 3.8 \qquad ....(1)$$

Now consider the small circle. For this circle we can also write using same logic,

$$PR = PT$$

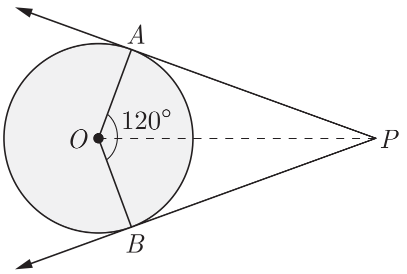
But we have $PT = 3.8$ cm

Thus $PR = PT = 3.8$ cm

Now $QR = QP + PR$

$= 3\cdot 8 + 3\cdot 8 = 7.6$ cm.

11. In the figure, PA and PB are tangents to a circle with centre O. If $\angle AOB = 120°$, then find $\angle OPA$.



Ans : [Delhi, Set 2014], [Board Term-2, 2012 Set (44)]

Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90°$$

Due to symmetry we have

$$\angle AOP = \frac{\angle AOB}{2} = \frac{120°}{2} = 60°$$

Now in right ΔAOP we have

$$\angle APO + \angle OAP + \angle AOP = 180°$$

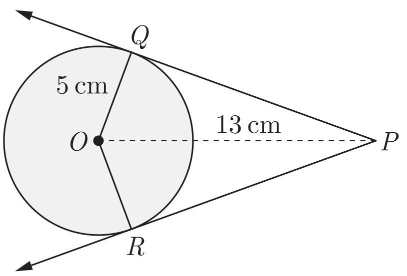
$$\angle APO + 90° + 60° = 180°$$

$$\angle APO = 180° - 150° = 30°.$$

12. From a point P, which is at a distant of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ are drawn to the circle, then the area of the quadrilateral $PQOR$ (in cm^2).

Ans : [Board Term-2, 2012 Set (31)]

As per the given question we draw the figure as below.



Here OQ is radius and QP is tangent at Q, since radius is always perpendicular to tangent at point of contact, ΔOQP is right angle triangle.

Now $PQ = \sqrt{OP^2 - OR^2}$

$= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm

Area of triangle ΔOQP

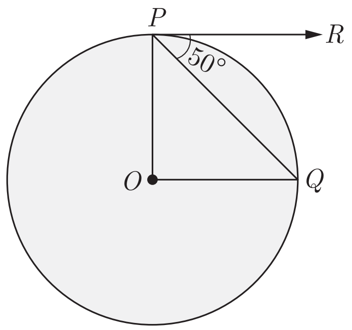
$\Delta = \frac{1}{2}(OQ)(QP)$

$= \frac{1}{2} \times 12 \times 5 = 30$

Area of quadrilateral $PQOR$,

$2 \times \Delta POQ = 2 \times 30 = 60$ cm^2

13. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of $50°$ with PQ, find $\angle POQ$.



Ans : [Board Term-2, 2012 Set (26)]

We have $\angle RPQ = 50°$

Since $\angle OPQ + \angle QPQ$ is right angle triangle,

$$\angle OPQ = 90° - 50° = 40°$$

Since, $OP = OQ$ because of radii of circle, we have

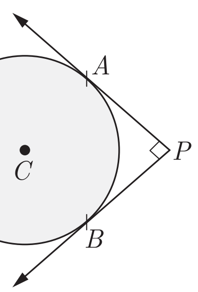
$$\angle OPQ = \angle OQR = 40°$$

In ΔPOQ we have

$$\angle POQ = 180° - (40° + 40°)$$

$$= 100°$$

14. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then find the length of each tangent.



Ans : [Board Term-2, 2013]

Here tangent drawn on circle from external point P are at aright angle, $CAPB$ will be a square

Thus $CA = AP = PB = BC = 4$ cm

Thus length of tangent is 4 cm.

15. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

Ans : [Board Term-2, 2012, Set (22)]

As per the given question we draw the figure as below.

$$\text{Length of the tangent} = \sqrt{d^2 - r^2} = \sqrt{8^2 - 6^2} = \sqrt{64 - 36} = \sqrt{28} = 2\sqrt{3} \text{ cm.}$$

16. If the angel between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

Ans : [Board Term II, 2012 Set (22)]

Sum of the angles between radii and between intersection point of tangent is always 180°.
Thus angle at the point of intersection of tangents

$$= 180° - 130° = 50°$$

17. To draw a pair of tangents to a circle which are inclined to each other at an angle of 30°, it is required to draw tangents at end points of two radii of the circle, what will be the angle between them ?

Ans : [Board Term II, 2012 Set (31)]

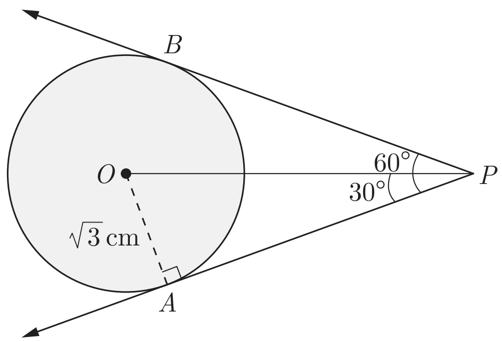
Sum of the angles between radii and between intersection point of tangent is always 180°.

Angle between the radii $= 180° - 30° = 150°$

18. Two tangents making an angle af 60° between them are drawn to a circle of radius $\sqrt{3}$ cm, then find the length of each tangent.

Ans : [Board, Term-2, 2013]

As per the given question we draw the figure as below.



Since, $\tan\theta = \dfrac{Altitude}{Base}$

So. $\tan 30° = \dfrac{OA}{AP}$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$$

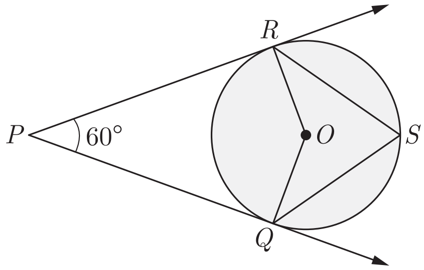
$$AP = \sqrt{3} \times \sqrt{3} = 3 \text{ cm.}$$

19. If a line intersects a circle in two distinct points, what is it called ?

Ans : [Board Term-2, 2012 Set (17)]

The line which intersects a circle in two distinct points is called secant.

20. In the given figure, find $\angle QSR$.



Ans : [Board Term-2, 2012 Set (5)]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus $\angle ROQ + \angle RPQ = 180°$

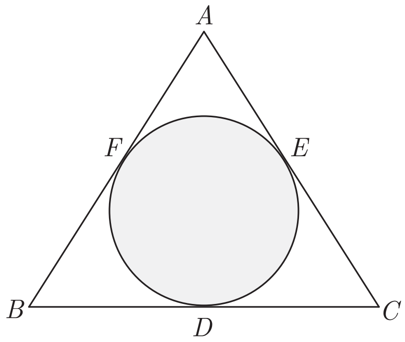
$\angle ROQ = 180° - 60° = 120°$

We know that angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus $\angle QSR = \frac{1}{2}\angle ROQ = \frac{1}{2} \times 120°$

$= 60°$

21. A triangle ABC is drawn to circumscribe a circle. If $AB = 13$ cm, $BC = 14$ cm and $AE = 7$ cm, then find AC.



Ans : [Board Term-2, 2012 Set (26)]

Since AF and AE are tangent of the circle, $AF = AE$

Thus $AF = AE = 7$ cm

Now $BF = AB - AF = 13 - 7 = 6$ cm

Since BF and BD are tangent of the circle, $BF = BD$

Thus $BD = BF = 6$ cm

Now $CD = BC - BD = 14 - 6 = 8$ cm

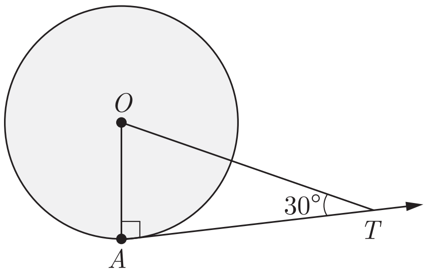
Since CD and CE are tangent of the circle, $CD = CE$

Thus $CE = CD = 8$ cm

Now $AC = AE + EC$

$= 7 + 8 = 15$ cm.

22. In given figure, if AT is a tangent to the circle with centre O, such that $OT = 4$ cm and $\angle OTA = 30^\circ$, then find the length of AT (in cm).



Ans : [Board Term-2, 2012 Set (13)]

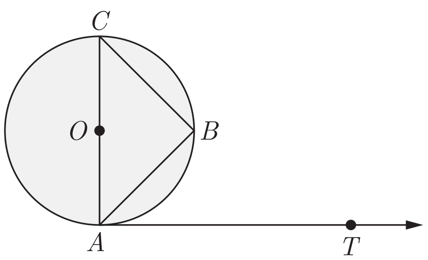
Since AT is a tangent to the circle, ΔOAT is right angle triangle

Now $\cos 30^\circ = \frac{AT}{OT}$ $\quad \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$$AT = OT\cos 30^\circ$$

or, $$AT = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3} \text{ cm.}$$

23. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A, find $\angle BAT$.



Ans : [Board Term-2, 2012 Set (32)]

We have $\angle ACB = 50^\circ$

Since $\angle CBA$ is angle in semi-circle.

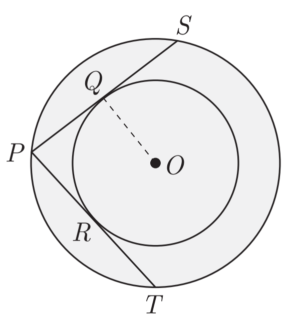
$$\angle CBA = 90^\circ$$

Now $$\angle OAB = 180^\circ - 90^\circ - 50^\circ = 40^\circ$$

$$\angle BAT = 90^\circ - \angle OAB$$

$$= 90^\circ - 40^\circ = 50^\circ$$

24. In the figure there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm find the length of PS.



Ans : [Delhi Compt. Set I, II, III 2017]

Since PQ and PR are tangent of the circle, $PQ = PR$

$$PQ = PR = 5 \text{ cm}$$

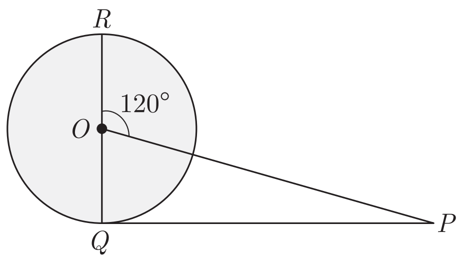
Since PS is chord of circle and point Q bisect it, thus

$$PQ = QS$$

$$PS = 2PQ$$

$$= 2 \times 5 = 10 \text{ cm}$$

25. PQ is a tangent drawn from an external point P to a circle with centre O, QOR is the diameter of the circle. If $\angle POR = 120^\circ$, What is the measure of $\angle OPQ$?



Ans : [Foreign Set I, II, III, 2017]

Since PQ is a tangent to the circle, ΔOQP is right angle triangle

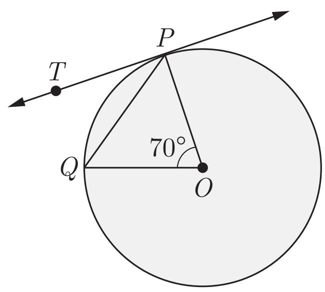
In ΔOQP $\quad \angle POR = \angle OQP + \angle OPQ$

(Exterior angle)

Thus $$\angle OPQ = \angle POR - \angle OQP$$

$$= 120^\circ - 90^\circ = 30^\circ$$

26. In figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P.



Ans : [Outside Delhi Set I, II, III 2017]

We have $\angle OPQ = \angle OQP$

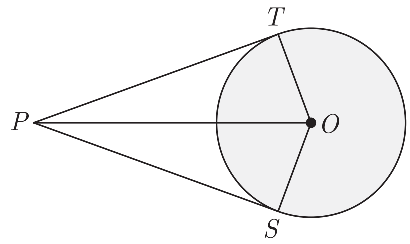
$$= \frac{180 - 70}{2} = 55^\circ$$

Thus $$TPQ = 90^\circ - 55^\circ = 35^\circ$$

SHORT ANSWER TYPE QUESTIONS - I

1. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that

$\angle SPT = 120^\circ$, Prove that $OP = 2PS$.



Ans : [Foreign Set I, II, III, 2016]

Given that $\angle SPT = 120^\circ$

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

Now in right triangle POS, we have

$$\cos 60^\circ = \frac{PS}{OP}$$

$$\frac{1}{2} = \frac{PS}{OP}$$

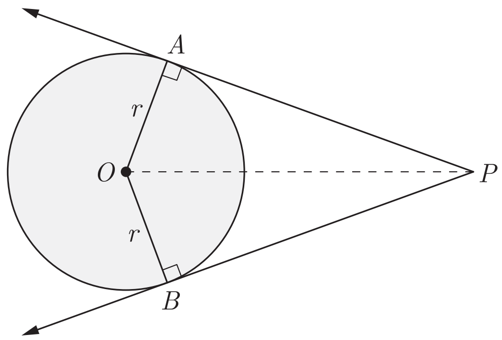
$$OP = 2PS \qquad \text{Hence proved.}$$

2. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Ans : [Outside Delhi Set, II, 2017]

Consider is circle of radius r and centre at O as shown in figure below. Here we have drwan two tangent from P at A and B. We have to prove that

$$AP = PB$$



We join OA, OB and OP.

Proof :

In ΔPAO and ΔPBO, OP is common and

$$OA = OB \qquad \text{radius of same circle}$$

Since radius is always perpendicular to tangent, at point of contact,

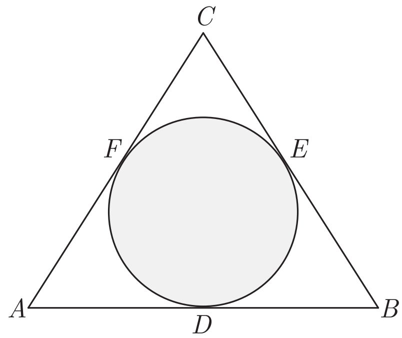
$$\angle OAP = \angle OBP = 90^\circ$$

Thus $\Delta PAO \cong \Delta PBO$.

and hence, $AP = BP$

Thus length of 2 tangents drawn from an external point to a circle are equal.

3. In the given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sodes AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



Ans : [Delhi Set I, II, III, 2016]

Since AF and AD are tangent of the circle, $AF = AD$

Let $AF = AD = x$

Now $DB = AB - AD = 12 - x$

Since BD and BE are tangent of the circle, $BD = BE$

Thus $BE = BD = 12 - x$

Now $CE = CB - BE = 8 - (12 - x)$

Since CF and CE are tangent of the circle, $CF = CE$

Thus $CF = CE = 8 - (12 - x)$ cm

But $AC = CF + FA$

Substituting values we have

$$10 = 8 - (12 - x) + x$$

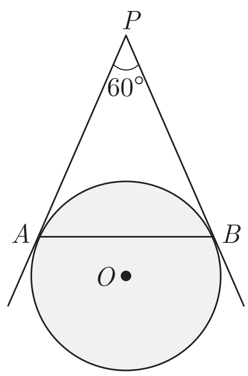
$$10 = 2x - 4$$

$$2x = 10 + 4 = 14$$

$$x = 7$$

Thus $AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm

4. In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Ans : [Delhi Set I, II, III, 2016]

Since length of 2 tangents drawn from an external point to a circle are equal, we have

$$PA = PB$$

Thus $\angle PAB = \angle PBA = 60^\circ$

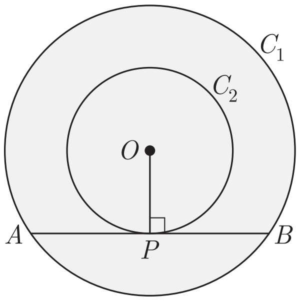
Hence ΔPAB is an equilateral triangle.

Therefore $AB = PA = 5$ cm.

5. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.

Ans : [Board Term-2, 2012 Set (17, 40)]

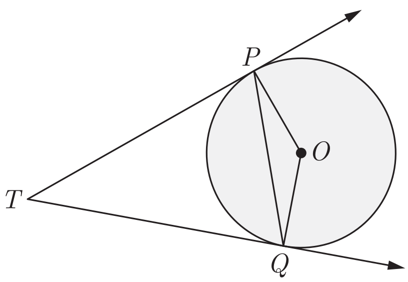
As per the given question we draw the figure as below.



Since OP is radius and APB is tangent, $OP \perp AB$.
Now for bigger circle, O is centre and AB is chord such that $OP \perp AB$.

Thus OP bisects AB.

6. In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.



Ans : [CBSE S.A.2 2016 Set HODM4OL]

We have $PQ = 6$ cm, $OP = OQ = 6$ cm

Since $PQ = OP = OQ$, triangle ΔPQO is an equilateral triangle.

Thus $\angle POQ = 60^\circ$

Now we know that $\angle POQ$ and $\angle PTQ$ are supplementary angle,

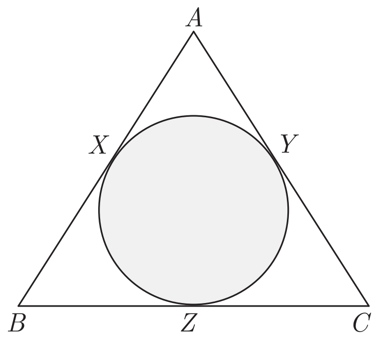
$$\angle POQ + \angle PTQ = 180^\circ$$

$$\angle PTQ = 180^\circ - \angle POQ$$

$$= 180^\circ - 60^\circ = 120^\circ$$

Thus $\angle PTQ = 120^\circ$

7. ABC is an isosceles triangle in which $AB = AC$ which is circumscribed about a circle as shown in the figure. Show that BC is bisected at the point of contact.



Ans : [Board Term-2, 2012 Set (22)]

We have $AB = AC$
Since tangents from an external point to a circle are equal,

$$AX = AY$$

$$BX = BZ$$

$$CZ = CY$$

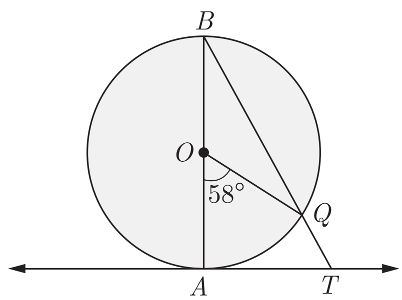
Now $AX + XB = AY + YC$

or $XB = YC$ $(AX = AY)$

or $BZ = CZ$

Thus Z is the mid-point of BC and Z is the point of contact. Hence, BC is bisected at he point of contact.

8. In given figure, AB is the diameter of a circle with center O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.



Ans : [Board Term-2, 2015 Set I, II, III]

We have $\angle AOQ = 58^\circ$
Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58^\circ = 29^\circ$$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90^\circ$

$\angle ABQ = \angle ABT$

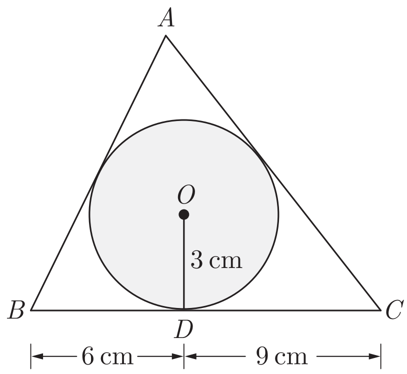
Now in ΔBAT,

$$\angle ATB = 90^\circ - \angle ABT$$

$$= 90^\circ - 29^\circ = 61^\circ$$

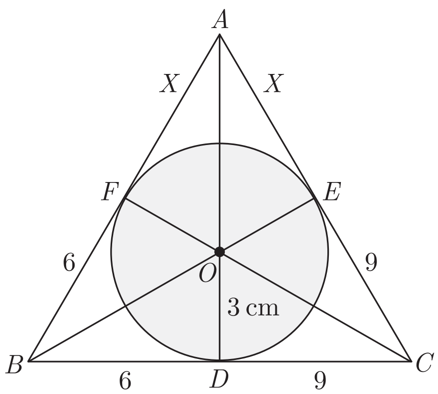
Thus $\angle ATQ = \angle ATB = 61^\circ$

9. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC.



Ans : [Outside Delhi CBSE, 2015, Set I, II, III]

We redraw the given circle as shown below.



Since tangents from an external point to a circle are equal,

$$AF = AE$$

$$BF = BD = 6 \text{ cm}$$

$$CE = CD = 9 \text{ cm}$$

Let $AF = AE = x$

Now $AB = AF + FA = 6 + x$

$$AC = AE + EC = x + 9$$

$$BC = 6 + 9 = 15 \text{ cm}$$

Perimeter of ΔABC,

$$p = 15 + 6 + x + 9 + x$$

$$= 30 + 2x$$

Now area $\Delta ABC = \frac{1}{2}rp$

Here $r = 3$ is the radius of circle. Substituting all values we have

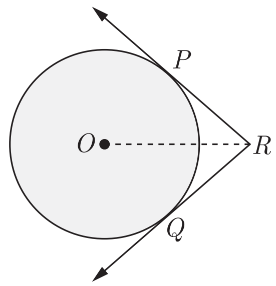
$$54 = \tfrac{1}{2} \times 3 \times (30 + 2x)$$

$$54 = 45 + 3x$$

or $x = 3$

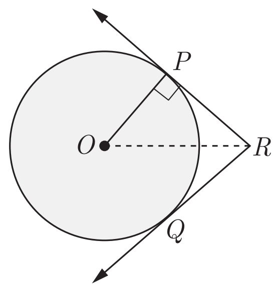
Thus $AB = 9$ cm, $AC = 12$ cm and $BC = 15$ cm.

10. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

We redraw the given figure by joining O to P as shown below.



$$\angle PRO = \tfrac{1}{2}\angle PRQ$$

$$= \frac{120^\circ}{2} = 60^\circ$$

Here ΔOPR is right angle triangle, thus

$$\angle POR = 90^\circ - \angle PRO$$

$$= 90^\circ - 60^\circ = 30^\circ$$

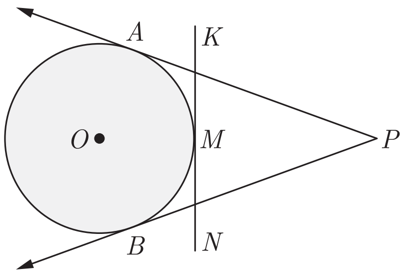
Now $$\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$$

or, $OR = 2PR = PR + PR$

Since $PR = QR$,

$$OR = PR + QR \qquad \text{Hence Proved}$$

11. PA and PB are tangents from point P to the circle with centre O as shown in figure. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN = AK + BN$



Ans : [Board Term-2, 2012 Set (28)]

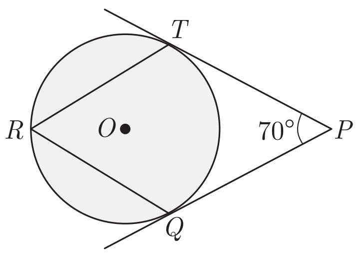
Since length of tangents from an external point to a circle are equal,

$$PA = PB, KA = KM, NB = NM,$$

$$KA + NB = KM + NM$$

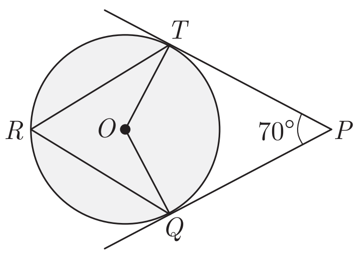
$$AK + BN = KN. \qquad \text{Hence Proved}$$

12. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ = 70°$, find $\angle TRQ$.



Ans : [Foreign Set I, II, III, 2015]

We redraw the given figure by joining O to T and Q as shown below.



Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $\angle TOQ = 180° - \angle TPQ$

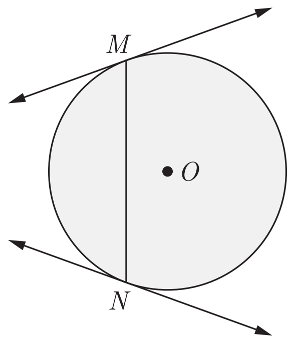
$$= 180° - 70° = 110°$$

Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle TRQ = \frac{1}{2}\angle TOQ$$

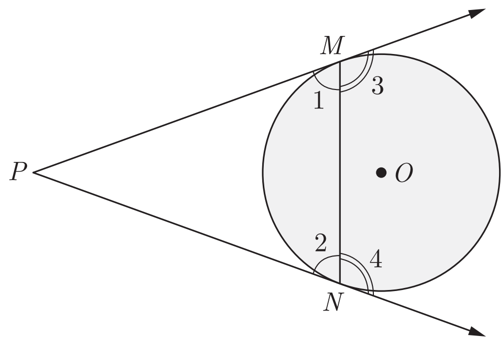
$$= \frac{1}{2} \times 110° = 55°$$

13. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Ans : [Delhi Term-2, 2015]

We redraw the given figure by joining M and N to P as shown below.



Since length of tangents from an external point to a circle are equal,

$$PM = PN$$

Since angles opposite to equal sides are equal,

$$\angle 1 = \angle 2$$

Now using property of linear pair we have

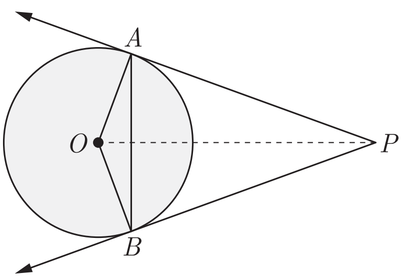
$$180° - \angle 1 = 180° - \angle 2$$

$$\angle 3 = \angle 4 \qquad \text{Hence Proved}$$

14. Two tangents PA and PB are drawn from an external point P to a circle inclined to each other at an angle of 70°, then what is the value of $\angle PAB$?

Ans : [Board Term-2, 2012 Set (26, 34)]

As per question we draw the given circle as shown below.



Here angle $\angle AOB$ and $\angle APB$ are supplementary angle.

Thus $\angle AOB = 180^\circ - \angle APB$

$= 180^\circ - 70^\circ = 110^\circ$

OA and OB are radius of circle and equal in length, thus angle $\angle OAB$ and $\angle OBA$ are also equal. Thus in triangle ΔOAB we have

$$\angle OBA + \angle OAB + \angle AOB = 180^\circ$$
$$\angle OAB + \angle OAB = 180^\circ - \angle AOB$$
$$2\angle OAB = 180^\circ - 110^\circ$$
$$\angle OAB = 35^\circ$$

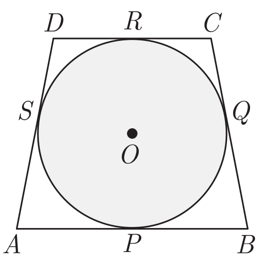
Since OA is radius and AP is tangent at A, $OA \perp AP$

$$\angle OAP = 90^o$$

Now $\angle PAB = \angle OAP - \angle OAB$

$= 90^o - 35^o = 55^o$

15. In Figure a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD, and DA touch the circle at the points P, Q, R and S respectively. Prove that. $AB + CD = BC + DA$.



Ans : [Outside Delhi Set, II, 2016]

Since length of tangents from an external point to a circle are equal,

At A, $AP = AS$ (1)

At B $BP = BQ$ (2)

At C $CR = CQ$ (3)

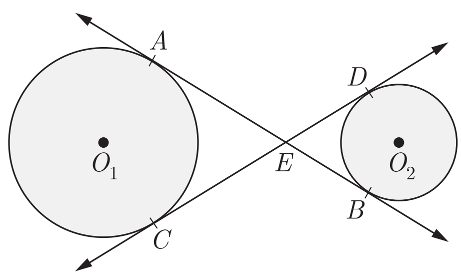
At D $DR = DS$ (4)

Adding eqn. (1), (2), (3), (4)

$$AP + BP + DR + CR = AS + DS + BQ + CQ$$
$$AP + BP + DR + RC = AS + SD + BQ + QC$$
$$AB + CD = AD + BC$$

Hence Proved

16. In Figure, common tangents AB and CD to the two circle with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.



Ans : [CBSE O.D. 2014]

Since EA and EC are tangents from point E to the circle with centre Q_1

$$EA = EC \quad ...(1)$$

and EB and ED are tangents from point E to the circle with center O_2

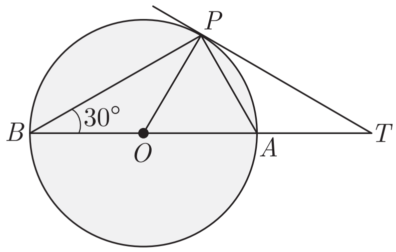
$$EB = ED \quad (2)$$

Adding eq (1) and (2) we have

$$EA + BE = CE + ED$$
$$AB = CD$$

Hence Proved

17. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^o$, what is the measure of $\angle PTA$?



Ans : [Board Term-2, 2012 Set (21)]

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^o$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^o$

Now $\angle POA = \angle OBP + \angle OPB$

$= 30^\circ + 30^\circ = 60^\circ$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^o$$

Now in right angle ΔOPT,

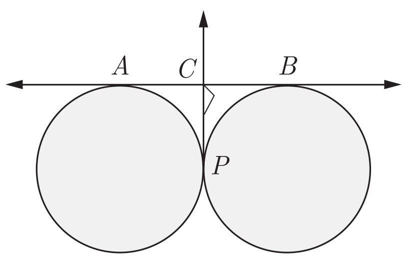
$$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$$

Substituting $\angle OPT = 90^o$ and $\angle POT = 60^\circ$ we have

$$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$$
$$= 180^\circ - 150^\circ = 30^o$$

Thus $\angle PTA = \angle PTO = 30^o$

18. In the given figure, if $BC = 4.5$ cm, find the length of AB.



Ans : [Board Term-2, 2012 Set (59)]

Since length of tangents from an external point to a circle are equal,

$$CB = CP = 4.5 \text{ cm}$$

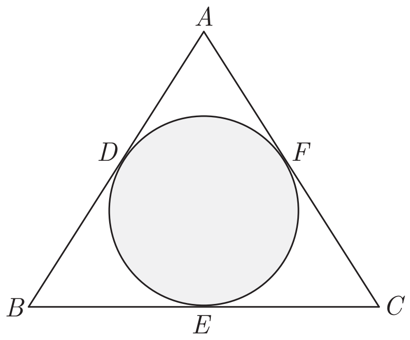
and $\quad CA = CP$

Now $\quad AB = AC + CB$

$$= CP + CP = 2CP$$

$$= 2 \times 4.5 = 9 \text{ cm}$$

19. In the given figure, if $AB = AC$, prove that $BE = CE$.



Ans : [Outside Delhi Set I, II, III 2017]

Since tangents from an external point to a circle are equal,

$$AD = AF \quad (1)$$

$$BD = BE \quad (2)$$

$$CE = CF \quad (3)$$

From $AB = AC$ we have

$$AD + DB = AF + FC$$

or $\quad DB = FC \quad (AD = AF)$

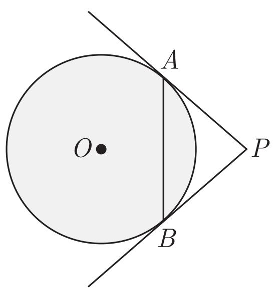
From eq (2) and (3) we have

$BE = EC$ Hence Proved

20. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Ans : [Outside Delhi Set I, II, III 2017]

As per question we draw figure shown below.



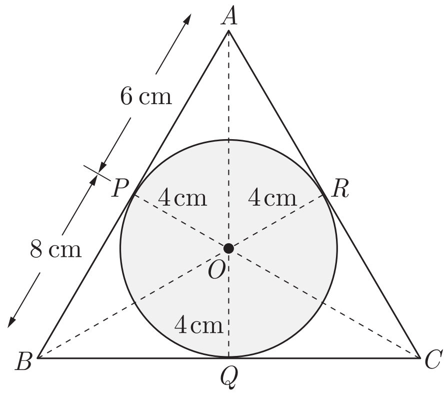
Since length of tangents from an external point to a circle are equal,

$$PA = PB$$

Since angles opposite to equal sides are equal,

$$\angle PAB = \angle PBA$$

21. In Figure the radius of incircle of ΔABC of area 84 cm^2 and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



Ans : [Outside Delhi Compt. Set I, II, III 2017]

Since length of tangents from an external point to a circle are equal,

At A, $\quad AP = AR = 6$ cm (1)

At B $\quad BP = BQ = 8$ cm (2)

At C $\quad CR = CQ = x$ (3)

Perimeter of ΔABC,

$$p = AP + PB + BQ + QC + CR + RA$$

$$= 6 + 8 + 8 + x + x + 6$$

$$= 28 + 2x$$

Now area $\quad \Delta ABC = \frac{1}{2} rp$

Here $r = 4$ is the radius of circle. Substituting all values we have

$$84 = \frac{1}{2} \times 4 \times (28 + 2x)$$

$$84 = 56 + 4x$$

$$21 = 14 + x$$

or $\quad x = 7$

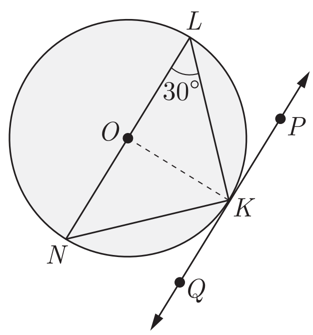
Thus $\quad AC = AR + RC$

$$= 6 + 7 = 13$$

$$BC = BQ + QC$$

$$= 8 + 7 = 15 \text{ cm}$$

22. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30°$, find $\angle PKL$.



Ans : [Outside Delhi Cmpt. Set I, II, III 2017]

Since OK and OL are radius of circle, thus

$$OK = OL$$

Angles opposite to equal sides are equal,

$$\angle OKL = \angle OLK = 30^o$$

Tangent is perpendicular to the end point of radius,

$$\angle OKP = 90^o \qquad \text{(Tangent)}$$

Now $\quad \angle PKL = \angle OKP - \angle OKL$

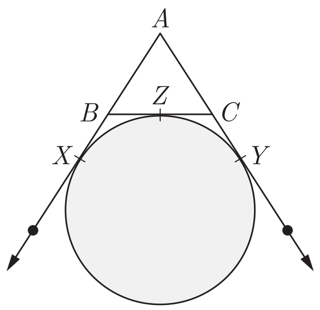
$$= 90^o - 30^o = 60^o$$

SHORT ANSWER TYPE QUESTIONS - II

1. ABC is a triangle. A circle touches sides AB and AC produced and side BC at BC at X, X, Y and Z respectively. Show that $AX = \frac{1}{2}$ perimeter of ΔABC.

Ans : [Board Term-2, 2016, Set-HODM4OL]

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $\quad AX = AY \qquad (1)$

At B $\quad BX = BZ$ cm $\qquad (2)$

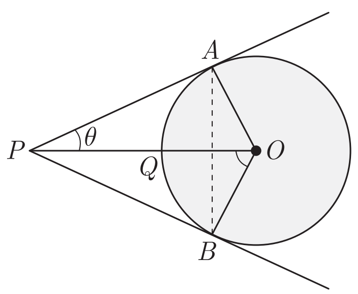
At C $\quad CY = CZ \qquad (3)$

Perimeter of ΔABC,

$$p = AB + AC + BC$$
$$= (AX - BX) + (AY - CY) + BZ + ZC)$$
$$= AX + AY - BX + BZ + ZC - CY$$
$$= AX + AY = 2AX$$

Thus $\quad AX = \frac{1}{2}p \qquad$ Hence Proved

2. In the given figure, OP is equal to the diameter of a circle with center O and PA and PB are tangents. Prove that ABP is an equilateral triangle.



Ans : [Board Term-2, 2014]

We redraw the given figure by joining A to B as shown below.

Since OA is radius and PA is tangent at A, $OA \perp AP$. Now in right angle triangle ΔOAP, OP is equal to diameter of circle, thus

$$OP = 2OA$$

$$\frac{OA}{OP} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Since PO bisect the angle $\angle APB$,

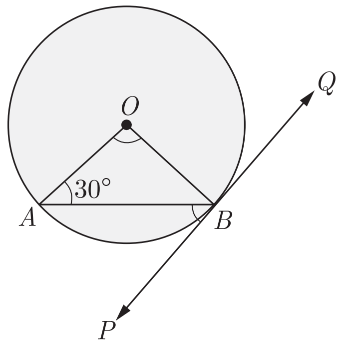
Hence, $\quad \angle APB = 2 \times 30^\circ = 60^o$

Now, in ΔAPB,

$$AP = AB$$
$$\angle PAB = \angle PBA$$
$$= \frac{180^\circ - 60^\circ}{2} = 60^o$$

Thus ΔAPB is an equilateral triangle.

3. In the figure, PQ is a tangent to a circle with center O. If $\angle OAB = 30^o$, find $\angle ABP$ and $\angle AOB$.



Ans : [Board Term-2 2014]

Here OB is radius and QT is tangent at B, $OB \perp PQ$

$$\angle OBP = 90^o$$

Since the tangent is perpendicular to the end point of radius,

Here OA and OB are radius of circle and equal. Since angles opposite to equal sides are equal,

$$\angle OAB = \angle OBA = 30^o$$

Now $\quad \angle AOB = 180^o - (30^o + 30^o)$

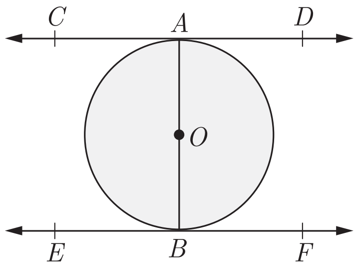
$$= 120^o$$

$$\angle ABP = \angle OBP - \angle OBA$$
$$= 90^o - 30^o = 60^o$$

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Ans : [Foreign Set I, II, III, Delhi CBSE, Term-2, 2014]
[Board Term-2, 2012 Set (12)]
[Delhi Set I, II, III 2017]

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^o$ and $\angle ABF = 90^o$

Hence $\angle CAB = \angle ABF$

and $\angle ABE = \angle BAD$

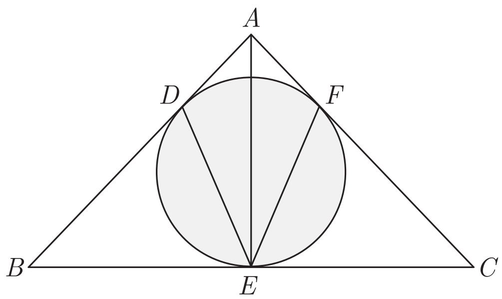
Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

$CD || EF$ Hence Proved

5. In $\Delta ABD, AB = AC$. If the interior circle of ΔABC touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC.

Ans : [Board Term-2, 2014 Delhi Set, 2012 Set (40)]

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AF = AD$ (1)

At B $BE = BD$ (2)

At C $CE = CF$ (3)

Now we have $AB = AC$

$AD + DB = AF + FC$

$BD = FC$ $(AD = AF)$

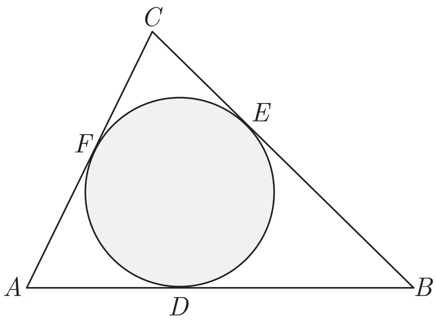
$BE = EC$ $(BD = BE, CE = CF)$

Thus E bisects BC.

6. A circle is inscribed in a ΔABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF.

Ans : [Board Term-2, 2012 set (21); Delhi 2013]

As per question we draw figure shown below.



We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$12 = 10 - x + 8 - x$

$2x = 18 - 12 = 6$

or $x = 3$

Now $AD = 3$ cm,

$BE = 10 - 3 = 7$ cm

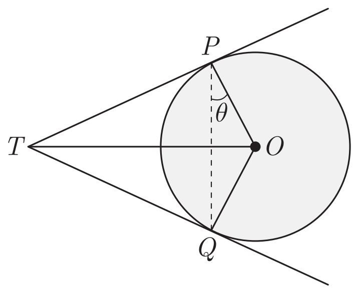
and $CF = 8 - 3 = 5$

7. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that

$$\angle PTO = \angle OPQ$$

Ans : [Delhi Set I, II, III, 2017]
[Delhi Compt Setm I, II, III, 2017]

As per question we draw figure shown below.



Let $\angle TPQ$ be θ. the tangent is perpendicular to the end point of radius,

$\angle TPO = 90^o$

Now $\angle TPQ = \angle TPO - \theta$

$= (90^o - \theta)$

Since, $TP = TQ$ and opposite angels of equal sides are always equal, we have

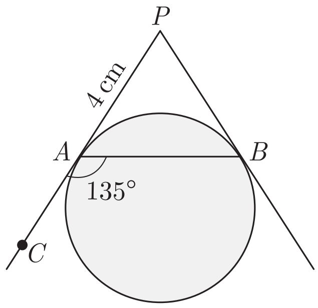
$$\angle TQP = (90^o - \theta)$$

Now in ΔTPQ we have

$$\angle TPQ + \angle TQP + \angle PTQ = 180^o$$
$$90^o - \theta + 90^o - \theta + \angle PTQ = 180^o$$
$$\angle PTQ = 180^o - 180^o + 2\theta = 2\theta$$

Hence $\angle PTQ = 2\angle OPQ$.

8. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4cm$ and $\angle BAC = 135^o$. Find the length of chord AB.



Ans : [Outside Delhi Set I, II, III, 2017]

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

Inb triangle ΔAPB we have

$$\angle APB = 180^\circ - \angle ABP - \angle BAP$$
$$= 180^\circ - 45^\circ - 45^\circ = 90^\circ$$

Thus ΔAPB is a isosceles right angled triangle

Now $$AB^2 = AP^2 + BP^2 = 2AP^2$$
$$= 2 \times 4^2 = 32$$

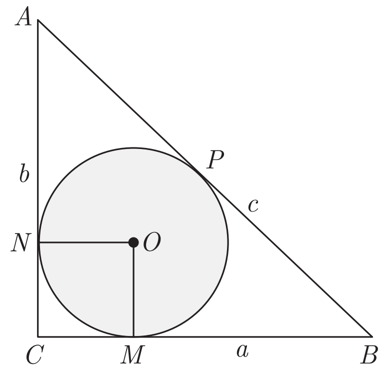
Hence $$AB = \sqrt{32} = 4\sqrt{2} \text{ cm}$$

LONG ANSWER TYPE QUESTIONS

1. a, b and c are the sides of a right triangle, where c is the hypotenuse. A circle, of radius r, touches the sides of the triangle. Prove that $r = \frac{a+b-c}{2}$.

Ans : [CBSE S.A.2 2016 Set HODM4OL]

As per question we draw figure shown below.



Let the circle touches CB at M, CA at N and AB at P.

Now $OM \perp CB$ and $ON \perp AC$ because radius is always perpendicular to tangent

OM and ON are radius of circle, thus

$$OM = ON$$

CM and CN are tangent from C, thus

$$CM = CN$$

Therefore $OMCN$ is a square. Let

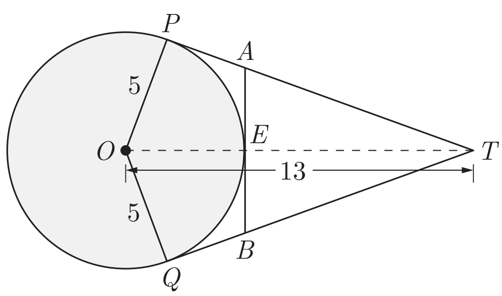
Let $OM = r = CM = CN = ON$

Since length of tangents from an external point to a circle are equal,

$$AN = AP, CN = CM, BM = BP$$
$$AN = AP$$
$$AC - CN = AB - BP$$
$$b - r = c - BM$$
$$b - r = c - (a - r)$$
$$b - r = c - a + r$$
$$2r = a + b - c$$
$$r = \frac{a+b-c}{2}$$

Hence Proved.

2. In figure O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Ans : [Delhi Set I, II, III, 2016]

Here ΔOPT is right angled triangle because PT is tangent on radius OP.

Thus $PT = \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE$

$= 13 - 5 = 8$ cm

Since length of tangents from an external point to a circle are equal,

Let $PA = AE = x$

Here ΔAET is right angled triangle because AB is tangent on radius OE.

In ΔAET, $TA^2 = TE^2 + EA^2$

$(TP - PA)^2 = 8^2 + x^2$

$(12 - x)^2 = 64 + x^2$

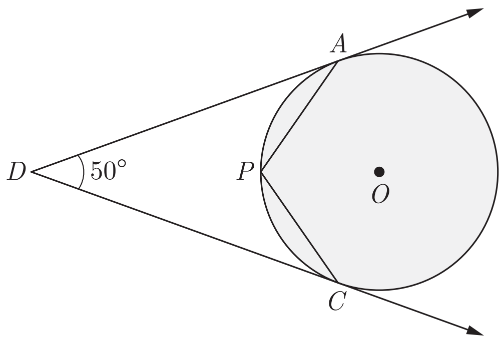
$144 - 24x + x^2 = 64 + x^2$

$24x = 144 - 64 = 80$

or, $x = 3.3$ cm.

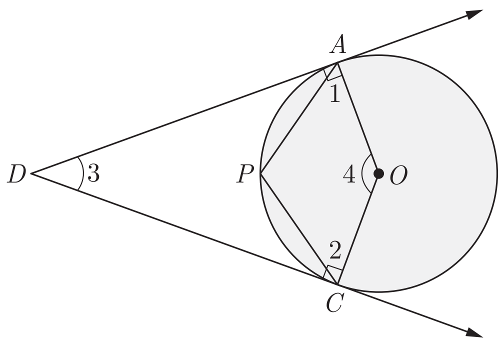
Thus $AB = 2 \times x = 2 \times 3.3 = 6.6$ cm.

3. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50°$.



Ans : [Board Term-2, 2015]

We redraw the given figure by joining A and C to O as shown below.



Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$\angle DAO = \angle DCO = 90°$

and

$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360°$

$50° + 90° + 90° + \angle AOC = 360°$

$230° + \angle AOC = 360°$

$\angle AOC = 360° - 230° = 130°$

Now Reflex $\angle AOC = 360° - 130° = 230°$

$\angle APC = \frac{1}{2}$ reflex $\angle AOC$

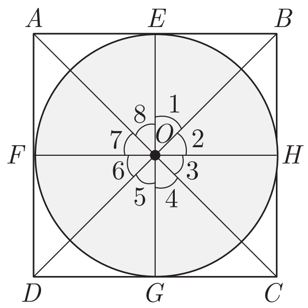
(angle subtended at centre...)

$\angle APC = \frac{1}{2} \times 230° = 115°$

4. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans : [Foreign Set I, II, III 2017][CBSE O. D. 2014]

A circle centre O is inscribed in a quadrilateral $ABCD$ as shown in figure given below.



Since OE and OF are radius of circle

$OE = OF$ (radii of circle)

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle OEA = \angle OFA = 90°$

Now in ΔAEO and ΔAFO

$OE = OF$

$\angle OEA = \angle OFA = 90°$

$OA = OA$ (Common side)

Thus $\Delta AEO \cong \Delta AFO$ (SAS congruency)

$\angle 7 = \angle 8$

Similarly, $\angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$

Since angle around a point is $360°$,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360°$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

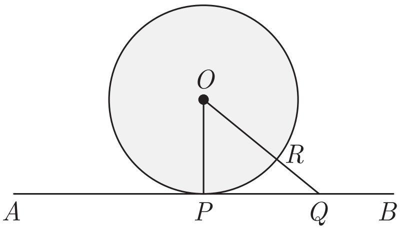
$\angle AOB + \angle COD = 180°$

Hence Proved.

5. Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

Ans : [Outside Delhi Set II, 2016]

Consider a circle with centre O with tangent AB at point of contact P as shown in figure below



Let Q be point on AB and we join OQ. Suppose it touch the circle at R.

We $OP = OR$ (Radius)

Clearly $OQ > OR$

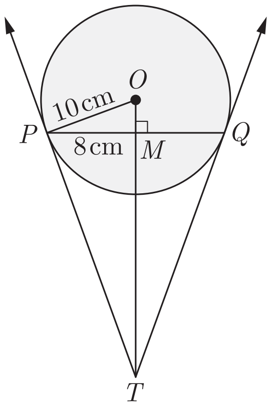
$OQ > OP$

Same will be the case with all other points on circle. Hence OP is the smallest line that connect AB and smallest line is perpendicular.

Thus $OP \perp AB$

or, $OP \perp PQ$ Hence Proved

6. In figure, PQ, is a chord of length 16 cm, of a circle of radius 10 cm. the tangents at P and Q intersect at a point T. Find the length of TP.



Ans : [Delhi CBSE, Term-2, 2014]

Here PQ is chord of circle and OM will be perpendicular on it and it bisect PQ. Thus $\Delta\, OMP$ is a right angled triangle.

We have $OP = 10$ cm (Radius)

$PM = 8$ cm $(PQ = 16$ cm)

Now in $\Delta\, OMP, OM = \sqrt{10^2 - 8^2}$

$= \sqrt{100 - 64} = \sqrt{36}$

$= 6$ cm

Now $\angle TPM + \angle MPO = 90^\circ$

Also, $\angle TPM + \angle PTM = 90^\circ$

$\angle MPO = \angle PTM$

$\angle TMP = \angle OMP = 90^\circ$

$\Delta\, TMP \sim \Delta\, PMO (AA)$

or, $\frac{TP}{PO} = \frac{MP}{MO}$

$\frac{TP}{10} = \frac{8}{6}$

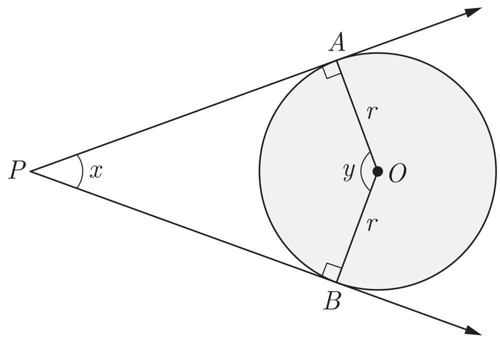
$TP = \frac{80}{6} = \frac{40}{3}$

Hence length of TP is $\frac{40}{3}$ cm.

7. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove than opposite angles are supplementary.

Ans : [Board Term-2, 2011 (B1]

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle A = \angle B = 90^\circ$

Since, $AOBP$ is a quadrilateral,

So, $\angle A + \angle B + x + y = 360^\circ$

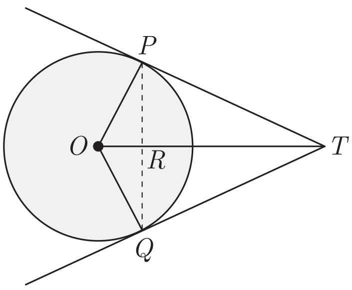
$90^\circ + 90^\circ + x + y = 360^\circ$

$180 + x + y = 360^\circ$

$x + y = 180^\circ$

Therefore opposite angle are supplementary.

8. In figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents drawn at P and Q intersect at T. Find the length of TP.



Ans : [Outside Delhi Compt. Set I, II, III 2017]

Since length of tangents from an external point to a circle are equal,

$PT = QT$

Thus ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTQ$.

Thus $OT \perp PQ$ and OT also bisects PQ.

Thus $PR = PQ = \frac{8}{2} = 4$ cm

Since $\Delta\, OPR$ is right angled isosceles triangle,

$OR = \sqrt{OP^2 - PR^2}$

$$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$$
$$= 3 \text{ cm}$$

Now, Let $TP = x$ and $TR = y$ then we have

$$x^2 = y^2 + 16 \quad (1)$$

Also in ΔOPT,

$$x^2 + (5)^2 = (y+3)^2 \quad (2)$$

Solving (1) and (2) we get

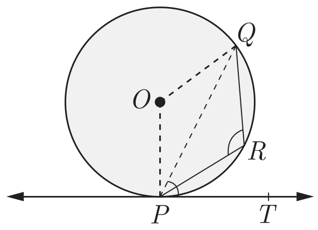
$$y = \frac{16}{3} \text{ and } x = \frac{20}{3}$$

Hence, $TP = \frac{20}{3}$

HOTS QUESTIONS

1. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.

Ans : [Outside Delhi CBSE Board, 2015 Set I, II, III 2017]



We have $\angle QPT = 60°$

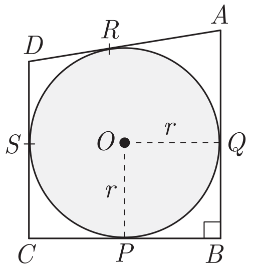
Here $\angle OPT = 90°$ because of tangent at radius.

Now
$$\angle OPQ = \angle OQP$$
$$= \angle OPT - \angle QTP$$
$$= 90° - 60° = 30°$$
$$\angle POQ = 180° - (\angle OPQ + \angle OQP)$$
$$= 180° - (30° + 30°)$$
$$= 180° - 60° = 120°$$

Now Reflex $\angle POQ = 360° - 120° = 240°$

$$\angle PRQ = \frac{1}{2} \text{ Reflex } \angle POQ$$
$$= \frac{1}{2} \times 240° = 120°$$

2. In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90°$ and $DS = 5$ cm, then find the radius of the circle (in cm).



Ans : [Board Term-2, 2013]

Since length of tangents from an external point to a circle are equal,

$$DR = DS = 5 \text{ cm}$$
$$AR = AQ$$
$$BQ = BP$$

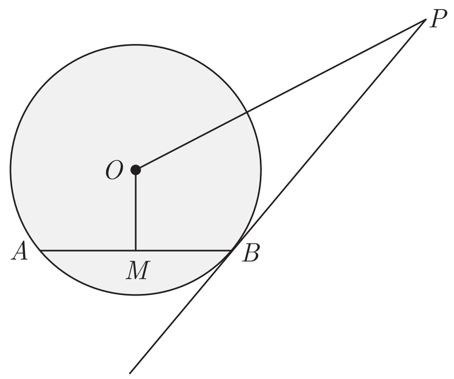
Now
$$AR = AD - DR$$
$$= 23 - 5 = 18 \text{ cm}$$
$$AQ = AQ = 18 \text{ cm}$$
$$QB = AB - AQ$$
$$= 29 - 18 = 11 \text{ cm}$$
$$PB = QB = 11$$

Now $\angle OQB = \angle OPB = 90°$ because radius is always perpendicular to tangent.

Thus $OP = OQ = PB = BQ$

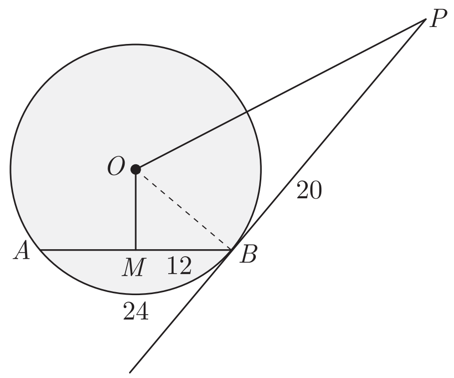
So, $POQB$ is a square. Hence, $r = OP = PB = 11$ cm

3. PB is a tangent to the circle with centre O to $B.AB$ is a chord of length 24 cm at a distance of 5 cm from the centre. It the tangent is length 20 cm, find the length of PO.



Ans : [Delhi Board Term-2, 2015]

We redraw the given figure by joining O to B as shown below.



Here ΔOMB right angled triangle because AB is chord and OM is perpendicular on it.

In right angled triangle ΔOMB we have,

$$OB^2 = OM^2 + MB^2$$
$$= 5^2 + 12^2 = 13^2$$

Thus $OB = 13$

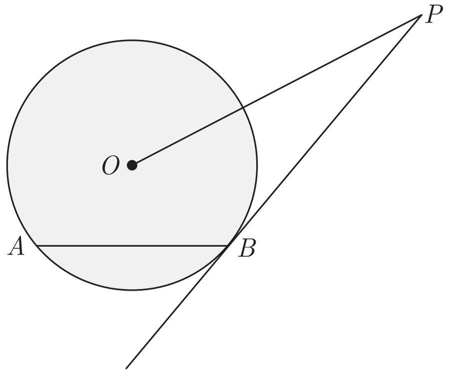
Here ΔOBP right angled triangle because PB is tangent on radius OB.

This in right angled triangle ΔOBP we have,

$$OP^2 = OB^2 + BP^2$$
$$= 13^2 + 20^2$$
$$= 569$$

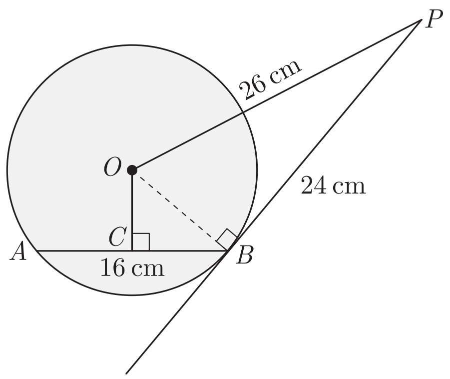
Thus $$OP = \sqrt{569} = 23.85 \text{ cm}$$

4. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distanace of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



Ans : [Board Term-2, 2012 Set (40, 2014)]

We redraw the given figure by joining O to B as shown below.



Here we have drawn perpendicular OC on chord AB. Thus Triangle ΔOCB is also right angled triangle, We have $PB = 24$ cm, $OP = 26$ cm.
Triangle ΔOPB is right angled triangle because PB is tangent at radius OB and $\angle OPB = 90°$.
In right angled ΔOPB, we have

$$OB = \sqrt{OP^2 - BP^2}$$
$$= \sqrt{26^2 - 24^2}$$
$$= \sqrt{676 - 576} = \sqrt{100}$$
$$= 10 \text{ cm}$$

Since perpendicular drawn from the centre to a chord bisect it, we have

$$BC = \frac{1}{2}AB = \frac{16}{2} = 8 \text{ cm}$$

Now in ΔOBC, $OC^2 = OB^2 - BC^2$

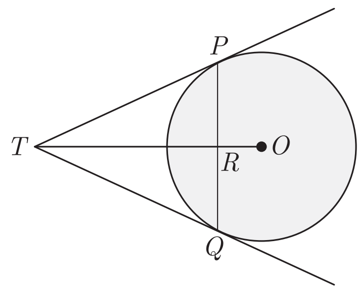
$$= 10^2 - 8^2 = 36$$
$$OC = 6 \text{ cm}$$

Thus distance of the chord from the centre is 6 cm.

5. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the rigth bisector of line segment PQ.

Ans : [Delhi CBSE Term-2, 2015 Set I, II, III]

A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle as shown in figure below.



Since length of tangents from an external point to a circle are equal,

$$TP = TQ$$

Angle $\angle TPR$ and $\angle TQR$ are opposite angle of equal sides, thus

$$\angle TPR = \angle TQR$$

Now in ΔPTR and ΔQTR

$$TP = TQ$$
$$TR = TR \quad \text{(Common)}$$
$$\angle TPR = \angle TQR$$

Thus $\Delta PTR \cong \Delta QTR$

and $PR = QR$

and $\angle PRT = \angle QRT$

But $\angle PRT + QRT = 180°$ as PQ is line segment,

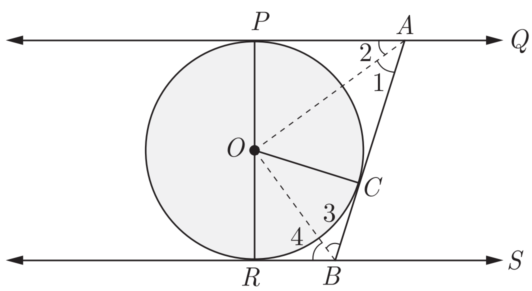
$$\angle PRT = \angle QRT = 90°$$

Therefore TR or OT is the right bisector of line segment PQ. Hence proved.

6. Prove that the intercept of a tangent between a pair of parallel tangents to a circle subtend a right angle at the centre of the circle.

Ans : [Delhi CBSE, Term-2, 2014]
[Board Term-2, 2012 Set (22, 5)]

As per question we draw figure shown below.



Here PQ and RS are two parallel tangents to a circle with centre O.

AB is tangent to a circle at C, intersecting PQ and RS at A and B respectively.

Since $PA \,||\, RS$ and AB is transversal,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

By congruency $\angle 1 = \angle 2, \angle 3 = \angle 4$, thus we have

$$2\angle 1 + 2\angle 3 = 180^\circ$$

$$\angle 1 + \angle 3 = 90^\circ$$

In ΔAOB, by angle sum property of a triangle,

$$\angle AOB = 180^\circ - (\angle 1 + \angle 3)$$

$$= 180^\circ - 90^\circ$$

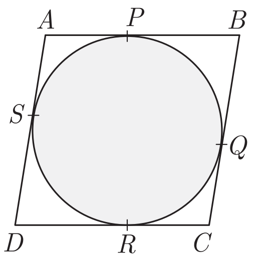
Thus $\angle AOB = 90^\circ$

7. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans : [Delhi CBSE, Term-2, 2014]
[Board Term-2, 2012 Set (1); Delhi 2013]

Let $ABCD$ be the parallelogram.

$$AB = CD, AD = BC \qquad (1)$$



Since length of tangents from an external point to a circle are equal,

At A, $AP = AS$ (2)

At B $BP = BQ$ (3)

At C $CR = CQ$ (4)

At D $DR = DS$ (5)

Adding above 4 equation we have

$$AP + PB + CR + DR = AS + BQ + CQ + DS$$

or, $AB + CD = AD + BC$

From (1) $2AB = 2AD$

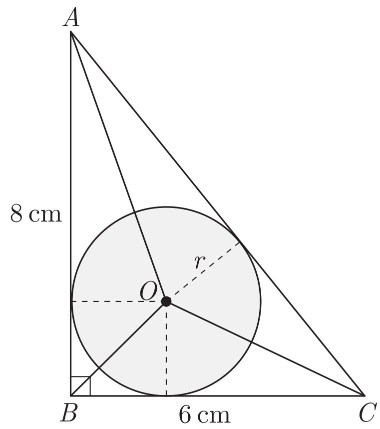
or $AB = AD$

Thus $ABCD$ is a rhombus.

8. ABC is a right triangle in which $\angle B = 90^\circ$. A circle is inscribed in the triangle. It $AB = 8$ cm and $BC = 6$ cm, find the radius r of the circle.

Ans : [Board Term II, 2012 Set (44)]

As per question we draw figure shown below.



Area of triangle ΔABC,

$$\Delta ABC = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

and $AC = \sqrt{8^2 + 6^2} = 10$ cm

Here we have joined AO, BO and CO

For area of triangle we have

$$\Delta ABC = \Delta OBC + \Delta OCA + \Delta OAB$$

$$24 = \tfrac{1}{2}rBC + \tfrac{1}{2}rAC + \tfrac{1}{2}rAB$$

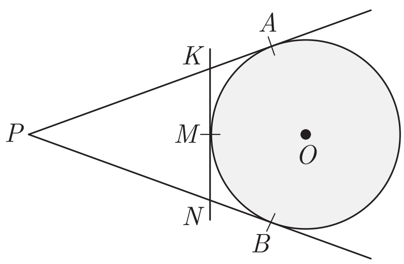
$$= \tfrac{1}{2}r(BC + AC + AB)$$

$$= \tfrac{1}{2}r(6 + 10 + 8) = 12r$$

or $12r = 24$

Thus $r = 2$ cm.

9. In given figure, PA and PB are tangents from a point P to the circle wisth centre O. At the point M, nother tangent to the circle is drawn cutting PA and PB aat K and N. Prove that the perimeter of $\Delta PNK = 2PB$.



Ans : [Board Term-2, 2012 Set (1, 25)]

Since length of tangents from an external point to a circle are equal,

$$PA = PB$$

$$KM = KA$$

$$MN = BN$$

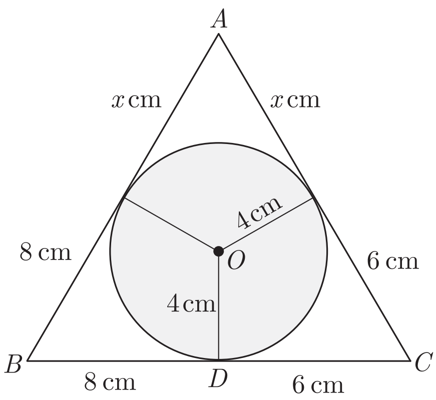
Now $KN = KM + MN$

$$= KA + BN$$

Now perimeter of ΔPNK

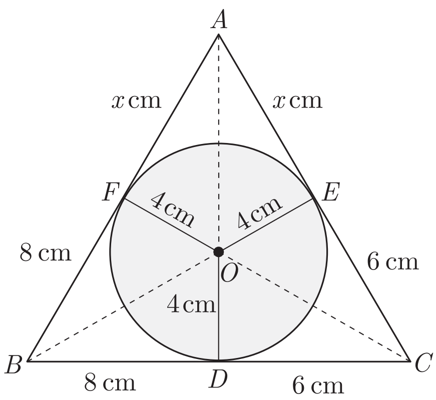
$$\begin{aligned} p &= PN + KN + PK \\ &= PN + BN + KA + PK \\ &= PB + PA \\ &= 2PB \qquad (PA = PB) \end{aligned}$$

10. In the figure, the ΔABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find AB and AC.



Ans : [Board Term-2, 2012(34); Delhi CBSE Term II, 2014]

We redraw the given circle by joining AO, BO and CO shown in figure below. Let length of AF be x.



Since length of tangents from an external point to a circle are equal,

At A, $\quad AF = AE = x \qquad (2)$

At B $\quad BF = BD = 8$ cm $\qquad (3)$

At C $\quad CD = CE = 6$ cm $\qquad (4)$

Now $\quad AB = x + 8$

$AC = x + 6$

$BC = 8 + 6 = 14$ cm

Perimeter of circle

$$\begin{aligned} p &= AB + BC + CA \\ &= x + 8 + 14 + x + 6 \\ &= 2(x + 14) \end{aligned}$$

Semi-perimeter of circle

$$s = \tfrac{1}{2}p = x + 14$$

Area or triangle ΔABC

$$\begin{aligned} \Delta ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{48x^2 + 672x} \qquad (1) \end{aligned}$$

Area or triangle ΔABC

$$\begin{aligned} \Delta ABC &= \tfrac{1}{2} rp \\ &= \tfrac{1}{2} \times 4 \times 2(x + 14) \\ &= 4(x + 14) \qquad (2) \end{aligned}$$

From equation (1) and (2) we have

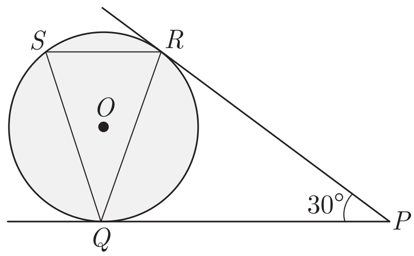
$$\begin{aligned} 48x^2 + 672x &= 16(x + 14)^2 \\ 48x(x + 14) &= 16(x + 14)^2 \\ 3x &= x + 14 \end{aligned}$$

or, $\quad x = 7$

Thus $\quad AC = 6 + 7 = 13$ cm

and $\quad AB \; 8 + 7 = 15$ cm.

11. In the figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



Ans : [Delhi CBSE Term-2, 2015, (Set I, II, III)]

Since length of tangents from an external point to a circle are equal,

$$PR = PQ$$

Now $\quad \angle PRQ = \angle PQR = \dfrac{180° - 30°}{2}$

$$= \frac{150°}{2} = 75°$$

Since $SR \parallel QP$, $\angle SRQ$ and $\angle RQP$ are alternate angle

$$\angle SRQ = \angle RQP = 75°$$

Thus $\quad SQ = RQ$

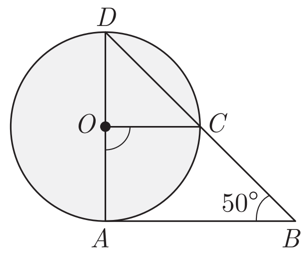
and $\quad \angle RSQ = \angle SRQ = 75°$

In triangle ΔAQR

$$\begin{aligned} \angle SQR + \angle QSR + \angle QRS &= 180° \\ \angle SQR + 75° + 75° &= 180° \\ \angle SQR &= 180° - 150° = 30° \end{aligned}$$

Thus $\angle SQR = 30°$.

12. In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and $\angle ABC = 50°$. Find $\angle AOC$.



Ans : [Board Term-2, 2015]

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Therefore $\angle A = 90°$

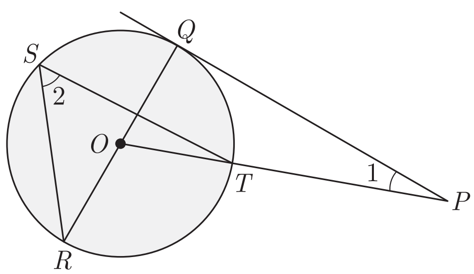
Now in ΔDAB we have

$$\angle D + \angle A + \angle B = 180°$$
$$\angle D + 90° + 50° = 180°$$
$$\angle D = 40°$$

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle AOC = 2\angle ADC = 2\angle D$$
$$= 2 \times 40° = 80°$$

13. In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. It $\angle POR = 130°$ and S is a point on the circle, find $\angle 1 + \angle 2$.



Ans : [Delhi Compt. Set I, II, III 2017]

Here $\angle OQP = 90°$ because radius is always perpendicular to tangent at point of contact.
Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

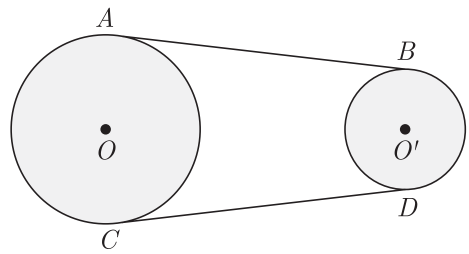
$$\angle 2 = \frac{1}{2}\angle TOR = \frac{1}{2}\angle POR$$
$$= \frac{1}{2} \times 130° = 65°$$

Now $\angle POQ = 180° - 130° = 50°$

$$\angle 1 = 180° - \angle OQP - \angle POQ$$
$$= 180° - 90° - 50° = 40°$$

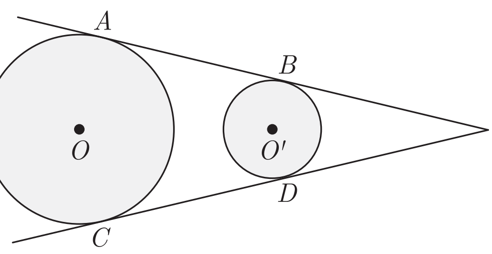
Now $\angle 2 + \angle 1 = 65° + 40° = 105°$

14. In the figure AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



Ans : [Delhi Compt. Set III 2017]

We redraw the given figure by extending AB and BD which intersect at P as shown in figure below



Since length of tangents from an external point to a circle are equal,

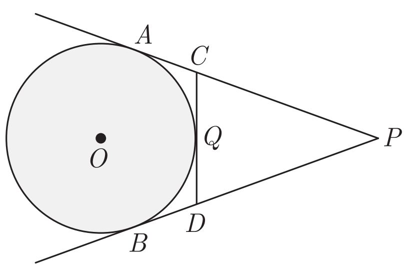
$$PA = PC$$

and $PB = PD$

Now, $PA - PB = PC - PD$

$AB = CD$ Hence Proved

15. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PI$.



Ans : [Delhi Compt. Set I, II, III 2017]

Since length of tangents from an external point to a circle are equal,

$$CA = CQ = 3 \text{ cm}$$
$$DQ = DB = 3 \text{ cm}$$

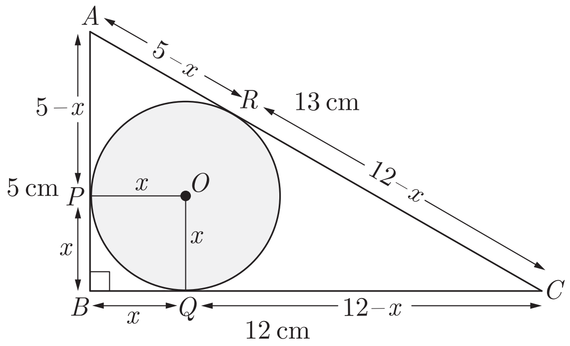
and $PB = PA = 12$ cm

$$PA + PB = PC + CA + PD + DA$$
$$PC + PD = PA - CA + PB - DB$$
$$= 12 - 3 + 12 - 3 = 18 \text{ cm}$$

16. In a right angle $\Delta ABC.$, $BC = 12\,cm$ and $AB = 5\,cm$. Find the radius of the circle inscribed in this triangle.

Ans : [Delhi CBSE Term-2, 2014]

Let the radius of circle be x. As per given in question we draw the figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AP = AR = 5 - x$ (1)

At B $BP = BQ = x$ (2)

At C $CR = CQ = 12 - x$ (3)

Here, $AB = 5$ cm, $BC = 12$ cm and $\Delta B = 90^{\circ}$

Now $AC = \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169} = 13$ cm

Now $AC = AR + RC$

$13 = 5 - x + 12 - x$

$2x = 17 - 13 = 4$

$x = \frac{4}{2} = 2$ cm

Hence, radius of the circle is 2 cm.