Introduction of Trignometry

VERY SHORT ANSWER TYPE QUESTIONS

1. In a triangle ABC, write $\cos\left(\frac{B+C}{2}\right)$ in terms of angle A.

Ans : [Board Term-1, 2016, Set-OYPG7]

In a triangle $A+B+C = 180^\circ$

or, $B+C = 180^\circ - A$

Thus $\cos\left(\frac{B+C}{2}\right) = \cos\left[\frac{180^\circ - A}{2}\right]$

$$= \cos\left[90 - \frac{A}{2}\right]$$

$$= \sin\frac{A}{2}$$

2. If $\sec\theta.\sin\theta = 0$, then find the value of θ.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

We have $\sec\theta.\sin\theta = 0$

$$\frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = 0 = \tan 0^\circ$$

Thus $\theta = 0^\circ$

3. If $A+B = 90^\circ$ and $\sec A = \frac{2}{3}$, then find the value of $\operatorname{cosec} B$.

Ans : [Board Term-1, 2016, Set-ORDAWEZ]

We have $A+B = 90^\circ$

and $\sec A = \frac{5}{3}$

or, $\sec(90^\circ - B) = \frac{5}{3}$.

Thus $\operatorname{cosec} B = \frac{5}{3}$

4. If $\tan 2A = \cot(A+60^\circ)$, find the value of A where $2A$ is an acute angle.

Ans : [Board Term-1, 2016, Set-LGRKRO]

We have $\tan 2A = \cot(A+60^\circ)$

$$\cot(90-2A) = \cot(A+60^\circ)$$

$$90 - 2A = A + 60^\circ$$

$$3A = 30^\circ$$

Thus $A = 10^\circ$

5. Find the value of $\frac{\sin 25^\circ}{\cos 65^\circ} + \frac{\tan 23^\circ}{\cot 67^\circ}$

Ans : [Board Term-1, 2015, Set-FHN8MGD]

$$\frac{\sin 25^\circ}{\cos 65^\circ} + \frac{\tan 23^\circ}{\cot 67^\circ} = \frac{\sin 25^\circ}{\sin(90^\circ - 65^\circ)} + \frac{\tan 23^\circ}{\tan(90^\circ - 67^\circ)}$$

$$= \frac{\sin 25^\circ}{\sin 25^\circ} + \frac{\tan 23^\circ}{\tan 23^\circ}$$

$$= 1 + 1 = 2$$

6. If $\cos 2A = \sin(A-15)$, find A.

Ans : [Board Term1, 2015, Set-FHN8MGD]

We have $\cos 2A = \sin(A-15)$

$$\sin(90^\circ - 2A) = \sin(A - 15^\circ)$$

$$90^\circ - 2A = A - 15$$

$$3A = 105^\circ$$

$$A = 35^\circ$$

7. If $\tan(3x+30^\circ) = 1$ them find the value of x.

Ans : [Board Term-1, 2015, Set-WjQZQBN]

We have $\tan(3x+30^\circ) = 1 = \tan 45^\circ$

$$3x + 30^\circ = 45^\circ$$

$$x = 5^\circ$$

8. What happens to value of $\cos\theta$ when θ increases from 0° to 90°.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

$\cos\theta$ decreases from 1 to θ.

9. Find the value of $\tan^2 10^\circ - \cot^2 80^\circ$.

Ans : [DDE-M, 2015]

We have

$$\tan^2 10^\circ - \cot^2 80^\circ = \tan^2(90^\circ - 80^\circ) - \cot^2 80^\circ$$

$$[\therefore \tan(90^\circ - \theta) = \cot\theta]$$

$$= \cot^2 80^\circ - \cot^2 80^\circ$$

$$= 0$$

10. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A+B$.

Ans : [Board Term-1, 2016, Set-MV98HN3]

We have $\sin A = \cos B$

$$\sin A = \sin(90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

11. If A and B are acute angles and $\operatorname{cosec} A = \sec B$, then find the value of $A+B$.

Ans : [DDE-E, 2015]

We have $\operatorname{cosec} A = \sec B$

$$\operatorname{cosec} A = \operatorname{cosec}(90^\circ - B)$$

$$A = 90^\circ - B$$

$$A+B = 90^\circ$$

12. Find the value of $\cot 10^\circ . \cot 30^\circ . \cot 80^\circ$

Ans : [CTOQ, 2015]

$$\cot 10^\circ \cot 30^\circ \cot 80^\circ = \cot(90^\circ - 80^\circ)\cot 30^\circ \cot 80^\circ$$

$$= \tan 80^\circ \cot 30^\circ \frac{1}{\tan 80^\circ}$$

$$= \cot 30^\circ = \sqrt{3}$$

SHORT ANSWER TYPE QUESTIONS - I

1. Evaluate :

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

Ans : [Board term-1, 2016, Set-MV98HN3]

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

$$= \frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + 2 - 1}{(1)^2}$$

$$= \frac{3\times\frac{1}{3} + 3 + 2 - 1}{1}$$

$$= 1+3+2-1 = 5$$

2. If $\sin(A+B) = 1$ and $\sin(A-B) = \frac{1}{2}$, $0 \le A+B = 90^\circ$ and $A > B$, then find A and B.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

We have $\sin(A+B) = 1 = \sin 90^\circ$

$$A+B = 90^\circ \quad ...(1)$$

and $\sin(A-B) = \frac{1}{2} = \sin 30^\circ$

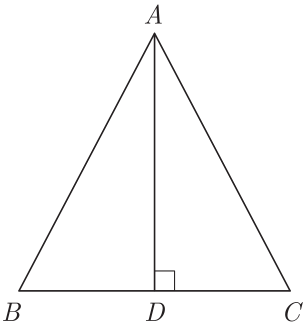
$$A-B = 30^\circ \quad ...(2)$$

Solving eq. (1) and (2), we obtain

$$A = 60^\circ \text{ and } B = 30^\circ$$

3. Find $\operatorname{cosec} 30^\circ$ and $\cos 60^\circ$ geometrically.

Ans : [Board Term-1, 2015, Set-FHN8MGD]



Let a triangle ABC with each side equal to $2a$.

$$\angle A = \angle B = \angle C = 60^\circ$$

Draw AD perpendicular to BC

$$\Delta BDA \cong \Delta CDA \text{ by } RHS$$

$$BD = CD$$

$$\angle BAD = CAD = 30^\circ \text{ by } CPCT$$

$$AD = \sqrt{3a}$$

In ΔBDA, $\operatorname{cosec} 30^\circ = \frac{AB}{BD} = \frac{2a}{a} = 2$

and $\cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

4. Evaluate : $\frac{\operatorname{cosec} 13^\circ}{\sec 77^\circ} - \frac{\cot 20^\circ}{\tan 70^\circ}$

Ans : [JTOQ, 2015]

$$\frac{\operatorname{cosec} 13^\circ}{\sec 77^\circ} - \frac{\cot 20^\circ}{\tan 70^\circ} = \frac{\operatorname{cosec}(90^\circ - 77^\circ)}{\sec 77^\circ} - \frac{\cot(90^\circ - 70^\circ)}{\tan 70^\circ}$$

$$= \frac{\sec 77^\circ}{\sec 77^\circ} - \frac{\tan 70^\circ}{\tan 70^\circ}$$

$$= 1 - 1 = 0$$

5. Evaluate : $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ}$

Ans : [Board Term-1, 2013, Set-FFC]

We have $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2}$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2}$$

6. If $\sin(36+\theta)^\circ = \cos(16+\theta)^\circ$, then find θ, where $(36+\theta)^\circ$ and $(16+\theta)^\circ$ are both acute angles.

Ans : [Board Term-1, 2012, Set-68]

We have $\sin(36+\theta)^\circ = \cos(16+\theta)^\circ$

$$\cos\left[90^\circ - (36+\theta)^\circ\right] = \cos(16+\theta)^\circ$$

$$90^\circ - 36^\circ - \theta = 16^\circ + \theta$$

$$2\theta \; 90^\circ - 36^\circ - 16^\circ = 38^\circ$$

$$\theta = \frac{38^\circ}{2} = 19^\circ.$$

7. If $\sqrt{2}\sin\theta = 1$, find the value of $\sec^2\theta - \operatorname{cosec}^2\theta$.

Ans : [Board Term-1, 2012, Set-67]

We have $\sqrt{2}\sin\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

Thus $\theta = 45^\circ$

Now $\sec^2\theta - \operatorname{cosec}^2\theta = \sec^2 45^\circ - \operatorname{cosec}^2 45^\circ$

$$= (\sqrt{2})^2 - (\sqrt{2})^2$$

$$= 0$$

8. If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

Ans : [Board Term-1, 2012, Set-50]

We have $4\cos\theta = 11\sin\theta$

or, $\cos\theta = \frac{11}{4}\sin\theta$

Now $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11\times\frac{11}{4}\sin\theta - 7\sin\theta}{11\times\frac{11}{4}\sin\theta + 7\sin\theta}$

$$= \frac{\sin\theta\left(\frac{121}{4} - 7\right)}{\sin\theta\left(\frac{121}{4} + 7\right)}$$

$$= \frac{121-28}{121+28} = \frac{93}{149}$$

9. If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0^\circ < A+B$

$\leq 90^o, A > B$, then find A and B.

Ans : [Board Term-1, 2012, Set-69]

We have $\tan(A+B) = \sqrt{3} = \tan 60^o$

$$A+B = 60^o \quad ...(1)$$

Again $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^o$

or, $A - B = 30^o \quad ...(2)$

Adding equations (1) and (2), we obtain,

$$2A = 90^o$$

$$A = \frac{90^o}{2} = 45^o$$

Putting this value of A in equation (1), we get

$$B = 60^o - A = 60^o - 45^o = 15^o$$

Hence, $A = 45^o$ and $B = 15^o$

10. If $\cos(A-B) = \frac{\sqrt{3}}{2}$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, find $\sin A$ and B, where $(A+B)$ and $(A-B)$ are acute angles.

Ans : [Board Term-1, 2012, Set-70]

We have $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^o$

$$A - B = 30^o \quad ...(1)$$

Also $\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60^o$

$$A + B = 60^o \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^o$$

$$A = 45^o$$

Putting this value of A in equation (1), we get $B = 15^o$

11. Express $\cos 68^o + \tan 76^o$ in terms of the angles between 0^o and 45^o.

Ans : [Board Term-1, 2012, Set-64]

Here we will use $\cos(90^o - \theta) = \sin\theta$ and $\tan(90^o - \theta) = \cot\theta$.

$$\cos 68^o + \tan 76^o = \cos(90^o - 22^o) + \tan(90^o - 14^o)$$

$$= \sin 22^o + \cot 14^o$$

12. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$.

Ans : [Board Term-1, 2012, Set-25]

We have $2\sin 2\theta = \sqrt{3}$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^o$$

$$2\theta = 60^o$$

Hence, $\cos 2\theta = \cos 60^o = \frac{1}{2}$.

13. Find the value of $\sin 30^o . \cos 60^o + \cos 30^o . \sin 60^o$ is it equal to $\sin 90^o$ or $\cos 90^o$?

Ans : [Board Term-1, 2016, Set-ORDAWEZ]

$$\sin 30^o \cos 60^o + \cos 30^o \sin 60^o = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^o = 1$ but not equal to $\cos 90^o$ as $\cos 90^o = 0$.

14. Evaluate : $\frac{6\sin 23^o + \sec 79^o + 3\tan 48^o}{\operatorname{cosec} 11^o + 3\cot 42^o + 6\cos 67^o}$

Ans : [Board Term-1, 2012, Set-55]

$$\frac{6\sin 23^o + \sec 79^o + 3\tan 48^o}{\operatorname{cosec} 11^o + 3\cot 42^o + 6\cos 67^o}$$

$$= \frac{6\sin(90^o - 23)^o + \operatorname{cosec}(90^o - 79^o) + 3\cot(90^o - 48^o)}{\operatorname{cosec} 11^o + 3\cot 42^o + 6\cos 67^o}$$

$$= \frac{6\cos 67^o + \operatorname{cosec} 11^o + 3\cot 42^o}{\operatorname{cosec} 11^o + 3\cot 42^o + 6\cos 67^o}$$

$$= 1$$

15. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^o < \theta < 90^o$, find the value of θ.

Ans : [Boar Term-1, 2012, Set-35]

We have $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^o < \theta < 90^o$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^o \quad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\theta = 30^o$$

16. Evaluate : $\frac{\cos 45^o}{\sec 30^o} + \frac{1}{\sec 60^o}$

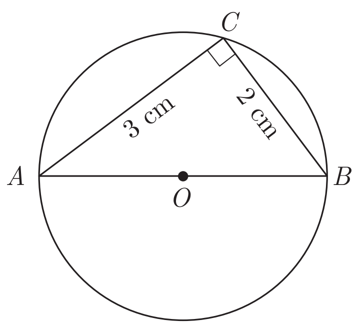
Ans : [Board Term-1, 2012, Set-63]

We have $\frac{\cos 45^o}{\sec 30^o} + \frac{1}{\sec 60^o} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \frac{1}{2}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{1}{2}$$

$$= \frac{\sqrt{6}+2}{4}$$

17. In the given figure, AOB is a diameter of a circle with center O. fing $\tan A \tan B$.



Ans : [Board Term-1, 2012, Set-52]

In ΔABC, $\angle C$ in a semi-circle angle, thus

$$\angle C = 90^o$$

$$\tan A = \frac{BC}{AC} = \frac{2}{3}$$

and $\tan B = \frac{AC}{BC} = \frac{3}{2}$

$$\tan A . \tan B = \frac{2}{3} \times \frac{3}{2} = 1$$

18. If $\sin\phi = \frac{1}{2}$, show that $3\cos\phi - 4\cos^3\phi = 0$.

Ans :

We have $\sin\theta = \frac{1}{2}$

$\phi = 30^{o}$

Now substituting this value of θ in LHS we have

$$3\cos\phi - 4\cos^3\phi = 3\cos 30^{o} - 4\cos^3 30^{o}$$
$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$
$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{2}}{2}$$
$$= 0$$

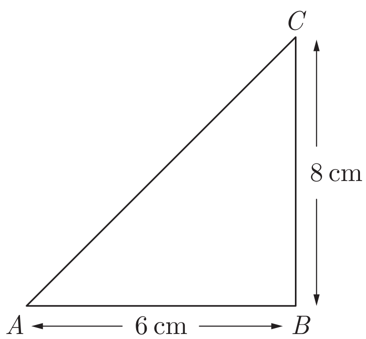
Hence Prove

SHORT ANSWER TYPE QUESTIONS - II

1. If in a triangle ABC right angled at $B, AB = 6$ units and $BC = 8$ units, then find the value of $\sin A . \cos C + \cos A . \sin C$.

Ans : [Board Term-1, 2016, set-O4YP6G7]

As per question statement figure is shown below.



We have $AC^2 = 8^2 + 6^2 = 100$

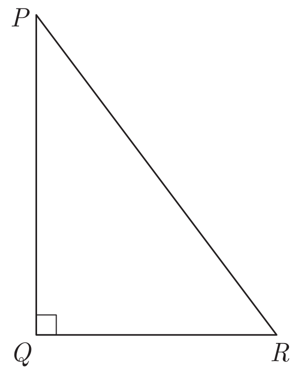
$AC = 10$

Now $\sin A = \frac{8}{10}, \cos A = \frac{6}{10}$

and $\sin C = \frac{6}{10}, \cos C = \frac{8}{10}$

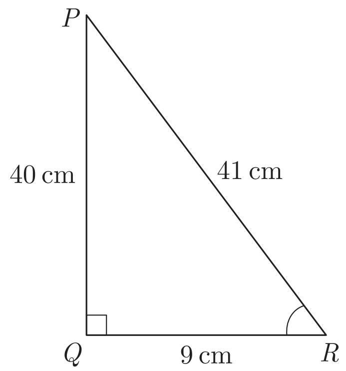
Thus $\sin A\cos C + \cos A\sin C = \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$
$$= \frac{64}{100} + \frac{36}{100}$$
$$= \frac{100}{100} = 1$$

2. In the given $\angle PQR$, right-angled at $Q, QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$.



Ans : [Board Term-1, 2015, Set-FHN8MGD]

We redraw the figure as shown below.



Using Pythagoras theorem we have

$$PQ^2 + QR^2 = PR^2$$
$$PQ^2 + 9^2 = (PQ+1)^2$$
$$PQ^2 + 81 = (PQ+1)^2$$
$$PQ^2 + 81 = PQ^2 + 1 + 2PQ$$
$$PQ = 40$$

Since $PR - PQ = 1$, thus

$$PR = 1 + 40 = 41$$
$$\sin R + \cos R = \frac{40}{41} + \frac{9}{41} = \frac{49}{41}$$

3. Express $\cos 71^{o} - \sin 57^{o} + \tan 63^{o}$ in terms of trigonometric ratios of angles between 0^{o} and 45^{o}.

Ans : [JTOQ, 2015]

$\cos 71^{o} - \sin 57^{o} + \tan 63^{o}$
$$= \cos(90^{o} - 19^{o}) - \sin(90^{o} - 33^{o}) + \tan(90^{o} - 27^{o})$$
$$= \sin 19^{o} - \cos 33^{o} + \cot 27^{o}$$

4. If $\cos(40^{o} + x) = \sin 30^{o}$, find the value of x.

Ans : [DDE-E, 2015]

We have $\cos(40^{o} - x) = \sin 30^{o}$
$$\cos(40^{o} + x) = \sin(90^{o} - 60^{o})$$
$$\cos(40^{o} + x) = \cos 60^{o}$$
$$40^{o} + x = 60^{o}$$

$$x = 60^o - 40^o = 20^o$$

Thus $x = 20^o$

5. Evaluate : $\dfrac{5\cos^2 60^o + 4\cos^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 60^o}$

Ans : [Board Term-1, 2013, Set-Lk-59]

$$\frac{5\cos^2 60^o + 4\cos^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 60^o}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

6. If $\sin 30^o = \cos(\theta - 6^o)$, where 3θ and $\theta - 6^o$ are both acute angles, find the value of θ.

Ans : [board Term-1, 2011, Set-21]

According to the question,

$$\sin 3\theta = \cos(\theta - 6^o)$$
$$\cos(90^o - 3\theta) = \cos(\theta - 6^o)$$
$$90^o - 3\theta = \theta - 6^o$$
$$4\theta = 90^o + 6^o = 96^o$$

Thus $\theta = \dfrac{96^o}{4} = 24^o$

7. Simplify :
$$\frac{\sin\theta\sec(90^o - \theta)\tan\theta}{\text{cosec}(90^o - \theta)\cos\theta\cot(90^o - \theta)} - \frac{\tan(90^o - \theta)}{\cot\theta}$$

Ans : [Board Term-1, 2011, Set-66]

$$\sec(90^o - \theta) = \text{cosec}\,\theta,$$
$$\tan(90^o - \theta) = \cot\theta,$$
$$\cot(90^o - \theta) = \tan\theta,$$
$$\text{cosec}\,(90^o - \theta) = \sec\theta$$

Hence,
$$\frac{\sin\theta\sec(90^o - \theta)\tan\theta}{\text{cosec}(90^o - \theta)\cos\theta\cot(90^o - \theta)} - \frac{\tan(90^o - \theta)}{\cot\theta}.$$

$$= \frac{\sin\theta\,\text{cosec}\,\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} - \frac{\cot\theta}{\cot\theta}$$

$$= \frac{\sin\theta \times \frac{1}{\sin\theta} \times \tan\theta}{\frac{1}{\cos\theta} \times \cos\theta\tan\theta} - 1$$

$$= 1 - 1 = 0$$

8. Verify : $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$, for $\theta = 60^o$

Ans :

$$\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos 60^o}{1+\cos 60^o}}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \qquad \left(\cos 60^o = \frac{1}{2}\right)$$

$$\text{RHS} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 60^o}{1+\cos 60^o}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{RHS} = \text{LHS}$$

Hence, relation is verified for $\theta = 60^o$.

9. If $\tan A + \cot A = 2$, then find the value of $\tan^2 A + \cot^2 A$.

Ans : [DDE-M, 2015]

We have $\tan A + \cot A = 2$

Squaring both sides, we have

$$(\tan A + \cot A)^2 = (2)^2$$
$$\tan^2 A + \cot^2 A + 2\tan A.\cot A = 4$$
$$\tan^2 A + \cot^2 A + 2\tan A \times \frac{1}{\tan A} = 4$$
$$\tan^2 A + \cot^2 A + 2 = 4$$
$$\tan^2 A + \cot^2 A = 4 - 2$$
$$\tan^2 A + \cot^2 A = 2$$

10. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$.

Ans : [Board Term-1, 2011, Set-74]

We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$$\sin\theta = \cos\theta(\sqrt{2} - 1)$$
$$= \frac{\cos\theta(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}$$

or, $$\sin\theta = \frac{\cos\theta(2-1)}{\sqrt{2}+1}$$

$$(\sqrt{2}+1)\sin\theta = \cos\theta$$
$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$$
$$co\theta - \sin\theta \;\; \sqrt{2}\sin\theta$$

Hence proved.

11. Prove that : $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$.

Ans : [Board Term-1, 2013, FFC: 2011, Set-74]

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$= \frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

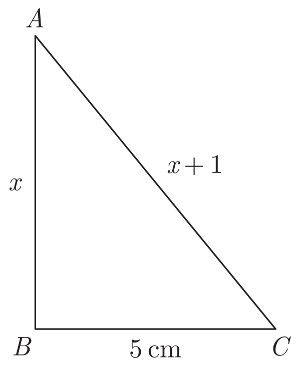
$$= \cos A + \sin A$$
$$= \sin A + \cos A$$
$$= \text{RHS}$$

Hence proved.

12. In ΔABC, $\angle B = 90^o$, $BC = 5$ cm, $AC - AB = 1$, Evaluate : $\frac{1+\sin C}{1+\cos C}$.

Ans : [Board Term-1, 2011, Set-52]

As per question we have drawn the figure given below.



We have $AC - AB = 1$

Let $AB = x$, then we have

$$AC \quad x + 1$$

Now
$$AC^2 = AB^2 + BC^2$$
$$(x+1)^2 = x^2 + 5^2$$
$$x^2 + 2x + 1 = x^2 + 25$$
$$2x = 24$$
$$x = \frac{24}{2} = 12 \text{ cm}$$

Hence, $AB = 12$ cm and $AC = 13$ cm

Now
$$\sin C = \frac{AB}{AC} = \frac{12}{13}$$
$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now
$$\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$$

LONG ANSWER TYPE QUESTIONS

1. Evaluate :

$\tan^2 30^o \sin 30^o + \cos 60^o \sin^2 90^o \tan^2 60^o - 2\tan 45^o \cos^2 0^o \sin 90^o$

Ans : [Board Term-1, 2015, WJQZQBN]

$\tan^2 30^o \sin 30^o + \cos 60^o \sin^2 90^o \tan^2 60^o - 2\tan 45^o \cos^2 0^o \sin 90^o$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1$$
$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 1 \times 3 \times - 2 \times 1 \times 1 \times 1$$
$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1+9-12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

2. Given that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, find the values of $\tan 75^o$ and $\tan 90^o$ by taking suitable values of A and B.

Ans : [NCERT]

We have $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

(i)
$$\tan 75^o = \tan(45^o + 30^o)$$
$$= \frac{\tan 45^o + \tan 30^o}{1 \cdot \tan 45^o \cdot \tan 30^o}$$
$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= \frac{3+2\sqrt{3}+1}{(\sqrt{3})^2-(1)^2} = \frac{4+2\sqrt{3}}{2}$$

Hence $\tan 75^o = 2 + \sqrt{3}$

(ii)
$$\tan 90^o = \tan(60^o + 30^o)$$
$$= \frac{\tan 60^o + \tan 30^o}{1 - \tan 60^o \tan 30^o}$$
$$= \frac{\sqrt{3}+\frac{1}{\sqrt{3}}}{1-\sqrt{3}\times\frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$$

Hence, $\tan 90^o = \infty$

3. Evaluate :

$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin 90^o - 2\cos^2 90^o + \frac{1}{24}$

Ans : [Board Term-1, 2013, LK-59]

$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin 90^o - 2\cos^2 90^o + \frac{1}{24}$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$$
$$= \frac{1}{4}\left(\frac{1}{2}\right) + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$
$$= \frac{3+32+12+1}{24} = \frac{48}{24} = 2$$

4. Evaluate : $4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45 - \sin^2 90^o)$

Ans : [Board Term-1, 2013, Set-FFC]

$4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45 - \sin^2 90^o)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$
$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$
$$= 4 \times \frac{2}{16} - 3 \times -\frac{1}{2}$$
$$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

5. If $15\tan^2\theta + 4\sec^2\theta = 23$, then find the value of $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$.

Ans : [Board Term-1, 2012, Set-38]

We have
$$15\tan^2\theta + 4\sec^2\theta = 23$$
$$15\tan^2\theta + 4(\tan^2\theta + 1) = 23 \quad (\sec^2\theta = 1 + \tan^2\theta)$$
$$15\tan^2\theta + 4\tan^2\theta + 4 = 23$$
$$19\tan^2\theta = 19$$
$$\tan\theta = 1 = \tan 45^o$$

Thus
$$\theta = 45^o$$

Now, $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$
$$= (\sec 45^o + \operatorname{cosec} 45^o)^2 - \sin^2 45^o$$
$$= (\sqrt{2} + \sqrt{2})^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= (2\sqrt{2})^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2}$$

6. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Ans : [board Term-1, 2012, Set-48]

We have $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$, then we have

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$
$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$
$$(x - \sqrt{3})(\sqrt{3}x - 1) = 0$$

Thus $x = \sqrt{3}$ or $\frac{1}{\sqrt{3}}$

or $\cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

Therefore $\theta = 30^\circ$ or $\theta = 60^\circ$

If $\theta = 30^\circ$, then

$$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$
$$= 3 + \frac{1}{3} = \frac{10}{3}$$

If $\theta = 60^\circ$, then

$$\cot^2 60^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$
$$= \frac{1}{3} + 3 = \frac{10}{3}.$$

7. Evaluate the following :
$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

Ans : [Board Term-1, 2012, Set-43]

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$
$$= \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$
$$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} = \frac{10}{3}$$

8. Evaluate : $\frac{\cos 65^\circ}{\sin 25^\circ} - \frac{\tan 20^\circ}{\cot 70^\circ} - \sin 90^\circ + \tan 5^\circ \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 85^\circ$.

Ans : [Board Term-1, 2012, Set-50]

We have $\frac{\cos 65^\circ}{\sin 25^\circ} = \frac{\cos 65^\circ}{\sin(90^\circ - 65^\circ)} = \frac{\cos 65^\circ}{\cos 65^\circ} = 1$,

$$\frac{\tan 20^\circ}{\cot 70^\circ} = \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} = \frac{\cot 70^\circ}{\cot 70^\circ} = 1$$

and $\sin 90^\circ = 1$

$\tan 5^\circ \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 85^\circ$

$$= \tan(90^\circ - 85^\circ)\tan(90^\circ - 55^\circ) \tan 55^\circ \tan 60^\circ \tan 85^\circ.$$
$$= \cot 85^\circ \tan 85^\circ \cot 55^\circ \tan 55^\circ \cdot \sqrt{3}$$
$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$$

Now given expression $= 1 - 1 - 1 + \sqrt{3} = \sqrt{3} - 1$

9. Prove that : $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta$.

Ans : [Board Term-1, 2012, Set-48]

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta}$$
$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1 - \tan\theta)\tan\theta}$$
$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$
$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$
$$[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$
$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$
$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$
$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

10. In an acute angled triangle ABC, if $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$, find $\angle A, \angle B$ and $\angle C$.

Ans : [Board Term-1, 2012, Set-39]

We have $\sin(A + B - C) = \frac{1}{2} = \sin 30^\circ$

or, $A + B - C = 30^\circ$ ...(1)

and $\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

or, $B + C - A = 45^\circ$ ...(2)

Adding equation (1) and (2), we get

$$2B = 75^\circ$$

or, $B = 37.5^\circ$

Now subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$

or, $A - C = 7.5^\circ$ ...(3)

Now $A + B + C = 180^\circ$

$$A + B + C = 180^\circ$$

$A + C = 180^\circ - 37.5^\circ = 142.5^\circ$ ...(4)

Adding equation (3) and (4), we have

$$2A = 135^\circ$$

or, $A = 67.5^\circ$

and, $C = 75^\circ$

Hence, $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$

VERY SHORT ANSWER TYPE QUESTIONS

1. If $k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$, then find the value of k.

Ans : [Board Term-1, 2015, Set-JJOQ]

We have $k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$

$$= \sec^2\theta(1-\sin^2\theta)$$
$$= \sec^2\theta.\cos^2\theta$$
$$= \sec^2\theta \times \frac{1}{\sec^2\theta}$$

or, $k+1 = 1$

or, $k = 1-1 = 0$

Thus $k = 0$

2. Find the value of $\sin^2 41^\circ + \sin^2 49^\circ$

Ans : [DDE-M, 2015][NCERT]

We have

$$\sin^2 41 + \sin^2 49 = \sin^2(90^\circ - 49^\circ) + \sin^2 49^\circ$$
$$= \cos^2 49 + \sin^2 49^\circ$$
$$= 1 \qquad [\cos^2\theta + \sin^2\theta = 1]$$

SHORT ANSWER TYPE QUESTIONS - I

1. Express the trigonometric ratio of $\sec A$ and $\tan A$ in terms of $\sin A$.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

We have $\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}}$

and $\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$

2. Prove that : $\frac{(\sin^4\theta + \cos^4\theta)}{1-2\sin^2\theta\cos^2\theta} = 1$

Ans : [Board Term-1, 2015, Set-WJQZQBN]

$$\frac{(\sin^4\theta + \cos^4\theta)}{1-2\sin^2\theta\cos^2\theta} = \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1-2\sin^2\theta\cos^2\theta}$$
$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1-2\sin^2\theta\cos^2\theta}$$
$$= \frac{1-2\sin^2\theta\cos^2\theta}{1-2\sin^2 + \cos^2\theta}$$
$$= 1 \qquad \text{Hence Prove}$$

3. Prove that : $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Ans : [DDE-M, 2015]

To prove $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

We have $\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1)$

$$[[1+\tan^2\theta = \sec^2\theta]]$$
$$= \sec^2\theta(\tan^2\theta)$$
$$= (1+\tan^2\theta)\tan^2\theta$$
$$= \tan^2\theta + \tan^4\theta$$

Hence Proved.

4. Find the value of θ, if, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4; \theta \leq 90^\circ$

Ans : [DDE-E, 2015]

We have $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

$$\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$$
$$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin\theta} = 4$$
$$\frac{\cos\theta(2)}{\cos^2\theta} = 4$$
$$\frac{1}{\cos\theta} = 2$$
$$\cos\theta = \frac{1}{2}$$
$$\cos\theta = \cos 60^\circ$$

Thus $\theta = 60^\circ$

5. Prove that : $-1 + \frac{\sin A\sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$

Ans : [Board Term-1, 2012, Set-62]

$$-1 + \frac{\sin A\sin(90^\circ - A)}{\cot(90^\circ - A)} = -1 + \frac{\sin A\cos A}{\tan A}$$
$$= -1 + \sin A\cos A \times \cot A$$
$$= -1 + \sin A\cos A \times \frac{\cos A}{\sin A}$$
$$= -1 + \cos^2 A = -(1-\cos^2\theta)$$
$$= -\sin^2 A \qquad \text{Hence Proved.}$$

6. Prove that : $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

Ans : [Board Term-1,2012, Set-74]

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}}$$
$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}}$$
$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$
$$= \operatorname{cosec} A - \cot A \qquad \text{Hence Proved.}$$

7. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$.

Ans : [board Term-1, 2013, Set-FFC]

We have $\sin\theta - \cos\theta = \frac{1}{2}$

Squaring both sides, we get

$$(\sin\theta - \cos\theta)^2 = \left(\frac{1}{2}\right)^2$$
$$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{1}{4}$$
$$1 - 2\sin\theta\cos\theta = \frac{1}{4}$$
$$2\sin\theta\cos\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

Again, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$$= 1 + 2\sin\theta\cos\theta$$
$$= 1 + \frac{3}{4} = \frac{7}{4}$$

Thus $\sin\theta + \cos\theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

8. If θ be an acute angle and $5\operatorname{cosec}\theta = 7$, Then evaluate $\sin\theta + \cos^2\theta - 1$.

Ans : [Board Term-1, 2012, Set-43]

We have
$$5\operatorname{cosec}\theta = 7$$
$$\operatorname{cosec}\theta = \frac{7}{5}$$
$$\sin\theta = \frac{5}{7} \qquad [\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}]$$
$$\sin\theta + \cos^2\theta - 1 = \sin\theta - (1 - \cos^2\theta)$$
$$= \sin\theta - \sin^2\theta \quad [\sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{5}{7} - \left(\frac{5}{7}\right)^2 = \frac{35-25}{49} = \frac{10}{49}$$

9. If $\sin A = \frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$.

Ans : [Board Term-1, 2012, Set-21]

Using $\because \cot^2\theta = -1 + \operatorname{cosec}^2\theta$ we have
$$2\cot^2 A - 1 = 2(\operatorname{cosec}^2 A - 1) - 1$$
$$= \frac{2}{\sin^2 A} - 3$$
$$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3 = \frac{8}{3} - 3 = \frac{-1}{3}$$

Thus
$$2\cot^2 A - 1 = \frac{-1}{3}$$

SHORT ANSWER TYPE QUESTIONS - II

1. Prove that : $\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A} = \cos A - \sin A$

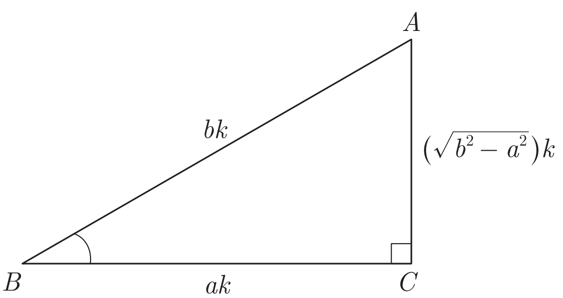
Ans : [Board Term-1, 2016, Set-MV98HN3]

$$\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A} = \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$$
$$= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A}$$
$$= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)}$$
$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$
$$= \cos A - \sin A \qquad \text{Hence Proved.}$$

2. If $b\cos\theta = a$, then prove that $\operatorname{cosec}\theta + \cot\theta = \sqrt{\frac{b+a}{b-a}}$.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

Consider the triangle shown below.



$$b\cos\theta = a$$
$$AC^2 = AB^2 - BC^2$$
or,
$$\cos\theta = \frac{a}{b}$$
$$AC = \sqrt{b^2 - a^2}k$$
$$\operatorname{cosec}\theta = \frac{b}{\sqrt{b^2-a^2}}, \cot\theta = \frac{1}{\sqrt{b^2-a^2}}$$
$$\operatorname{cosec}\theta + \cot\theta = \frac{b+a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$$

3. Prove that : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

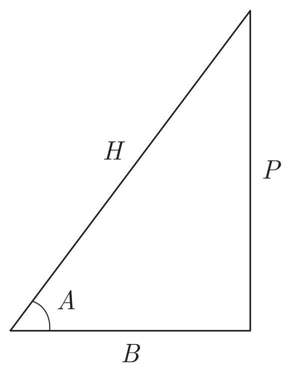
Ans : [Bard Term-1, 2015, Set-WJQZQBN, FHN8MGD]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$
$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$
$$= \tan\theta \qquad \text{Hence Proved}$$

4. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$.

Ans : [DDE-E, 2015][NCERT]

Consider the triangle shown below.



Let $\tan A = \frac{P}{B}$ and $\sec A = \frac{H}{B}$
$$H^2 = P^2 + B^2$$
Now
$$1 + \tan^2 A = 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{P^2}{B^2}$$
$$= \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2}$$
$$= \left(\frac{H}{B}\right)^2$$
$$= \sec^2 A \qquad \text{Hence Proved.}$$

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement.

5. Prove that : $(\cot\theta - \operatorname{cosec}\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Ans : [Board Term-1, 2015, Set JTOQ, 2015]

To prove $(\cot\theta - \text{cosec}\,\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

$$(\cot\theta - \text{cosec}\,\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \quad [[\sin^2\theta + \cos^2\theta = 1]]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$ Hence Proved.

6. Prove that :
$(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Ans : [DDE-M, 2015]

$$\text{LHS} = (\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \left(\frac{1}{\sin\theta.\cos\theta}\right) \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \cos\theta\sin\theta \times \frac{1}{\sin\theta\cos\theta} = 1$$ Hence Proved.

7. Show that :
$\text{cosec}^2\theta - \tan^2(90^\circ - \theta) = \sin^2\theta + \sin(90^\circ - \theta)$

Ans : [Board Term-1, 2013, LK-59]

$$\text{cosec}^2\theta - \tan^2(90^\circ - \theta) = \frac{1}{\sin^2} - \frac{\sin^2(90^\circ - \theta)}{\cos^2(90 - \theta)}$$

$$= \frac{1}{\sin^2\theta} - \frac{\sin^2(90^\circ - \theta)}{\sin^2\theta}$$

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta}$$

$$= 1$$

$$= \sin^2\theta + \cos^2\theta$$

$$= \sin^2\theta + \sin^2(90^\circ - \theta)$$

Hence Proved

8. Prove that : $\frac{\text{cosec}^2\theta}{\text{cosec}\,\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta + 1} = 2\sec^2\theta$

Ans : [Board Term-1, 2013, FFC]

We have

$$\frac{\text{cosec}^2\theta}{\text{cosec}\,\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta + 1} = \text{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$$

$$= \text{cosec}^2\theta\left[\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta}\right]$$

$$= \frac{1 \times \sin\theta}{\sin^2\theta}\left[\frac{(1+\sin\theta)-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}\right]$$

$$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1-\sin^2\theta}\right]$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$ Hence Proved

9. Prove that :
$$\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}.$$

Ans : [Board Term-1, 2011, Set-66]

$$\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}$$

$$\frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

$$\frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A} = \frac{2}{\sin A}$$

$$\frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A} = \frac{2}{\sin A}$$

$$\frac{\text{cosec}\,A + \cot A + \text{cosec}\,A - \cot A}{(\text{cosec}\,A - \cot A)(\text{cosec}\,A + \cot a)} = \frac{2}{\sin A}$$

$$\frac{2\,\text{cosec}\,A}{\text{cosec}^2 A - \cot^2 A} = \frac{2}{\sin A}$$

$$\frac{2}{\frac{\sin A}{1}} = \frac{2}{\sin A}$$

$$\frac{\sin A}{2} = \frac{2}{\sin A}$$ Hence Proved.

10. If $\sec\theta = x + \frac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$.

Ans : [Board Term-1, 2011, Set-55]

We have $\sec\theta = x + \frac{1}{4x}$

Squaring both side we have

$$\sec^2\theta = x^2 + \frac{1}{16x^2} + 2.x\frac{1}{4x}$$

$$1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\tan^2\theta = x^2 + \frac{1}{16x^2} + 2 - 1$$

$$= x^2 + \frac{1}{16x^2} - 1$$

$$= x^2 + \frac{1}{16x^2} - 2.x.\frac{1}{4x}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

Taking square root both sides we obtain

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

Now take $\tan\theta = x - \frac{1}{4x}$

Now $\sec\theta = x + \frac{1}{4x}$ Given

$$\tan\theta + \sec\theta = 2x$$

Now take $\tan\theta = -\left(x - \frac{1}{4x}\right) = -x + \frac{1}{4x}$

$$\sec\theta = x + \frac{1}{4x}$$

$$\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x} \quad \text{Hence proved.}$$

11. Prove that : $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$

Ans : [Board Term-1, 2011, Set-39]

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin\theta - 1} = \text{RHS}$$

Hence Proved.

12. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, Prove that $x^2 + y^2 = 1$.

Ans : [Board Term-1, 2011, Set-44]

We have $\quad x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \quad (1)$

and $\quad x\sin\theta = y\cos\theta$

or, $\quad x = \frac{y\cos\theta}{\sin\theta} \quad (2)$

Eliminating x from eqn. (1) and eqn. (2) we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$

$$y\cos\theta \times 1 = \sin\theta\cos\theta$$

$$y = \sin\theta \quad ...(3)$$

Substituting this value of y in eqn. (2) we have,

$$x = \cos\theta \quad (4)$$

Squaring and adding eqn. (3) and eqn. (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1 \quad \text{Hence Proved.}$$

13. Prove that $\frac{\cos^2\theta + \sin^2\theta}{\cos\theta + \sin\theta} + \frac{\cos^2\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

Ans : [Board Term-1, 2011, Set-40]

$$\text{LHS} = \frac{\cos^2\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$+ \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$$

$$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 2 = \text{RHS} \quad \text{Hence Proved.}$$

14. Evaluate the following :

$$\frac{\sec^2(90^\circ - \theta) - \cot^2\theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} - \frac{2\cos^2 60^\circ\tan^2 28^\circ\tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$$

Ans : [Board Term-1, 2011, Set-60]

$$\frac{\sec^2(90^\circ - \theta) - \cot^2\theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} - \frac{2\cos^2 60^\circ\tan^2 28^\circ\tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$$

$$= \frac{(\text{cosec}^2\theta - \cot^2\theta)}{2(\sin^2 25^\circ + \cos^2 25^\circ)} - \frac{2 \times \frac{1}{2} \times \frac{1}{2}\tan^2 28^\circ \times \cot^2 28^\circ}{3(\sin^2 43^\circ - \tan^2 43^\circ)}$$

$$= \frac{1}{2 \times 1} - \frac{\frac{1}{2} \times \tan^2 28^\circ \times \frac{1}{\tan^2 28^\circ}}{3}$$

$$= \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

15. Evaluate :

$$\frac{\sec 41^\circ\sin 49^\circ + \cos 29^\circ\,\text{cosec}\,61^\circ - \frac{2}{\sqrt{3}}(\tan 20^\circ\tan 60^\circ\tan 70^\circ)}{3(\sin^2 31^\circ + \sin^2 59^\circ)}$$

Ans : [Board Term-1, 2011, Set-25]

$$\frac{\sec 41^\circ\sin 49^\circ + \cos 29^\circ\,\text{cosec}\,61^\circ - \frac{2}{\sqrt{3}}(\tan 20^\circ\tan 60^\circ\tan 70^\circ)}{3(\sin^2 31^\circ + \sin^2 59^\circ)}$$

$$= \frac{\sec(90^\circ - 41^\circ)\sin 49^\circ + \cos 29^\circ\,\text{cosec}(90^\circ - 29^\circ) - \frac{2}{\sqrt{3}}[\tan 20^\circ\sqrt{3}\tan(90^\circ - 20^\circ)]}{3[\sin^2 31^\circ\sin^2(90^\circ - 31^\circ)]}$$

$$= \frac{\text{cosec}\,49^\circ\sin 49^\circ + \cos 29^\circ\sec 29^\circ - \frac{2}{\sqrt{3}}[\tan 20^\circ\sqrt{3}\tan(90^\circ - 20^\circ)]}{3[\sin^2 31^\circ + \sin^2(90^\circ - 31^\circ)]}$$

$$= \frac{1 + 1 - 2[\tan 20^\circ\cot 20^\circ]}{3[\sin^2 31^\circ + \cos^2 31^\circ]} = \frac{1+1-2}{3} = \frac{2-2}{3} = 0$$

16. Evaluate :

$$\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta)\tan(30^\circ - \theta)} + \text{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta)$$

Ans : [Board Term-1, 2011, Set-40]

$$\frac{\cos^2(15^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta)\tan(30^\circ - \theta)}$$

$$+ \text{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta)$$

$$= \frac{\cos^2(45^\circ + \theta) + \sin^2(45^\circ + \theta)}{\tan(60^\circ + \theta)\cot(60^\circ + \theta)}$$

$$+ \text{cosec}(75^\circ + \theta) - \text{cosec}(90^\circ - 15^\circ + \theta)$$

$$= \frac{\cos^2(45^\circ + \theta) + \sin^2(45^\circ + \theta)}{\tan(60^\circ + \theta)\cot(60^\circ + \theta)} +$$

$$+ \text{cosec}(75^\circ + \theta) - \text{cosec}(75^\circ + \theta)$$

$$= \frac{1}{1} = 1$$

17. Express : $\sin A, \tan A$ and $\text{cosec}\,A$ in terms of $\sec A$.

Ans : [Board Term-1, 2011, Set-25]

(1) $\quad \sin^2 A + \cos^2 A = 1$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(2) $$\tan A = \frac{\sin A}{\cos A} = \sin A \sec A$$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \sec A$$

$$= \sqrt{\sec^2 A - 1}$$

(iii) $$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

18. Find the value of the following without using trigonometric tables :

$$\frac{\cos 50^\circ}{2\sin 40^\circ} + \frac{4(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}\tan 12^\circ \tan 78^\circ . \sin 90^\circ$$

Ans : [Board Term-1, 2011, Set-21]

We have $\cos 50^\circ = \cos(90^\circ - 40^\circ) = \sin 40^\circ$

$$\operatorname{cosec}^2 59^\circ = \operatorname{cosec}^2(90^\circ - 31^\circ) = \sec^2 31^\circ$$

and $\tan 78^\circ = \tan(90^\circ - 12^\circ) = \cot 12^\circ$

Hence,

$$\frac{\cos 50^\circ}{2\sin 40^\circ} + \frac{4(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}\tan 12^\circ \tan 78^\circ . \sin 90^\circ$$

$$= \frac{\sin 40^\circ}{2\sin 40^\circ} + \frac{4(\sec^2 31^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}\tan 12^\circ \cot 12^\circ \times 1$$

$$= \frac{1}{2} + \frac{4}{3} - \frac{2}{3} = \frac{7}{6}$$

19. Evaluate :

$$\frac{\operatorname{cosec}^2 63^\circ + \tan 24^\circ}{\cos^2 66^\circ + \sec^2 27^\circ} + \frac{\sin^2 63^\circ + \cos 63^\circ . \sin 27^\circ + \sin 27^\circ \sec 63^\circ}{2(\operatorname{cosec}^2 65^\circ - \tan^2 25^\circ)}$$

Ans : [Sample Question Paper 2017-18]

$$\frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\cot^2(90^\circ - 24^\circ) + \sec^2(90^\circ - 63^\circ)} + \frac{\sin^2 63^\circ + \cos 63^\circ . \sin(90^\circ - 63^\circ) + \sin 27^\circ . \sec(90 - 27)}{2(\operatorname{cosec}^2 65^\circ - \tan^2(90^\circ - 65^\circ))}$$

$$= \frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\tan^2 24^\circ + \operatorname{cosec}^2 63^\circ} + \frac{\sin^2 63^\circ + \cos 63^\circ . \cos 63^\circ + \sin 27^\circ . \operatorname{cosec} 27^\circ}{2(\operatorname{cosec}^2 65^\circ - \cot^2 65^\circ)}$$

$$= 1 + \frac{\sin^2 63^\circ + \cos^2 63^\circ + \sin 27^\circ \times \frac{1}{\sin 27^\circ}}{2 \times 1}$$

$$= 1 + \frac{1+1}{2} = 1 + 1 = 2$$

20. If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate $\tan\theta + \cot\theta$.

Ans : [Sample Question Paper 2017-18]

We have $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

$$2\sin\theta\cos\theta \quad 2 - 1 = 1$$

$$\frac{1}{\sin\theta\cos\theta} = 2$$

Now, $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} = 2$$

LONG ANSWER TYPE QUESTIONS

1. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :
(1) $x = a\sec\theta, y = b\tan\theta$, or
(2) $x = a\operatorname{cosec}\theta, y = b\cot\theta$

Ans : [Board Term-1, 2015, WJQZQBN]

(1) We have $x = a\sec\theta, y = b\tan\theta$,

$$\frac{x^2}{a^2} = \sec^2\theta, \ \frac{y^2}{b^2} = \tan^2\theta$$

or, $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

(ii) We have $x = a\operatorname{cosec}\theta, y = b\cot\theta$

$$\frac{x^2}{a^2} = \operatorname{cosec}^2\theta, \ \frac{y^2}{b^2} = \cot^2\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \operatorname{cosec}^2\theta - \cot\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

2. If $\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$.

Ans : [Board Term-1, 2015, WJQZQBN]

We have $\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta$

Squaring both sides we have

$$\operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta = 2\cot^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta)$$

$$(\operatorname{cosec}\theta + \cot\theta)\sqrt{2}\cot\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$$

Hence Proved.

3. Prove that :

$$\frac{\cot^3\theta . \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta . \cos^2\theta}{(\cos\theta + \sin\theta)} = \frac{\sec\theta\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Ans : [Set-FHN8MGD, 2015]

$$\frac{\cot^3\theta . \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta . \cos^2\theta}{(\cos\theta + \sin\theta)}$$

$$= \frac{\frac{\cos^3\theta}{\sin^3\theta} \times \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta} \times \cos^3\theta}{(\cos\theta + \sin\theta)^2}$$

$$= \frac{\cos^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta + \sin\theta)^2}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)^2}$$

$$= \frac{1 - \sin\theta\cos\theta}{\cos\theta + \sin\theta} = \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\operatorname{cosec}\theta\sec\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$ Hence Proved

4. Prove that : $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta.$

Ans : [Board Terim-1, 2012, Set-9]

$$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}}$$

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta.1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

$$(\tan^2\theta = \sec^2\theta - 1)$$

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= 2 \times \frac{1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$ Hence Prove

5. Prove that : $\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}.$

Ans : [Board Term-1, 2012, Set-21]

We have
$$\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$$

$$= \frac{\sin\theta(\frac{1}{\cos\theta} + 1)}{\sin\theta(\frac{1}{\cos\theta} - 1)}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1}$$

Hence Proved.

6. Prove that : $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

Ans : [Board Term-1, 2012, Set-62]

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$$

$$= \frac{2\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A - 1} = \frac{2\operatorname{cosec}^2 A}{\cot^2 A}$$

$$= \frac{\frac{2}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{2}{\cos^2 A} = 2\sec^2 A$$ Hence Proved.

7. If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2 - 1}{p^2 + 1}$.

Ans : [Board Term-1, 2016, Set-MV98HN3]

$$\frac{p^2 - 1}{p^2 + 1} = \frac{(\operatorname{cosec}\theta + \cot\theta) - 1}{(\operatorname{cosec}\theta + \cot\theta)^2 + 1}$$

$$= \frac{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta + 1}$$

$$= \frac{1 + \cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta - 1 + 2\operatorname{cosec}\theta\cot\theta + 1}$$

$$= \frac{2\cot\theta(\cot\theta + \operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{\cos\theta}{\sin\theta} \times \sin\theta = \operatorname{cosec}\theta$$ Hence proved

8. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $m^2 + n^2 = a^2 + b^2$

Ans : [Board Term-1, 2012, Set-58]

We have

$$m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta \quad ...(1)$$

and, $$n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta \quad ...(2)$$

Adding equations (1) and (2) we get

$$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$$

$$= a^2(1) + b^2(1)$$

$$= a^2 + b^2$$ Hence Proved.

9. Prove that : $\frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta.$

Ans : [Board Term-1, 2012, Set-50]

$$\frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = \frac{\cos^2\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= 1 + \sin\theta\cos\theta$$ Hence Proved

10. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, prove that $q(p^2 - 1) = 2p$

Ans : [Board Term-1, 2012, Set-38]

We have $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

$$q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)[(\cos\theta + \sin\theta)^2 - 1]$$

$$= (\sec\theta + \operatorname{cosec}\theta)(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$$

$$= (\sec\theta + \operatorname{cosec}\theta)[1 + 2\sin\theta\cos\theta - 1]$$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$

$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\sin\theta\cos\theta$$

$$= 2(\sin\theta + \cos\theta) = 2p$$ Hence Proved.

11. If $x = r\sin A\cos C, y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2 + y^2 + z^2 = r^2$

Ans : [Board Term-1, 2012, Set-50]

Since, $$x^2 = r^2\sin^2 A\cos^2 C$$

$$y^2 = r^2\sin^2 A\sin^2 C$$

and $$z^2 = r^2\cos^2 A$$

$$x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$$

$$= r^2\sin^2 A(\cos^2 C + \sin^2 C) + r^2\cos^2 A$$

$$= r^2\sin^2 A + r^2\cos^2 A$$

$$= r^2(\sin^2 A + \cos^2 A)$$

$= r^2$ Hence Proved.

12. Prove that: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$.

Ans : [Board Term-1, 2012, Set-40]

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)} \times \frac{(1-\sin\theta)}{(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta} = \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta$$ Hence Prove

13. Prove that
$(1-\sin\theta+\cos\theta)^2 = 2(1+\cos\theta)(1-\sin\theta)$.

Ans : [board Term-1, 2012, Set-62]

$$(1-\sin\theta+\cos\theta)^2$$

$$= 1+\sin^2\theta+\cos^2\theta-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$

$$= 1+1-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$

$$= 2+2\cos\theta-2\sin\theta-2\sin\theta\cos\theta$$

$$= 2(1+\cos\theta)-2\sin\theta(1+\cos\theta)$$

$$= (1+\cos\theta)(2-2\sin\theta)$$

$$= 2(1+\cos\theta)(1-\sin\theta)$$ Hence Proved

14. Prove that : $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta-1} = \sec\theta+\tan\theta$

Ans : [Board Term-1, 2012, Set-43]

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta+\sec\theta)-[1-\sec\theta+\tan\theta]}{\tan\theta-\sec\theta+1}$$

$$= \tan\theta+\sec\theta$$ Hence Proved

15. Prove that :
$(\sin\theta+\text{cosec}\,\theta)^2+(\cos\theta+\sec\theta)^2 = 7+\tan^2\theta+\cot^2\theta\cot^2\theta$

Ans : [Board Term-1,2012, Set-52]

$$(\sin\theta+\text{cosec}\,\theta)^2+(\cos\theta+\sec\theta)^2$$

$$= \sin^2\theta+\text{cosec}^2\theta+2\sin\theta\,\text{cosec}\,\theta+\cos^2\theta$$
$$+\sec^2\theta+2\cos\theta\sec\theta$$

$$= (\sin^2\theta+\cos^2\theta)+\text{cosec}^2\theta+2\sin\theta\times\frac{1}{\sin\theta}$$
$$+\sec^2\theta+2\cos\theta\times\frac{1}{\sin\theta}$$

$$= 1+(1+\cot^2\theta)+2+(1+\tan^2\theta)+2$$

$$= 7+\tan^2\theta+\cot^2\theta$$ Hence Proved

16. If $\sin\theta = \frac{c}{\sqrt{c^2+d^2}}$ and $d>0$, find the value of $\cos\theta$ and $\tan\theta$.

Ans : [Board Term - 1, 2013 LK-59]

We have $\sin\theta = \frac{c}{\sqrt{c^2+d^2}}$

Now $\cos^2\theta = 1-\sin^2\theta$

$$= 1-\left(\frac{c}{\sqrt{c^2+d^2}}\right)^2$$

$$= 1-\frac{c^2}{c^2+d^2}$$

$$= \frac{c^2+d^2-c^2}{c^2+d^2} = \frac{d^2}{c^2+d^2}$$

Thus $\cos\theta = \frac{d}{\sqrt{c^2+d^2}}$

Again, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{c}{\sqrt{c^2+d^2}}}{\frac{d}{\sqrt{c^2+d^2}}}$

Thus $\tan\theta = \frac{c}{d}$

17. If $\tan\theta = \frac{1}{\sqrt{5}}$,

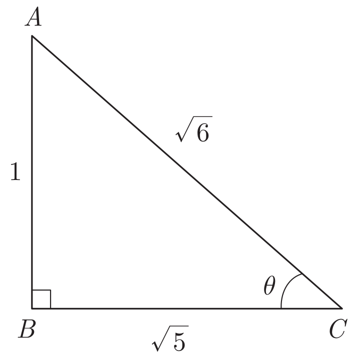
(1) Evaluate : $\frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}$

(2) Verify the identity : $\sin^2\theta+\cos^2\theta = 1$

Ans : [Board Term-1, 2012, Set-60]

We have $\tan\theta = \frac{1}{\sqrt{5}}$

We draw the triangle as shown below and write all dimensions.



(i) $\frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta} = \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$

$$= \frac{\cot^2\theta-\tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$

$$= \frac{(\sqrt{5})^2-(\frac{1}{5})^2}{2(\sqrt{5})^2+(\frac{1}{\sqrt{5}})^2}$$

$$= \frac{5-\frac{1}{5}}{2+5+\frac{1}{5}} = \frac{25-1}{35+1} = \frac{24}{36} = \frac{2}{3}$$

(2) $$\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$
$$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6}$$
$$= 1$$ Hence proved.

18. Evaluate :
$$\frac{\cot(90^\circ-\theta)\sin(90^\circ-\theta)}{\sin\theta} + \frac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ)$$

Ans : [Board Term-1, 2012, Set-35]

Given expression :
$$\frac{\cot(90^\circ-\theta)\sin(90^\circ-\theta)}{\sin\theta} + \frac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ)$$
$$= \frac{\tan\theta\cos\theta}{\sin\theta} + \frac{\cot(90^\circ-50^\circ)}{\tan 50^\circ} - [\cos^2 20^\circ + \cos^2(90^\circ-20^\circ)]$$
$$= \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} + \frac{\tan 50^\circ}{\tan 50^\circ} - [\cos^2 20^\circ + \sin^2 20^\circ]$$
$$= 1 + 1 - 1 = 1$$

19. Evaluate :
$$\frac{\text{cosec}^2(90^\circ-\theta) - \tan^2\theta}{4(\cos^2 40^\circ + \cos^2 50^\circ)} - \frac{2\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{3(\text{cosec}^2 70^\circ - \tan^2 20^\circ)}$$

Ans : [Board Terim-1, 2012, Set-52]

$$\text{cosec}^2(90^\circ-\theta) = \sec^2\theta$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\cos^2 40^\circ + \cos^2 50^\circ = \cos^2(90^\circ-50^\circ) + \cos^2 50^\circ$$
$$\sin^2 50^\circ + \cos^2 50^\circ = 1$$
$$\tan^2 30^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$
$$\sec^2 52^\circ \sin^2 38^\circ = \sec^2 52^\circ \sin^2(90^\circ - 52^\circ)$$
$$= \sec^2 52 \cos^2 52^\circ = 1$$

and
$$\text{cosec}^2 70^\circ - \tan^2 20^\circ = \text{cosec}^2(90^\circ-20^\circ) - \tan^2 20^\circ$$
$$= \sec^2 20^\circ - \tan^2 20^\circ = 1$$

Thus given expression becomes
$$= \frac{1}{4} - \frac{2\times\frac{1}{3}\times 1}{3(1)}$$
$$= \frac{1}{4} - \frac{2}{9} = \frac{9-8}{36} = \frac{1}{36}$$

20. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \frac{1}{p}$, Hence, find the values of $\cos\theta$ and $\sin\theta$.

Ans : [Board Term-1, 2015]

We have $\sec\theta + \tan\theta = p$

Now $$\frac{1}{p} = \frac{1}{\sec\theta+\tan\theta} \times \frac{(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta}$$
$$= \frac{\sec\theta-\tan\theta}{\sec^2\theta - \tan^2\theta} = \sec\theta - \tan\theta$$
$$= \sec\theta - \tan\theta$$

Solving $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \frac{1}{p}$,
$$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2+1}{2p}$$

and $$\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right) = \frac{p^2-1}{2p}$$

Thus $$\cos\theta = \frac{2p}{p^2+1}$$

and $$\sin\theta = \tan\theta\cos\theta = \frac{p^2-1}{p^2+1}$$

21. Prove that : $(\text{cosec}\,\theta + \cot\theta)^2 = \frac{\sec\theta+1}{\sec\theta-1}$

Ans :

$$(\text{cosec}\,\theta + \cot\theta)^2 = \text{cosec}^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta.\cot\theta$$
$$= \left(\frac{1}{\sin\theta}\right)^2 + \left(\frac{\cos\theta}{\sin\theta}\right) + \frac{2\times 1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}$$
$$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta}$$
$$= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{\sin^2\theta}$$
$$= \frac{(1+\cos\theta)(1+\cos\theta)}{1-\cos^2\theta}$$
$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$
$$= \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}$$
$$= \frac{\sec\theta+1}{\sec\theta-1}$$ Hence Prove.

22. Prove that :
$(\sin A + \sec A)^2 + (\cos A + \text{cosec}\,A)^2 = (1 + \sec A\,\text{cosec}\,A)^2$

Ans : [Board Term-I, 2012, Set 25]

$$\text{LHS} = (\sin A + \sec A)^2 + (\cos A + \text{cosec}\,A)^2$$
$$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$$
$$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A}$$
$$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + 2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$
$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A\cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}\right)$$
$$= 1 + \frac{1}{\sin^2 A\cos^2 A} + \frac{2}{\sin A\cos A}$$
$$= \left(1 + \frac{1}{\sin A\cos A}\right)^2$$
$$= (1 + \sec A.\,\text{cosec}\,A)^2$$ Hence Proved

23. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$
$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
Prove that each of the side is equal to ± 1.

Ans : [Board Term-I, 2012, Set-12]

We have

$(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)$
$\quad =(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$

Multiply both sides by

$(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$

or, $(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)\times$

$(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$

$=(\sec A-\tan A)^2(\sec B-\tan B)^2(\sec C-\tan C)^2$

or, $(\sec^2 A-\tan^2 A)(\sec^2 B-\tan^2 B)(\sec^2 C-\tan^2 C)$

$=(\sec A-\tan A)^2(\sec A-\tan B)^2(\sec C-\tan C)^2$

or, $1=\left[(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)\right]^2$

or, $(\sec A-\tan A)(\sec B-\tan B)(\sec C+\tan C)=\pm 1$

24. If $4\sin\theta=3$, find the value of x if $\sqrt{\dfrac{\text{cosec}^2\theta\cot^2\theta}{\sec^2\theta-1}}+2\cot\theta=\dfrac{\sqrt{7}}{x}+\cos\theta$

Ans : [Board Term-1, 2012, Set-40]

We have $\sin\theta=\dfrac{3}{4}$

or, $\sin^2\theta=\dfrac{9}{16}$

Since $\sin^2\theta+\cos^2=1$, we have

$$\cos^2\theta=1-\sin^2\theta=1-\frac{9}{16}=\frac{7}{16}$$

$$\cos\theta=\frac{\sqrt{7}}{4}$$

and $\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{\frac{3}{4}}{\frac{\sqrt{7}}{4}}=\dfrac{3}{\sqrt{7}}$

Thus $\sqrt{\dfrac{\text{cosec}^2\theta-\cot^2\theta}{\sec^2\theta-1}}+2\cot\theta=\dfrac{\sqrt{7}}{x}+\cos\theta$

$$\sqrt{\frac{1}{\tan^2\theta}}+2\times\frac{\sqrt{7}}{3}=\frac{\sqrt{7}}{x}+\frac{\sqrt{7}}{4}$$

$$\frac{1}{\tan\theta}+\frac{2\sqrt{7}}{3}=\frac{\sqrt{7}}{x}+\frac{\sqrt{7}}{4}$$

$$\frac{\sqrt{7}}{3}+\frac{2\sqrt{7}}{3}-\frac{\sqrt{7}}{4}=\frac{\sqrt{7}}{x}$$

$$\frac{4\sqrt{7}-\sqrt{7}}{4}=\frac{\sqrt{7}}{x}$$

$$\frac{3\sqrt{7}}{4}=\frac{\sqrt{7}}{x}$$

Thus $x=\dfrac{4}{3}$

HOTS QUESTIONS

1. Prove that $\sec^2\theta+\text{cosec}^2\theta$ can never be less than 2.

Ans :

Let $\sec^2\theta+\text{cosec}^2\theta=x$

$$1+\tan^2\theta+1+\cot^2\theta=x$$

$$2+\tan^2\theta+\cot^2\theta=x$$

$$2+\tan^2\theta+\cot^2\theta=x$$

$\tan^2\theta\geq 0$ and $\cot^2\theta\geq x$

Thus $x>2$

Thus $\sec^2\theta+\text{cosec}^2\theta>2$

Hence $\sec^2\theta+\text{cosec}^2\theta$ can never be less than 2.

2. (a) Solve for ϕ, if $\tan 5\phi=1$

(b) Solve for ϕ, if $\dfrac{\sin\phi}{1+\cos\phi}+\dfrac{1+\cos\phi}{\sin\phi}=4$

Ans :

(a) $\tan 5\phi=1$

$\tan 5\phi=\tan 45^\circ$

$5\phi=45^\circ$

Thus $\phi=9^\circ$

(b) $$\frac{\sin\phi}{1+\cos\phi}+\frac{1+\cos\phi}{\sin\phi}=4$$

$$\frac{\sin^2\phi+(1+\cos\theta)^2}{\sin\phi(1+\cos\phi)}=4$$

$$\frac{\sin^2\phi+1+\cos^2\phi+2\cos\phi}{\sin\phi+\sin\phi\cos\phi}=4$$

$$\frac{2+2\cos\phi}{\sin\phi(1+\cos\phi)}=4$$

$$\frac{2(1+\cos\phi)}{\sin\phi(1+\cos\phi)}=4$$

$$\frac{2}{\sin\phi}=4$$

$$\sin\phi=\frac{1}{2}$$

$$\sin\phi=\sin 30$$

Thus $\phi=30^\circ$

3. If $\tan A+\sin A=m$ and $\tan A-\sin A=n$, show that $m^2-n^2=4\sqrt{mn}$.

Ans :

We have $\tan A+\sin A=m$

and $\tan A-\sin A=n$

$m^2-n^2=(\tan A+\sin A)^2-(\tan A-\sin A)^2$

$=(\tan^2 A+\sin^2 A+2\sin A.\tan A)$
$\quad -(\tan^2 A+\sin^2 A-2\sin A.\tan A)$

$=\tan^2 A+\sin^2 A+2\sin A\tan A$
$\quad -\tan^2 A-\sin^2 A+2\sin A\tan A$

$=4\sin A\tan A$

$$4\sqrt{mn}=4\sqrt{(\tan A+\sin A)(\tan A-\sin A)}$$

$$=4\sqrt{\tan^2 A-\sin^2 A}$$

$$=4\sqrt{\frac{\sin^2 A-\sin^2 A\cos^2 A}{\cos^2 A}}$$

$$=4\sqrt{\frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}}$$

$$=4\sqrt{\frac{\sin^2 A\times\sin^2 A}{\cos^2 A}}$$

$$= 4\sin A\tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

4. If $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$, show that $(m^2+n^2)\cos^2\beta = n^2$.

Ans :

We have $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$

$$m^2 = \frac{\cos^2\alpha}{\cos^2\beta} \text{ and } n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

$$(m^2+n^2)\cos^2\beta = \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\frac{\sin^2\beta+\cos^2\beta}{\cos^2\beta\sin^2\beta}\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta}$$

$$= n^2$$ Hence Proved.

5. If $7\operatorname{cosec}\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\operatorname{cosec}\phi = 3$.

Ans :

We have $7\operatorname{cosec}\phi - 3\cot\phi = 7$

$$7\operatorname{cosec}\phi - 7 = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1) = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1)(\operatorname{cosec}\phi + 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7(\operatorname{cosec}^2\phi - 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot^2\phi = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot\phi = 3(\operatorname{cosec}\phi + 1)$$

$$7\cot\phi - 3\operatorname{cosec}\phi = 3$$ Hence Proved

6. Prove that : $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

Ans : [Sample Question Paper 2017-18]

$$\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta+1) - [(1-\cos\theta)(1+\cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\cos\theta + \sin\theta - 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta + \cot\theta$$ Hence Proved