CHAPTER 7

Coordinate Geometry

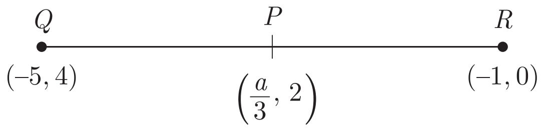
TOPIC 1 : DISTANCE BETWEEN TWO POINTS AND SECTION FORMULA

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the value of a, for which point $P(\frac{a}{3}, 2)$ is the midpoint of the line segment joining the Points $Q(-5,4)$ and $R(-1,0)$.

Ans : [Board Sample Paper, 2016]

As per question, line diagram is shown below.



Since P is mid-point of QR, we have

$$\frac{a}{3} = \frac{-5+(-1)}{2} = \frac{-6}{2} = -3$$

or, $$a = -9$$

2. The ordinate of a point A on y-axis is 5 and B has co-ordinates $(-3,1)$. Find the length of AB.

Ans : [Delhi CBSE, Term-2, 2014]

We have $A(0,5)$ and $B(-3,1)$.

Distance between A and B,

$$\begin{aligned} AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-0)^2+(1-5)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} = 5 \end{aligned}$$

3. Find the perpendicular distance of $A(5,12)$ from the y-axis.

Ans : [Board Terms-2, 2011 Set (A1)]

As per question, line diagram is shown below.
Perpendicular from point $A(5,12)$ on y-axis touch it at $(0,12)$.
Distance between $(5,12)$ and $(0,12)$ is,

$$\begin{aligned} d &= \sqrt{(0-5)^2+(12-12)^2} \\ &= \sqrt{25} \\ &= 5 \text{ units.} \end{aligned}$$

4. If the centre and radius of circle is $(3,4)$ and 7 units respectively,, then what it the position of the point $A(5,8)$ with respect to circle?

Ans : [Board Term-2, 2013]

Distance of the point, from the centre

$$\begin{aligned} a &= \sqrt{(5-3)^2+(8-4)^2} \\ &= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \end{aligned}$$

Since $2\sqrt{5}$ is less than 7, the point lies inside the circle.

5. Find the perimeter of a triangle with vertices $(0,4)$,$(0,0)$ and $(3,0)$.

Ans : [Board Term-2, 2011 Set (B1)]

We have $A(0,4)$, $B(0,0)$, and $C(3,0)$.

$$\begin{aligned} AB &= \sqrt{(0-2)^2+(0-4)^2} = \sqrt{16} = 4 \\ BC &= \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3 \\ CA &= \sqrt{(0-3)^2+(4-0)^2} \\ &= \sqrt{9+16} = \sqrt{25} = 5 \end{aligned}$$

Thus Perimeter of triangle $= 4+3+5 = 12$

6. To locate a point Q on line segment AB such that $BQ = \frac{5}{7} \times AB$. What is the ratio of line segment in which AB is divided?

Ans : [Board Term-2, 2013]

We have $$BQ = \frac{5}{7}AB$$

$$\frac{BQ}{AB} = \frac{5}{7} \Rightarrow \frac{AB}{BQ} = \frac{7}{5}$$

$$\frac{AB-BQ}{BQ} = \frac{AQ}{BQ} = \frac{7-5}{5} = \frac{2}{5}$$

$$AQ : BQ = 2:5$$

7. Find the distance of the point $(-4,-7)$ from the y-axis.

Ans : [Board Term-2, 2013]

As per question, line diagram is shown below.
Perpendicular from point $A(-4,-7)$ on y-axis touch it at $(0,-7)$.
Distance between $(-4,-7)$ and $(0,-7)$ is

$$\begin{aligned} d &= \sqrt{(0+4)^2+(-7+7)^2} \\ &= \sqrt{4^2+0} = \sqrt{16} = 4 \text{ units} \end{aligned}$$

8. If the distance between the points $(4,k)$ and $(1,0)$ is 5, then what can be the possible values of k.

Ans : [Delhi Set I, II, III 2017]

Using distance formula

$$\begin{aligned} \sqrt{(4-1)^2+(k-0)^2} &= 5 \\ 3^2+k^2 &= 25 \\ k &\pm 4 \end{aligned}$$

9. Find the coordinates of the point on y-axis which is nearest to the point $(-2,5)$.

Ans : [Sample Question Poper, 2017]

The point on y-axis that is nearest to the point $(-2,5)$ is $(0,5)$.

10. In what ratio does the x-axis divide the line segment joining the points $(-4,-6)$ and $(-1,7)$? Find the coordinates of the point of division.

Ans : [Board Sample Paper, 2017]

Let x-axis be divides the line-segment joining $(-4,-6)$ and $(-1,7)$ at the point $P(x,y)$ in the ratio $1:k$.

Now, the coordinates of point of division P,

$$(x,y) = \frac{1(-1)+k(-4)}{k+1}, \frac{1(7)+k(-6)}{k+1}$$

$$= \frac{-1-4k}{k+1}, \frac{7-6k}{k+1}$$

Since P lies on x axis, therefore $y=0$, which gives

$$\frac{7-6k}{k+1} = 0$$

$$7-6k = 0$$

$$k = \frac{7}{6}$$

Hence, the ratio is $1:\frac{7}{6}$ or, $6:7$ and the coordinates of P are $\left(-\frac{34}{13}, 0\right)$

SHORT ANSWER TYPE QUESTIONS - I

1. Find a relation between x and y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$.

Ans : [Board Sample Paper, 2016]

Let $P(x,y)$ is equidistant from $A(-5,3)$ and $B(7,2)$, then we have

$$AP = BP$$

$$\sqrt{(x+5)^2+(y-3)^2} = \sqrt{(x-7)^2+(y-2)^2}$$

$$(x+5)^2+(y-3)^2 = (x-7)^2+(y-2)^2$$

$$10x+25-6y+9 = -14x+49-4y+4$$

$$24x+34 = 2y+53$$

$$24x-2y = 19$$

Thus $24x-2y-19 = 0$ is the required relation.

2. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,-5)$ and $R(-3,6)$, find the co-ordinates of P.

Ans : [Delhi Set I, II, III, 2016]

Let the point P $(2y,y)$,

Since $PQ = PR$, we have

$$\sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$

$$(2y-2)^2+(y+5)^2 = (2y+3)^2+(y-6)^2$$

$$-8y+4+10y+25 = 12y+9-12y+36$$

$$2y+29 = 45$$

$$y = 8$$

Hence, coordinates of point P are $(16,8)$

3. Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the co-ordinates of the point of division.

Ans : [Delhi Set I, II, III, 2016]

Let y-axis be divides the line-segment joining $A(5,-6)$ and $B(-1,-4)$ at the point $P(x,y)$ in the ratio $AP:PB = k:1$

Now, the coordinates of point of division P,

$$(x,y) = \frac{k(-1)+1(5)}{k+1}, \frac{k(-4)+1(-6)}{k+1}$$

$$= \frac{-k+5}{k+1}, \frac{-4k-6}{k+1}$$

Since P lies on y axis, therefore $x=0$, which gives

$$\frac{5-k}{k+1} = 0$$

$$k = 5$$

Hence required ratio is $5:1$

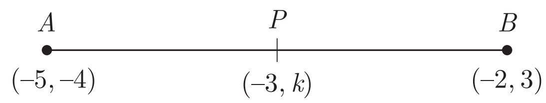
Now $$y = \frac{-4(5)-6}{6} = \frac{-13}{3}$$

Hence point on y-axis is $(0,-\frac{13}{3})$.

4. Find the ratio in which the point $(-3,k)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Also find the value of k.

Ans : [Foreign Set I, II, III, 2016]

As per question, line diagram is shown below.



Let AB be divides by P in ratio $n:1$.

x co-ordinate for section formula

$$-3 = \frac{(-2)n+1(-5)}{n+1}$$

$$-3(n+1) = -2n-5$$

$$-3n-3 = -2n-5$$

$$5-3 = 3n-2n$$

$$2 = n$$

Ratio $$\frac{n}{1} = \frac{2}{1} \text{ or } 2:1$$

Now, y co-ordinate,

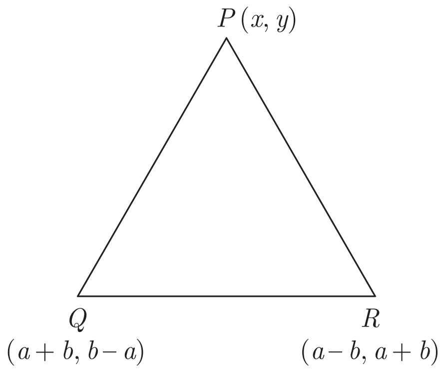
$$k = \frac{2(3)+1(-4)}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

5. If the point $P(x,y)$ is equidistant from the points $Q(a+b,b-a)$ and $R(a-b,a+b)$, then prove that $bx=ay$.

Ans : [O.D. Set I, II, III, 2016] [Board Term-2, 2012 Set (12)]

We have $|PQ| = |PR|$

$$\sqrt{[x-(a+b)]^2+[y-(b-a)]^2} = \sqrt{[x-(a-b)]^2+[y-(b+a)]^2}$$



$$[x-(a+b)]^2+[y-(b-a)]^2 = [x-(a-b)]^2+[y-(a+b)]^2$$

$$-2x(a+b)-2y(b-a) = -2x(a-b)-2y(a+b)$$

$$2x(a+b)+2y(b-a) = 2x(a-b)+2y(a+b)$$

$$2x(a+b-a+b)+2y(b-a-a-b) = 0$$

$$2x(2b)+2y(-2a) = 0$$

$$xb-ay = 0$$

$$bx = ay$$ Hence Proved

6. Prove that the point $(3,0)$, $(6,4)$ and $(-1,3)$ are the vertices of a right angled isosceles triange.

Ans : [O.D. Set I, II, III, 2016]

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now $AB^2 = (3-6)^2+(0-4)^2$

$= 9+16 = 25$

$BC^2 = (6+1)^2+(4-3)^2$

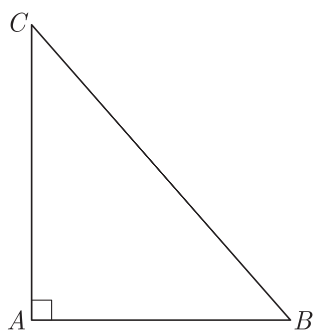
$= 49+1 = 50$

$CA^2 = (-1-3)^2+(3-0)^2$

$= 16+9 = 25$

$AB^2 = CA^2$ or, $AB = CA$

Hence triangle is isosceles.



Also, $25+25 = 50$

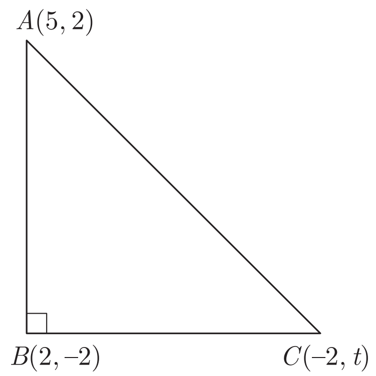
or, $AB^2+CA^2 = BC^2$

Since pythagoras theorem is verified, therefore triangle is a right angled triangle.

7. If $A(5,2)$, $B(2,-2)$ and $C(-2,t)$ are the vertices of a right angled triangle with $\angle B = 90°$, then find the value of t.

Ans : [Delhi CBSE Board, 2015][Set I, II, III]

As per question, triangle is shown below.



Now $AB^2 = (2-5)^2+(-2-2)^2 = 9+16 = 25$

$BC^2 = (-2-2)^2+(t+2)^2 = 16+(t+2)^2$

$AC^2 = (5+2)^2+(2-t)^2 = 49+(2+t^2)$

Since ΔABC is a right angled triangle

$$AC^2 = AB^2+BC^2$$

$$49+(2-t)^2 = 25+16+(t+2)^2$$

$$49+4-4t+t^2 = 41+t^2+4t+4$$

$$53-4t = 45+4t$$

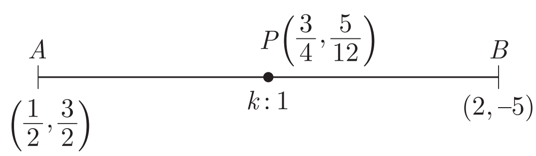
$$8t = 8$$

$$t = 1$$

8. Find the ratio in which the pont $P(\frac{3}{4},\frac{5}{12})$ divides the line segment joining the point A $(\frac{1}{2},\frac{3}{2})$ and $(2,-5)$.

Ans : [Delhi CBSE Term-2, 2015, Set I, II, III]

Let P divides AB in the ration $k{:}1$. Line diagram is shown below.



Now $$\frac{k(2)+1(\frac{1}{2})}{k+1} = \frac{3}{4}$$

$$8k+2 = 3k+3$$

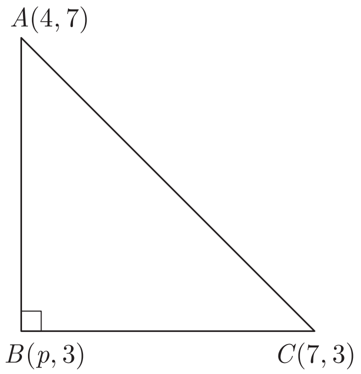
$$k = \frac{1}{5}$$

Thus required ratio is $\frac{1}{5}{:}1$ or 1:5.

9. The points $(4,7)$, $B(p,3)$ and $C(7,3)$ are the vertices of a right triangle, right-angled at B. Find the value of p.

Ans : [Outside Delhi CBSE, 2015, Set I, II]

As per question, triangle is shown below. Here ΔABC is a right angle triangle,



$$AB^2 + BC^2 = AC^2$$

$$(p-4)^2 + (3-7)^2 + (7-p)^2 + (3-3)^2 = (7-4)^2 + (3-4)^2$$

$$(p-4)^2 + (-4)^2 + (7-p)^2 + 0 = (3)^2 + (-4)^2$$

$$p^2 - 8p + 16 + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

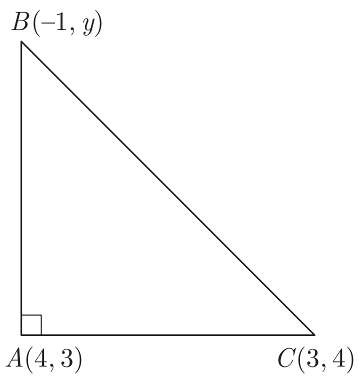
$$(p-4)(p-7) = 0$$

$$p = 7 \text{ or } 4$$

10. If $A(4,3)$, $B(-1,y)$, and $C(3,4)$ are the vertices of a right triangle ABC, right angled at A, then find the value of y.

Ans : [Outside Delhi Board, 2015, Set II]

As per question, triangle is shown below.



We have $AB^2 + AC^2 = BC^2$

$$(4+1)^2 + (3-y)^2 + (4-3)^2 = (3+1)^2 + (4-y)^2$$

$$(5)^2 + (3-y)^2 + (-1)^2 + (1)^2 = (4)^2 + (4-y)^2$$

$$25 + 9 - 6y + y^2 + 1 + 1 = 16 + 16 - 8y + y^2$$

$$36 + 2y - 32 = 0$$

$$2y + 4 = 0$$

$$y = -2$$

11. Show that the points (a,a), $(-a,-a)$ and $(-\sqrt{3}\,a, \sqrt{3}\,a)$ are the vertices of an equilateral triangle.

Ans : [Foreign Set I, II, III, 2015]

Let $A(a,a), B(-a,-a)$ and $C(-\sqrt{3}\,a, \sqrt{3}\,a)$

$$AB = \sqrt{(a+a)^2 + (a+a)^2}$$

$$= \sqrt{4a^2 + 4a^2}$$

$$= 2\sqrt{2}\,a$$

$$BC = \sqrt{(-a+\sqrt{3}\,a)^2 + (-a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 - 2\sqrt{3}\,a^2 + 3a^2 + a^2 + 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

$$AC = \sqrt{(a+\sqrt{3}\,a)^2 + (a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 + 2\sqrt{3}\,a^2 + 3a^2 + a^2 - 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

12. If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, find x, y.

Ans : [Delhi CBSE, Term II, 2014][Board Term-2, 2012 Set (1)]

If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, then at mid point,

$$\frac{\frac{x}{2} + x + 1}{2} = 5$$

$$\frac{3x}{2} + 1 = 10$$

$$3x = 18$$

or, $$x = 6$$

also $$\frac{\frac{y+1}{2} + y - 3}{2} = -2$$

$$\frac{y+1}{2} + y - 3 = -4$$

$$y + 1 + 2y - 6 = -8$$

$$y = -1$$

13. Find the point on the x-axis which is equidistant from the points $(2,-5)$ and $(-2,9)$.

Ans : [Board Term-2, 2012 Set (22)]

Let the point $P(x,0)$ on the x-axis is equidistant from points $A(2,-5)$ and $B(-2,9)$.

$$PA^2 = PB^2$$

$$(2-x)^2 + (-5-0)^2 = (-2-x)^2 + (9-0)^2$$

$$4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$$

$$-8x = 56$$

$$x = -7$$

Thus point is $(-7,0)$.

14. Show that $A(6,4)$, $B(5,-2)$ and $C(7,-2)$ are the vertices of an isosceles triangle.

Ans : [Board Term-2, 2012 Set (44)]

We have $A(6,4), B(5,-2), C(7,-2)$.

Now $AB = \sqrt{(6-5)^2+(4+2)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$BC = \sqrt{(5-7)^2+(-2+2)^2}$

$= \sqrt{(-2)^2+0^2} = 2$

$CA = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$AB = BC = \sqrt{37}$

Since two sides of a triangle are equal in length, triangle is an isosceles triangle.

15. If $P(2,-1), Q(3,4), R(-2,3)$ and $S(-3,-2)$ be four points in a plane, show that $PQRS$ is a rhombus but not a square.

Ans : [Board Term-2, 2012 (28)]

We have $P(2,-1), Q(3,4), R(-2,3), S(-3,-2)$

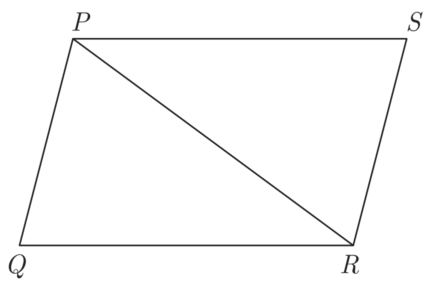
$PQ = \sqrt{1^2+5^2} = \sqrt{26}$

$QR = \sqrt{5^2+1^2} \ \sqrt{26}$

$RS = \sqrt{1^2+5^2} = \sqrt{26}$

$PS = \sqrt{5^2+1^2} = \sqrt{26}$

Since all the four sides are equal, $PQRS$ is a rhombus.



Now $PR = \sqrt{1^2+5^2} = \sqrt{26}$

$= \sqrt{4^2+4^2} = \sqrt{32}$

$PQ^2 + QR^2 = 2 \times 26 = 52 \neq (\sqrt{32})^2$

Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

16. Show that $A(-1,0), B(3,1), C(2,2)$ and $D(-2,1)$ are the vertices of a parallelogram $ABCD$.

Ans : [Board Term-2, 2012 Set (1)]

Mid-point of AC

$\left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Mid-point BD

$\left(\frac{3-2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2}, 1\right)$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

17. If $(3,2)$ and $(-3,2)$ are two vertices of an equilateral triangle which contains the origin, find the third vertex.

Ans : [Board Term-2, 2012 Set (12)]

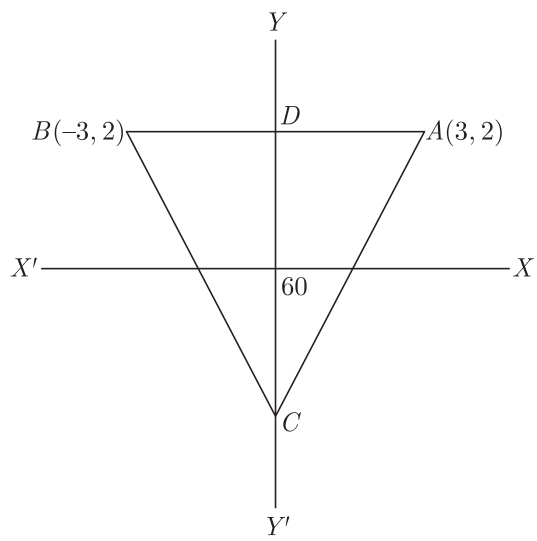
We have $A(3,2)$ and $B(-3,2)$.

It can be easily seen that mid-point of AB is lying on y-axis. Thus AB is equal distance from x-axis everywhere.

Also $OD \perp AB$

Hence 3^{rd} vertex of ΔABC is also lying on y-axis.

The digram of triangle should be as given below.



Let $C(x,y)$ be the coordinate of 3^{rd} vertex of ΔABC.

Now $AB^2 = (3+3)^2 + (2-2)^2 = 36$

$BC^2 = (x+3)^2 + (y-2)^2$

$AC^2 = (x-3)^2 + (y-2)^2$

Since $AB^2 = AC^2 = BC^2$

$(x+3)^2 + (y-2)^2 = 36 \quad (1)$

$(x-3)^2 + (y-2)^2 = 36 \quad (2)$

Since $P(x,y)$ lie on y-axis, substituting $x = 0$ in(1) we have

$3^2 + (y-2)^2 = 36 - 9 = 27$

$(y-2)^2 = 36 - 9 = 27$

Taking square root both side

$y - 2 = \pm 3\sqrt{3}$

$y = 2 \pm 3\sqrt{3}$

Since origin is inside the given triangle, coordinate of C below the origin,

$y = 2 - 3\sqrt{3}$

Hence Coordinate of C is $(0, 2-3\sqrt{3})$

18. Find a so that $(3,a)$ lies on the line represented by $2x - 3y - 5 = 0$. Also, find the co-ordinates of the point where the line cuts the x-axis.

Ans : [Board Term-2 Set (34)]

Since $(3,a)$ lies on $2x - 3y - 5 = 0$, it must satisfy this equation. Therefore

$2 \times 3 - 3a - 5 = 0$

$6 - 3a - 5 = 0$

$1 = 3a$

$a = \frac{1}{3}$

Line $2x - 3y - 5 = 0$ will cut the x-axis at $(x,0)$. and it must satisfy the equation of line.

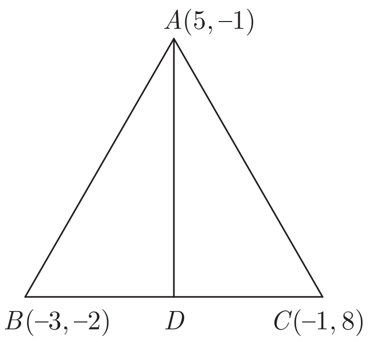
$$2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

Hence point is $\left(\frac{5}{2}, 0\right)$

19. If the vertices of ΔABC are $A(5,-1), B(-3,-2)$, $C(-1,8)$, Find the length of median through A.

Ans : [Board Term-2, 2012 Set (17)]

Let AD be the median. As per question, triangle is shown below.



Since D is mid-point of BC, co-ordinates of D,

$$(x_1, y_2) = \left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$$

$$= (-2, 3)$$

$$AD = \sqrt{(5+2)^2 + (-1-3)^2}$$

$$= \sqrt{(7)^2 + (4)^2}$$

$$= \sqrt{49+16}$$

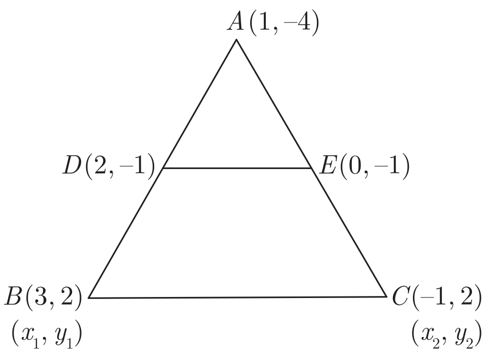
$$= \sqrt{65} \text{ units}$$

Thus length of median is $\sqrt{65}$

20. Find the mid-point of side BC of ΔABC, with $A(1,-4)$ and the mid-points of the sides through A being $(2,-1)$ and $(0,-1)$.

Ans : [Board Term-2, 2012 Set (21)]

Assume co-ordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively. As per question, triangle is shown below.



Now $\quad 2 = \frac{1+x_1}{2} \Rightarrow x_1 = 3$

and $\quad -1 = \frac{-4+y_1}{2} \Rightarrow y_1 = 2$

$$0 = \frac{1+x_2}{2} \Rightarrow x = -1$$

$$-1 = \frac{-4+y_2}{2} \Rightarrow y_2 = 2$$

Thus $\quad B(x_1, y_1) = (3, 2),$

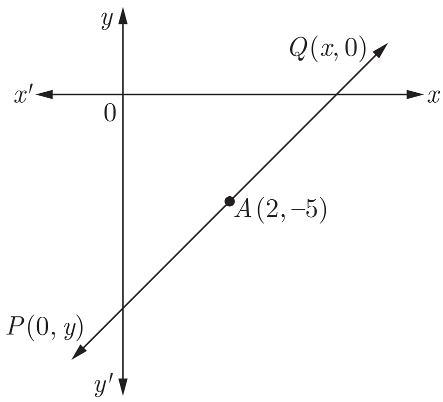
$$C(x_2, y_2) = (-1, 2)$$

So, mid-point of BC is $\left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1, 2)$

21. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2,-5)$ is the mid-point of PQ, then find the coordinates of P and Q.

Ans : [Outside Delhi, Set-III, 2017]

Let coordinates of P be $(0, y)$ and of Q be $(x, 0)$.
$A(2,-5)$ is mid point of PQ.
As per question, line diagram is shown below.



Using section formula,

$$(2, -5) = \left(\frac{0+x}{2} + \frac{y+0}{2}\right)$$

$$2 = \frac{x}{2} \Rightarrow x = 4$$

and $\quad -5 = \frac{y}{2} \Rightarrow y = -10$

Thus P is $(0, -10)$ and Q is $(4, 0)$

22. If $\left(1, \frac{p}{3}\right)$ is the mid point of the line segment joining the points $(2, 0)$ and $\left(0, \frac{2}{9}\right)$, then show that the line $5x + 3y + 2 = 0$ passes through the point $(-1, 3p)$.

Ans :

Since $\left(1, \frac{p}{3}\right)$ is the mid point of the line segment joining the points $(2, 0)$ and $\left(0, \frac{2}{9}\right)$, we have

$$\frac{p}{3} = \frac{0+\frac{2}{9}}{2} = \frac{1}{9}$$

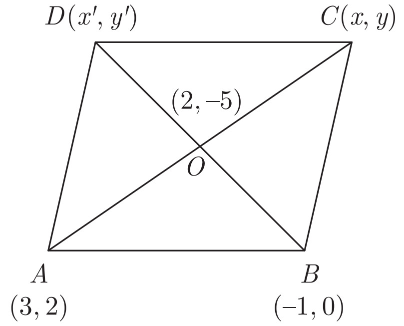
$$p = \frac{1}{3}$$

Now the point $(-1, 3p)$ is $(-1, 1)$.
The line $5x + 3y + 2 = 0$, passes through the point $(-1, 1)$ as $5(-5) + 3(1) + 2 = 0$

23. If two adjacent vertices of a parallelogram are $(3, 2)$ and $(-1, 0)$ and the diagonals intersect at $(2, -5)$ then find the co-ordinates of the other two vertices.

Ans : [Board Foreign Set I, II, III, 2017]

Let two other co-ordinates be (x, y) and (x', y') respectively using mid-point formula.
As per question parallelogram is shown below.



Now $\qquad 2 = \frac{x+3}{2} \Rightarrow x = 1$

and $\qquad -5 = \frac{2+y}{2} \Rightarrow y = -12$

Again, $\qquad \frac{-1+x'}{2} = 2 \Rightarrow x' = 5$

and $\qquad \frac{0+y'}{2} = -5 \Rightarrow y' = -10$

Hence, coordinates of $C(1, -12)$ and $D(5, -10)$

24. In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$?

Ans : [Delhi Compt. Set-I, II, III 2017]

Let $\qquad AP:PB = k:1$

Now $\qquad \frac{3k-6}{k+1} = -4$

$$3k - 6 = -4k - 4$$

$$7k = 2$$

$$k = \frac{7}{2}$$

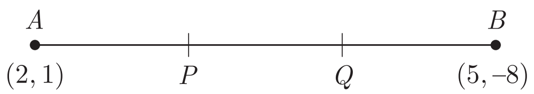
]

Hence, $\qquad AP:PB = 7:2$

25. If the line segment joining the points $A(2,1)$ and $B(5,-8)$ is trisected at the points P and Q, find the coordinates P.

Ans : [Outside Delhi Compt. Set-I, III, 2017]

As per question, line diagram is shown below.



Let $P(x,y)$ divides AB in the ratio $1:2$
Using section formula we get

$$x = \frac{1\times 5 + 2\times 2}{1+2} = 3$$

$$y = \frac{1\times -8 + 2\times 1}{1+2} = 2$$

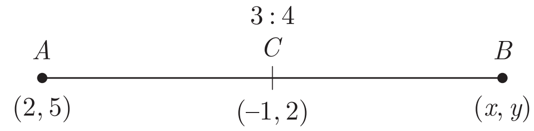
Hence coordinates of P are $(3,-2)$.

SHORT ANSWER TYPE QUESTIONS - II

1. If the point $C(-1,2)$ divides internally the line segment joining the points $A(2,5)$ and $B(x,y)$ in the ratio $3:4$, find the value of x^2+y^2.

Ans : [Foreign Set I, II, III, 2016]

As per question, line diagram is shown below.



We have $\qquad \frac{AC}{BC} = \frac{3}{4}$

Applying section formula for x co-ordinate,

$$-1 = \frac{3x + 4(2)}{3+4}$$

$$-7 = 3x + 8$$

$$x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y + 4(5)}{3+4}$$

$$14 = 3y + 20$$

$$y = 2$$

Thus (x,y) is $(-5,-2)$.

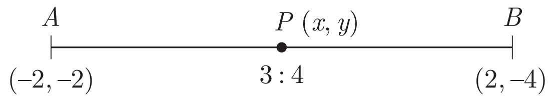
Now $\qquad x^2 + y^2 = (-5)^2 + (-2)^2$
$\qquad\qquad = 25 + 4 = 29$

2. If the co-ordinates of points A and B are $(-2,-2)$ and $(2,-4)$ respectively, find the co-ordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

Ans : [Outside Delhi, 2015, Set I, II]

We have $\qquad AP = \frac{3}{7}AB \Rightarrow AP:PB = 3:4$

As per question, line diagram is shown below.



Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{3\times 2 + 4\times -2}{3+4} = -\frac{2}{7}$$

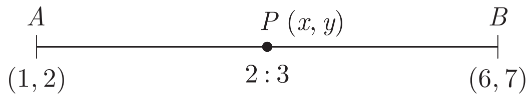
$$y = \frac{3\times -4 + 4\times -2}{3+4} = -\frac{20}{7}$$

Hence P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$

3. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$

Ans : [Outside Delhi, 2015, Set III]

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP : PB = 2:3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2\times 6 + 3\times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2\times 7 + 3\times 2}{2+3} = \frac{14+6}{5} = 4$$

Thus $P(x, y) = (3, 4)$

4. If the distance of $P(x,y)$ from $A(6,2)$ and $B(-2,6)$ are equal, prove that $y = 2x$.

Ans : [CBSE Board Term-2, 2015]

We have $P(x,y), A(6,2), B(-2,6)$

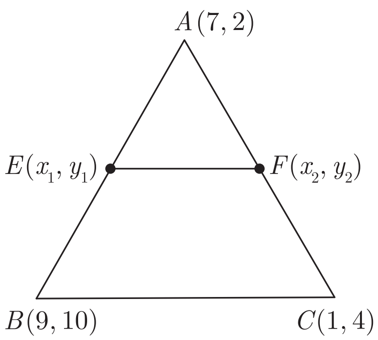
Now
$$PA = PB$$
$$PA^2 = PB^2$$
$$(x-6)^2 + (y-2)^2 = (x+2)^2 + (y-6)^2$$
$$-12x + 36 - 4y + 4 = 4x + 4 - 12y + 36$$
$$-12x - 4y = 4x - 12y$$
$$12y - 4y = 4x + 12x$$
$$8y = 16x$$
$$y = 2x$$

Hence Proved

5. The co-ordinates of the vertices of ΔABC are $A(7,2)$, $B(9,10)$ and $C(1,4)$. If E and F are the mid-points of AB and AC respectively, prove that $EF = \frac{1}{2}BC$.

Ans : [Board Term-2 2015]

Let the mid-points of AB and AC be $E(x_1, y_1)$ and $F(x_2, y_2)$. As per question, triangle is shown below.



Co-ordinates of point E

$$(x_1, y_1) = \left(\frac{9+7}{2}, \frac{10+2}{2}\right) = (8, 6)$$

Co-ordinates of point F

$$(x_2, y_2) = \left(\frac{7+1}{2}, \frac{2+4}{2}\right) = (4, 3)$$

Length, $$EF = \sqrt{(x-4)^2 + (6-3)^2}$$
$$= \sqrt{(4)^2 + (3)^2}$$
$$= 5 \text{ units} \quad ...(1)$$

Length $$BC = \sqrt{(9-1)^2 + (10-4)^2}$$
$$= \sqrt{(8)^2 + (6)^2}$$
$$= 10 \text{ units} \quad ...(2)$$

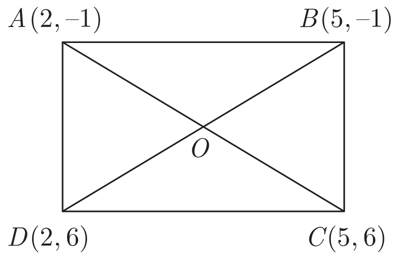
From equation (1) and (2) we get

$$EF = \frac{1}{2}BC$$ Hence proved.

6. Prove that the diagonals of a rectangle $ABCD$, with vertices $A(2,-1), B(5,-1), C(5,6)$ and $D(2,6)$ are equal and bisect each other.

Ans : [CBSE O.D. 2014]

As per question, rectangle $ABCD$, is shown below.



Now $$AC = \sqrt{(5-2)^2 + (6+1)^2} = \sqrt{3^2 + 7^2}$$
$$= \sqrt{9+49} = \sqrt{58}$$
$$BD = \sqrt{(5-2)^2 + (-1-6)^2} = \sqrt{3^2 + 7^2}$$
$$= \sqrt{9+49} = \sqrt{58}$$

Since $AC = BD = \sqrt{58}$ the diagonals of rectangle $ABCD$ are equal

Mid-point of AC

$$= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Mid-point of BD

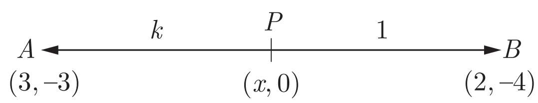
$$= \left(\frac{2+5}{2}, \frac{6+-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Since the mid-point of diagonal AC and mid-point of diagonal BD is same and equal to $(\frac{7}{5}, \frac{5}{2})$. Hence they bisect each other.

7. Find the ratio in which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Ans : [Delhi, Term-2, 2014]

y co-ordinate of any point on the x will be zero. Let $(x, 0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$.
Using section formula for y co-ordinate we have

$$0 = \frac{1(-3) + k(7)}{1+k}$$

$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have

$$x = \frac{1(3)+k(-2)}{1+k} = \frac{3-2\times\frac{3}{7}}{1+\frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $\left(\frac{3}{2},0\right)$.

8. Find the ratio in which $(11,15)$ divides the line segment joining the points $(15,5)$ and $(9,20)$

Ans : [board Term-2, 2014]

Let the two points $(15,5)$ and $(9,20)$ are divided in the ratio $k:1$ by point $P(11,15)$
Using Section formula, we get

$$x = \frac{m_2x_1+m_1x_2}{m_2+m_1}$$

$$11 = \frac{1(15)+k(9)}{1+k}$$

$$11+11k = 15+9k$$

$$k = 2$$

Thus ratio is 2 : 1.

9. Find the point on y-axis which is equidistant from the points $(5,-2)$ and $(-3,2)$.

Ans : [Delhi Set, 2014]
[Board Term-2, 2012 Set (13)]

Let point be $(0,y)$

$$5^2+(y+2)^2 = (3)^2+(y-2)^2$$

or, $$y^2+25+4y+4 = 9-4y+4$$

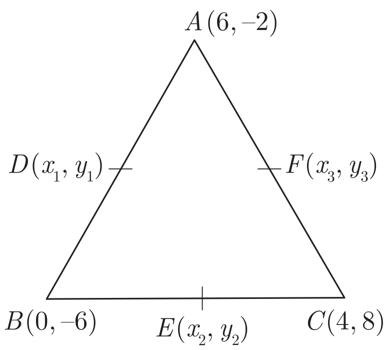
$$8y = -16 \text{ or, } y=-2$$

or, Point $(0,-2)$

10. The vertices of ΔABC are $A(6,-2), B(0,-6)$ and $C(4,8)$. Find the co-ordinates of mid-points of AB, BC and AC.

Ans : [Board Term-2, 2014]

Let mid-point of AB, BC and AC be $D(x_1,y_1)$, $E(x_2,y_2)$ and $F(x_2,y_3)$. As per question, triangle is shown below.



Using section formula, the co-ordinates of the points D,E,F are

For D, $$x_1 = \frac{6+0}{2} = 3$$

$$y_1 = \frac{-2-6}{2} = -4$$

For E, $$x_2 = \frac{0+4}{2} = 2$$

$$y_2 = \frac{-6+8}{2} = 1$$

For F, $$x_3 = \frac{4+6}{2} = 5$$

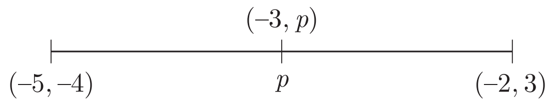
$$y_3 = \frac{-2+8}{2} = 3$$

The co-ordinates of the mid-points of AB, BC and AC are $D(3,-4)$, $E(2,1)$ and $F(5,3)$ respectively.

11. Find the ratio in which the point $(-3,p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Hence find the value of p.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let $X(-3,p)$ divides the line joining of $A(-5,-4)$ and $B(-2,3)$ in the ratio $k:1$.
The co-ordinates of p are $\left[\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right]$
But co-ordinates of P are $(-3,p)$. Therefore we get

$$\frac{-2k-5}{k+1} = -3 \Rightarrow k=2$$

and $$\frac{3k-4}{k+1} = p$$

Substituting $k=2$ gives

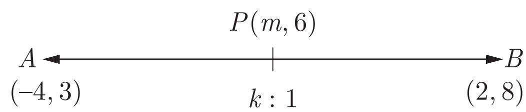
$$p = \frac{2}{3}$$

Hence ratio of division is 2:1 and $p = \frac{2}{3}$

12. Find the ratio in which the point $p(m,6)$ divides the line segment joining the points $A(-4,3)$ and $B(2,8)$. Also find the vlaue of m.

Ans : [Board Term-2, 2012 set (31)]

As per question, line diagram is shown below.



Let the ratio be $k:1$
Using section formula, we have

$$m = \frac{2k+(-4)}{k+1} \quad (1)$$

$$6 = \frac{8k+3}{k+1} \quad (2)$$

$$8k+3 = 6k+6$$

$$2k = 3$$

$$k = \frac{3}{2}$$

Thus ratio is $\frac{3}{2}:1$ or 3:2.
Substituting value of k in (1) we have

$$m = \frac{2(\frac{3}{2})+(-4)}{\frac{3}{2}+1} = \frac{3-4}{\frac{5}{2}} = \frac{-1}{\frac{5}{2}} = \frac{-2}{5}$$

13. If $A(4,-1), B(5,3), C(2,y)$ and $D(1,1)$ are the vertices of a parallelogram $ABCD$, find y.

Ans : [board Term-2, 2012 Set (5)]

Diagonals of a parallelogram bisect each other. Mid-points of AC and BD are same.

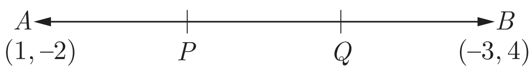
Thus $\left(3, \frac{-1+y}{2}\right) = (3,2)$

$$\frac{-1+y}{2} = 2 \Rightarrow y = 5$$

14. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Ans : [Board Term-2, 2012 Set(34)]

Let $P(x_1,y_1), Q(x_2,y_2)$ divides AB into 3 equal parts. Thus P divides AB in the ratio of 1:2.
As per question, line diagram is shown below.



Now $$x_1 = \frac{1(-3)+2(1)}{1+3} = \frac{-3+2}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1(4)+2(-2)}{1+2} = \frac{4-4}{3} = 0$$

Co-ordinates of P is $(-\frac{1}{3}, 0)$.

Here Q is mid-point of PB.

Thus $$x_2 = \frac{-\frac{1}{3}+(-3)}{2} = \frac{-10}{6} = \frac{-5}{3}$$

$$y_2 = \frac{0+4}{2} = 2$$

Thus co-ordinates of Q is $(-\frac{5}{2}, 2)$.

15. If (a,b) is the mid-point of the segment joining the points $A(10,-6)$ and $B(k,4)$ and $a-2b=18$, find the value of k and the distance AB.

Ans : [Board Term-2, 2012 Set(21)]

We have $A(10,-6)$ and $B(k,4)$.
If $P(a,b)$ is mid-point of AB, then we have

$$(a,b) = \left(\frac{k+10}{2}, \frac{-6+4}{2}\right)$$

$$a = \frac{k+10}{2} \text{ and } b = -1$$

From given condition we have

$$a - 2b = 18$$

Substituting value $b=-1$ we obtain

$$a + 2 = 18 \Rightarrow a = 16$$

$$a = \frac{k+10}{2} = 16 \Rightarrow k = 22$$

$$P(a,b) = (16,1)$$

$$AB = \sqrt{(22-10)^2+(4+6)^2}$$

$$= 2\sqrt{61} \text{ units}$$

16. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points $(8,-9)$ and $(2,1)$. Also find the co-ordinates of the point of division.

Ans : [Board Term-2, 2012 Set(21)]

Let a point $P(x,y)$ on line $2x+3y-5=0$ divides AB in the ratio $k:1$.

Now $$x = \frac{2k+8}{k+1}$$

and $$y = \frac{k-9}{k+1}$$

Substituting above value in line $2x+3y-5=0$ we have

$$2\left(\frac{2k+8}{k+1}\right)+3\left(\frac{k-9}{k+1}\right)-5 = 0$$

$$4k+16+3k-27-5k-5 = 0$$

$$2k-16 = 0$$

$$k = 8$$

Thus ratio is 8 : 1.

Substituting the value $k=8$ we get

$$x = \left(\frac{2\times 8+8}{8+1}\right) = \frac{8}{3}$$

$$y = \left(\frac{8-9}{8+1}\right) = -\frac{1}{9}$$

Thus $$P(x,y) = \left(\frac{8}{3}, -\frac{1}{9}\right)$$

17. Find the area of the rhombus of vertices $(3,0),(4,5),(-1,4)$ and $(-2,-1)$ taken in order.

Ans : [Board Term-2, 2012 Set (40)]

We have $A(3,0), B(4,5), C(-1,4), D(-2,-1)$

Diagonal AC, $$d_1 = \sqrt{(3+1)^2+(0-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32}$$
$$= \sqrt{16\times 2} = 4\sqrt{2}$$

Diagonal BD, $$d_2 = \sqrt{(4+2)^2+(5+1)^2}$$
$$= \sqrt{36+36} = \sqrt{72}$$
$$= \sqrt{36\times 2} = 6\sqrt{2}$$

Area of rhombus $$= \frac{1}{2}\times d_1 \times d_2$$
$$= \frac{1}{2}4\sqrt{2}\times 6\sqrt{2}$$
$$= 24 \text{ sq. unit.}$$

18. Find the ratio in which the line joining points $(a+b, b+a)$ and $(a-b, b-a)$ is divided by the point (a,b).

Ans : [Board Term-2, 2013]

Let $A(a+b, b+a), B(a-b, b-a)$ and $P(a,b)$ and P divides AB in $k:1$, then we have

$$a = \frac{k(a-b)+1(a+b)}{k+1}$$

$$a(k+1) = k(a-b)+a+b$$

$$ak+a = ak-bk+a+b$$

$$bk = b$$

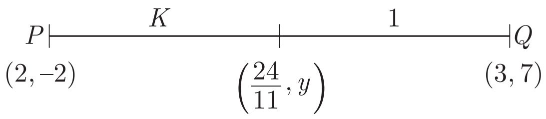
$$k = 1$$

Thus (a,b) divides $A(a+b,b+a)$ and $B(a-b,b-a)$ in 1:1 internally.

19. In what ratio does the point $(\frac{24}{11}, y)$ divides the line segment joining the points $P(2,-2)$ and $Q(3,7)$? Also find the value of y.

Ans : [CBSE Marking Scheme, 2017]

As per question, line diagram is shown below.



Let $P(\frac{24}{11}, y)$ divides the segment joining the points $P(2,-2)$ and $Q(3,7)$ in ratio $k:1$.

Using intersection formula $x = \frac{mx_2+nx_1}{m+1}$ we have

$$\frac{3k+2}{k+1} = \frac{24}{11}$$
$$33k+22 = 24k+24$$
$$9k = 2$$
$$k = \frac{2}{9}$$

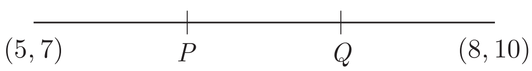
Hence, $$y = \frac{-18+14}{11} = -\frac{4}{11}$$

20. Find the co-ordinates of the points which divide the line segment joining the points $(5,7)$ and $(8,10)$ in 3 equal parts.

Ans : [Outside Delhi Compt. Set-II, 2017]

Let $P(x_1, y_2)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.



Now $$x = \frac{1(8)+2(7)}{3} = 6$$
$$y = \frac{1(10)+2(7)}{3} = 8$$

Thus $P(x_1, y_1)$ is $P(6,8)$. Since Q is the mid point of PB, we have

$$x_1 = \frac{6+8}{2} = 7$$
$$y_1 = \frac{8+10}{2} = 9$$

Thus $Q(x_2, y_2)$ is $Q(7,9)$

21. Find the co-ordinates of a point on the axis which is equidistant from the points $A(2,-5)$ and $B(-2,9)$.

Ans : [Delhi Compt. Set-I, 2017]

Let the point P on the x axis be $(x,0)$. Since it is equidistant from the given points $A(2,-5)$ and $B(-2,9)$

$$PA = PB$$
$$PA^2 = PB^2$$
$$(x-2)^2+[0-(-5)]^2 = (x-(-2))^2+(0-9)^2$$
$$x^2-4x+4+25 = x^2+4x+4+81$$
$$-4x+29 = 4x+85$$
$$x = -\frac{56}{8} = -7$$

Hence the point on x axis is $(-7,0)$

22. The line segment joining the points $A(3,-4)$ and $B(1,2)$ is trisected at the points P and Q. Find the coordinate of the PQ.

Ans : [Delhi Compt. Set-II, 2017]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.

Using intersection formula

$$x = \frac{1\times1+2\times3}{1+2} = \frac{7}{3}$$
$$y = \frac{1\times2+2\times-4}{1+2} = -2$$

Hence point P is $\left(\frac{7}{3}, -2\right)$

23. Show that ΔABC with vertices $A(-2,0), B(0,2)$ and $C(2,0)$ is similar to ΔDEF with vertices $D(-4,0), F(4,0)$ and $E(0,4)$.

Ans : [Board Foreign Set-I, II 2017], [Delhi Board Set-I, II, II, II 2017]

Using distance formula

$$AB = \sqrt{(0+2)^2+(2-0)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$
$$BC = \sqrt{(2-0)^2+(0-2)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$
$$CA = \sqrt{(-2,-2)^2+(0-0)^2} = \sqrt{16}$$
$$= 4 \text{ units}$$

and $$DE = \sqrt{(0+4)^2+(4-0)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$EF = \sqrt{(4-0)^2+(0-4)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$FD = \sqrt{(-4-4)^2+(0-0)^2} = \sqrt{64}$$
$$= 8 \text{ units}$$
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
$$\frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{4}{8} = \frac{1}{2}$$

Since Ratio of the corresponding sides of two similar Δ s is equal, we have

$\Delta ABC \sim \Delta DEF$ Hence Proved.

24. Find the co-ordinates of the point on the y-axis which is equidistant from the points $A(5,3)$ and $B(1,-5)$

Ans : [Delhi Compt. Set-III, 2017]

Let the points on y-axis be $P(0,y)$

Now $$PA = PB$$

$$PA^2 = PB^2$$
$$(0-5)^2 + (y-3)^2 = (0-1)^2 + (y+5)^2$$
$$5^2 + y^2 - 6y + 9 = 1 + y^2 + 10y + 25$$
$$16y = 8$$
$$y = \frac{1}{2}$$

Hence point on y-axis is $(0, \frac{1}{2})$.

25. In the given figure ΔABC is an equilateral triangle of side 3 units. Find the co-ordinates of the other two vertices.

Ans : [Board Foreign Set-I, II, 2017]

The co-ordinates of B will be $(2+3, 0)$ or $.(5, 0)$
Let co-ordinates of C be (x, y)

Since triangle is equilateral, we have

$$AC^2 = BC^2$$
$$(x-2)^2(y-0)^2 = (x-5)^2 + (y-0)^2$$
$$x^2 + 4 - 4x + y^2 = x^2 + 25 - 10x + y^2$$
$$6x = 21$$
$$x = \frac{7}{2}$$

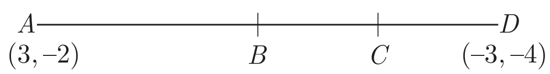
And $$(x-2)^2 + (y-0)^2 = 9$$
$$\left(\frac{7}{2} - 2\right)^2 + y^2 = 9$$
$$\frac{9}{4} + y^2 = 9 \text{ or, } y^2 = 9 - \frac{9}{4}$$
$$y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$$

Hence C is $\left(\frac{4}{3}, \frac{3\sqrt{3}}{2}\right)$.

26. Find the co-ordinates of the points of trisection of the line segment joining the points $(3, -2)$ and $(-3, -4)$.

Ans : [Board Foreign Set-I, II, III 2017]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect the line joining $A(3, -2)$ and $B(-3, -4)$.
As per question, line diagram is shown below.



Thus P divides AB in the ratio 1:2

Using intersection formula $x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + my_1}{m+n}$

$$x_1 = \frac{1(-3) + 2(3)}{1+2} = 1$$

and $$y_1 = \frac{1(-4) + 2(-2)}{1+2} = -\frac{8}{3}$$

Thus we have $x = 1$ and $y = -\frac{8}{3}$

Since Q is at the mid-point of PB, using mid-point formula

$$x_2 = \frac{1-3}{2} = -1$$

and $$y_2 = \frac{-\frac{8}{3} + (-4)}{2} = -\frac{10}{3}$$

Hence the co-ordinates of P and Q are $(1, -\frac{8}{3})$ and $(-1, -\frac{10}{3})$

27. If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$.

Ans : [Outside Delhi, Set-II, 2016]

Since $P(x, y)$ is equidistant from the given points $A(5, 1)$ and $B(-1, 5)$,

$$PA = PB$$
$$PA^2 = PB^2$$

Using distance formula,

$$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$$
$$(5-x)^2 + (1-y)^2 = (1+x)^2 + (5-y)^2$$
$$25 - 10x + 1 - 2y = 1 + 2x + 25 - 10y$$
$$-10x - 2y = 2x - 10y$$
$$8y = 12x$$
$$3x = 2y$$

Hence proved.

LONG ANSWER TYPE QUESTIONS

1. If $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8x, 5)$ in the ratio 3:1. Find the values of a and b.

Ans : [Board Sample Paper, 2016]

Using section formula we have

$$9a - 2 = \frac{3(8a) + 1 + (3a+1)}{3+1} \quad ...(1)$$

$$-b = \frac{3(5) + 1(-3)}{3+1} \quad ...(2)$$

Form (2) $$-b = \frac{15-3}{4} = 3 \Rightarrow b = -3$$

From (1), $$9a - 2 = \frac{24a + 3a + 1}{4}$$

$$4(9a-2) = 27a + 1$$
$$36a - 8 = 27a + 1$$
$$9a = 9$$
$$a = 1$$

2. Find the coordinates of the point which divide the line segment joining $A(2, -3)$ and $B(-4, -6)$ into three equal parts.

Ans : [Board Sample paper, 2016]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect the line joining $A(3, -2)$ and $B(-3, -4)$.
As per question, line diagram is shown below.
P divides AB in the ratio of 1:2 and Q divides AB in the ratio 2:1.
By section formula

$$x_1 = \frac{mx_2 + nx_1}{1+2} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$P(x_1, y_1) = \left(\frac{1(-4) + 2(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6-(-6)}{3}\right)$$

$$= (0, -4)$$

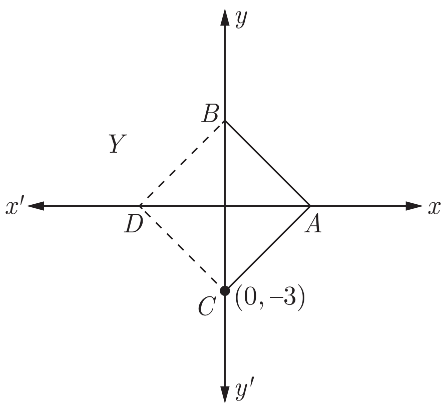
$$Q(x_2, y_2) = \left(\frac{2(-4) + 1(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-8+2}{3}, -\frac{12+(-3)}{3}\right) = (-2, -5)$$

3. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are $(0,3)$. The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that $BACD$ is a rhombus.

Ans : [Foreign Set I, II, 2015]

As per question, diagram of rhombus is shown below.



Co-ordinates of point B are $(0,3)$

Thus $BC = 6$ unit

Let the co-ordinates of point A be $(x,0)$

Now $AB = \sqrt{x^2 + 9}$

Since $AB = BC$, thus

$$x^2 + 9 = 36$$

$$x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

Co-ordinates of point A is $(3\sqrt{3}, 0)$

Since $ABCD$ is a rhombus

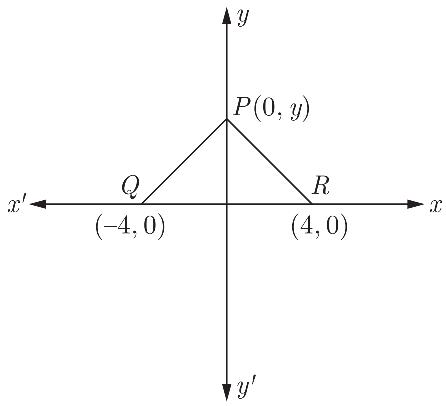
$$AB = AC = CD = DB$$

Thus co-ordinate of point D is $(-3\sqrt{3}, 0)$

4. The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are $(-4,0)$ and the origin is the mid-point of the base. find the co-ordinates of the point P and R.

Ans : [Foreign set III, 2015]

As per question, line diagram is shown below.



Co-ordinates of point R is $(4,0)$

Thus $QR = 8$ units

Let the co-ordinates of point P be $(0, y)$

Since $PQ = QR$

$$(-4-0)^2 + (0-y)^2 = 64$$

$$16 + y^2 = 64$$

$$y = \pm 4\sqrt{3}$$

Coordinates of P are $(0, 4\sqrt{3})$ or $(0, -4\sqrt{3})$

TOPIC 2 : AREA OF TRIANGLE

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the area of the triangle with vertices $(0,0)(6,0)$ and $(0,5)$

Ans : [Board Term-2, 2015]

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[0(0-5) + 6(5-0) + 0(0-0)]$$

$$= \frac{1}{2}[6 \times 5] = 15 \text{ sq. units}$$

2. If the points $A(x, 2)$, $B(-3, -4)$, $C(7, -5)$ are collinear, then find the value of x.
Since the points are collinear, then

$$\text{Area of triangle} = 0$$

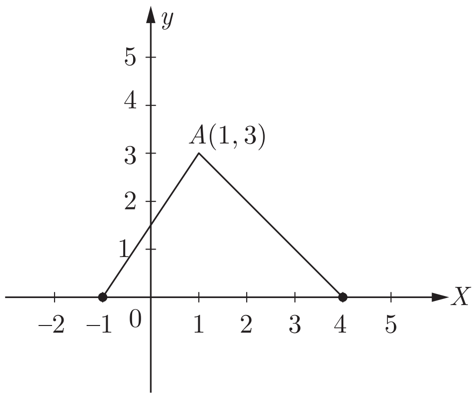
$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[x(-4+5) + (-3)(-5-2) + 7(2+4)] = 0$$

$$x + 21 + 42 = 0$$

$$x = -63$$

3. In Fig., find the area of triangle ABC (in sq. units)?



Ans : [Board Term-2, 2013]

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[1(0-0) + (-1)(0-3) + 4(3-0)]$$

$$= \frac{1}{2}[2+12] = \frac{15}{2} = 7.5 \text{ s, units}$$

4. If the point $(0,0),(1,2)$ and (x,y) are collinear, then find x.

Ans : [Board Term-2, 2011, Set A1]

The points are collinear, then area of triangle must be zero.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[0(2-y) + 1(y-0) + x(0-2)] = 0$$

$$[y - 2x] = 0$$

$$x = \frac{y}{2}$$

SHORT ANSWER TYPE QUESTIONS - I

1. Show that the points $A(0,1), B(2,3)$ and $C(3,4)$ are collinear.

Ans : [CBSE Term-2, 2016 Set-HODM4OL]

If the area of the triangle formed by the points is zero, then points are collinear.
We have $A(0,1), B(2,3)$ and $C(3,4)$

$$\Delta = \frac{1}{2}|0(3-4) + 2(4-1) + 3(1-3)|$$

$$= \frac{1}{2}|0 + (2)(3) + (3)(-2)|$$

$$= \frac{1}{2}|6-6| = 0$$

Thus given points are collinear.

2. Prove that the points $(2,-2),(-2,1)$ and $(5,2)$ are the vertices of a right angled triangle. Also find the area of this triangle.

Ans : [Foreign Set I, II, III, 2016]

We have $A(2,-2), B(-2,1)$ and $(5,2)$

Applying distance formula we get

$$AB^2 = (2+2)^2 + (-2-1)^2$$

$$= 16 + 9 = 25$$

Thus $AB = 5$

Similarly $AC^2 = (-2-5)^2 + (1-2)^2$

$$= 49 + 1 = 50$$

$$BC^2 = 50 \Rightarrow BC = 5\sqrt{2}$$

$$AC^2 = (2-5)^2 + (-2-2)^2$$

$$= 9 + 16$$

$$= 25$$

$$AC^2 = 25 \Rightarrow AC = 5$$

Clearly $AB^2 + AC^2 = BC^2$

$$25 + 25 = 50$$

Hence the triangle is right angled,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times Base \times Height$$

$$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2} \, sq.\ unit.$$

3. Find the relation between x and y, if the point $A(x,y), B(-5,7)$ and $C(-4,5)$ are collinear.

Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

If the area of the triangle formed by the points is zero, then points are collinear.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[x(7-5) - 5(5-y) - 4(y-7)] = 0$$

$$2x - 25 + 5y - 4y + 28 = 0$$

$$2x + y + 3 = 0$$

4. For what values of k are the points $(8,1),(3,-2k)$ and $(k,-5)$ collinear?

Ans : [Foreign Set I, II, III 2015]

Since points $(8,1),(3,-2k)$ and $(k,-5)$ are collinear, area of triangle formed must be zero.

$$\frac{1}{2}[8(-2k+5) + 3(-5,-1) + k(1+2k)] = 0$$

$$2k^2 - 15k + 22 = 0$$

$$k = 2, \frac{11}{2}$$

SHORT ANSWER TYPE QUESTIONS - II

1. Find the value of p, if the points $A(2,3), B(4,p)$, $C(6,-3)$ are collinear.

Ans : [Baord Term-2, 2012 sEt (17)]

Since points $A(2,3), B(4,p)$ and $C(6,-3)$ are collinear, area of triangle formed must be zero.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[2(p+3) + 4(-3-3) + 6(3-p)] = 0$$

$$[2p+6-24+18-6p] = 0$$
$$[-4p] = 0$$
$$4p = 0$$
$$p = 0$$

2. If $(5,2),(-3,4)$ and (x,y) are collinear, show that $x+4y-13=0$

Ans : [CBSE Board Term-2, 2015]

Since points $(5,2),(-3,4)$ and (x,y) are collinear, area of triangle formed must be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

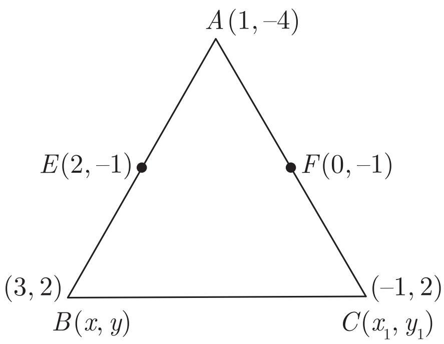
$$[5(4-y)+(-3)(y-2)+x(2-4)] = 0$$
$$[20-5y-3y+6+(-2x)] = 0$$
$$[-2x-8y+26] = 0$$
$$x+4y-13 = 0$$

Hence proved

3. Find the area of a triangle ABC with $A(1,-4)$ and mid-points of sides through A being $(2,-1)$ and $(0,-1)$.

Ans : [Delhi CBSE Board, 2015, Set I, III]

Let $B(x_1,y_1)$ and $C(x_2,y_2)$ be other vertices of triangle. As per question, triangle is shown below.



Let $E(2,-1)$ be the mid point of AB and $F(0,-1)$ be the mid point of AC.

Now $$\frac{x_1+1}{2} = 2 \Rightarrow x_1 = 3$$

and $$\frac{y_1+(-4)}{2} = -1 \Rightarrow y_2 = 2$$

Thus point B is $(3,2)$.

Again $$\frac{x_2-1}{2} = 0 \Rightarrow x_1 = -1$$

$$\frac{y_2+(-4)}{2} = -1 \Rightarrow y_1 = 2$$

Thus point C is $(-1,2)$

Now the co-ordinates are $A(1,-4), B(3,2), C(-1,2)$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

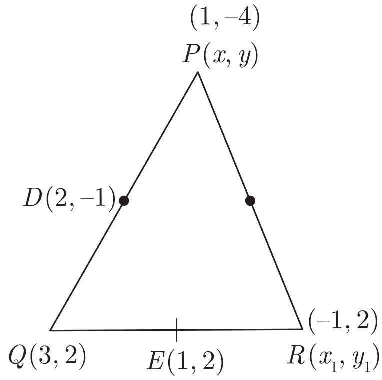
$$= \frac{1}{2}[1(2-2)+3(2+4)-1(-4-2)]$$

$$= \frac{1}{2}[0+18+6] = 12 \text{ sq. units}$$

4. Find the area of the triangle PQR with $Q(3,2)$ and mid-points of the sides through Q being $(2,-1)$ and $(1,2)$.

Ans : [Delhi CBSE Board, 2015 Set III]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ be other vertices of triangle. As per question, triangle is shown below.



Let $D(2,-1)$ be the mid point of PQ and $E(1,2)$ be the mid point of AC.

Let the co-ordinate of p be (x,y) and $R(x_1,y_1)$

Now $$\frac{x_1+3}{2} = 2 \Rightarrow x_1 = 1$$

$$\frac{y_1+2}{2} = -1 \Rightarrow y_2 = -4$$

Thus point is $P(1,-4)$

Again $$\frac{x_2+3}{2} = 1 \Rightarrow x_1 = -1$$

$$\frac{y_2+2}{2} = 2 \Rightarrow y_1 = 2$$

Thus point is $R(-1,2)$

Now we have $P(1,-4), Q(3,2), R(-1,2)$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$= \frac{1}{2}[1(2-2)+3(2+4)+(-1)(-4-2)]$$

$$= \frac{1}{2}[0+18+6] = \frac{1}{2}\times 24 = 12 \text{ sq. units}$$

5. If the points $A(-2,1), B(a,b)$ and $C(4,1)$ are collinear and $a-b=1$, find a and b.

Ans : [Delhi CBSE Term-2, 2014]

If three points are collinear, then area covered by given points must be zero.

Thus area

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\frac{1}{2}[-2(b-1)+a(1-1)+4(1-b)] = 0$$

$$[-2b+2+0+4(1-b)] = 0$$
$$-6b+6 = 0 \Rightarrow b = 1$$

Substituting $b=1$ in given condition $a-b=1$ we have

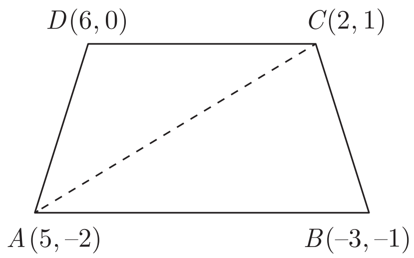
$$a - 1 = 1$$
$$a = 2$$

This $a = 2$ and $b = 1$.

6. Find the area of the quadrilateral $ABCD$, the co-ordinates of whose vertices are $A(5,-2), B(-3,-1)$, $C(2,1)$ and $D(6,0)$.

Ans : [Delhi Set, 2014], [Board Term-2, 2012 set (13)]

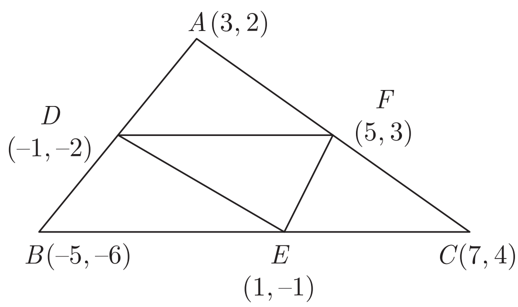
As per question the quadrilateral $ABCD$ is shown below.



Area of quadrilateral

$$= \Delta_{ABC} + \Delta_{ADC}$$
$$ABCD = ar(\Delta ABC) + ar(ADC)$$
$$\text{Area}_{ABCD} = \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2)]$$
$$+ (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)$$
$$= \frac{1}{2}[5(-1) - (-2)(-3) + (-3)(1)$$
$$-(-1)(2) + (2 \times 0 - 1 \times 6) + 6(-2) - (0 \times 5)$$
$$= \frac{1}{2}[-30] = |-15| = 15 \text{ sq. units}$$

7. In the given triangle ABC as shown in the diagram D, E and F are the mid-points of AB, BC and AC respectively. Find the area of ΔDEF.



Ans : [Board Term-2, 2012 Set (5)]

Mid-point BA $\quad x_D = \frac{3 + -5}{2} = -1$

and $\quad y_D = \frac{2-6}{2} = -2$

Thus point D is $(-1,-2)$

Mid-point BC, $\quad x_E = \frac{-5+7}{2} = 1$

and $\quad y_E \frac{-6+4}{2} = -1$

Thus point is E is $(1,-1)$.

Mid- Point CA, $\quad x_F = \frac{7+3}{2} = 5$

$$y_F = \frac{4+2}{2} = 3$$

Thus point F is $(5,3)$

Now, area ΔDEF

$$\Delta = \frac{1}{2}[-(-1-3) + 1(3+2) + 5(-2+1)]$$
$$= \frac{1}{2}[4 + 5 - 5]$$
$$= 2 \text{ Unit}$$

8. Find the area of the triangle formed by joining the mid-points of the sides of a triangle, whose co-ordinates of vertices are $(0,-1), (2,1)$ and $(0,3)$.

Ans : [Outside Delhi Compt. Set I, III 2017]

Let the vertices of given triangle be $A(0,-1), B(2,1)$ and $C(0,3)$. As per question the triangle is shown below.

Let the coordinates of mid-points

$$P = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1,0)$$
$$Q = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1,2)$$
$$Q = \left(\frac{0+0}{2}, \frac{-1+3}{2}\right) = (0,1)$$

Area of ΔPQR

$$\Delta = \frac{1}{2}[x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}[(2-1) + 1(1-0) + 0(0-2)]$$
$$= \frac{1}{2}(1 + 1 + 0) = 1 \text{ sq. units}$$

9. The area of a triangle is 5 sq. units. Two of its vertices are $(2,1)$ and $(3,-2)$. If the third vertex is $(\frac{7}{2}, y)$, Find the value of y.

Ans : [Delhi Set II 2017]

We have $\Delta ABC = 5$ sq. units

$$\frac{1}{2}\left[2(-2-y) + (y-1) + \frac{7}{2}(1+2)\right] = 5$$
$$\frac{1}{2}\left[-4 - 2y + 3y - 3 + \frac{21}{2}\right] = 5$$
$$y + \frac{7}{2} = 10$$
$$y = 10 - \frac{7}{2} = \frac{13}{2}$$

If we consider possibility of negative area then, we have

$$y + \frac{7}{2} = -10$$
$$y = -10 - \frac{7}{2} = -\frac{27}{2}$$

Hence the value of y is $\frac{13}{2}$ or $-\frac{27}{2}$

LONG ANSWER TYPE QUESTIONS

1. Prove that the area of a triangle with vertices $(t, t-2), (t+2, t+2)$ and $(t+3)$ is independent of t.

Ans : [Delhi Set I, II, III, 2016]

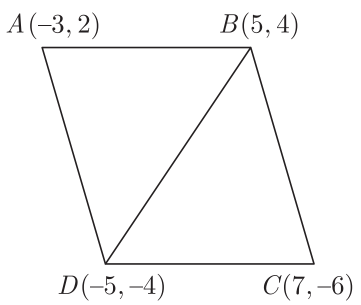
Area of the triangle

$$\Delta = \frac{1}{2} \mid t(t+2-t) + (t+2)(t-t+2) + $$
$$+(t+3)(t-2-t-2) \mid$$
$$= \frac{1}{2}[2t + 2t + 4 - 4t - 12]$$
$$= 4 \text{ sq. units. which is independent of } t.$$

2. Find the area of a quadrilateral $ABCD$, the co-ordinates of whose vertices are $A(-3,2), B(5,4)$, $C(7,-6)$ and $D(-5,-4)$.

Ans : [Foreign Set III, 2016]

As per question the quadrilateral is shown below.



Area of triangle ABD

$$\Delta_{ABD} = \frac{1}{2}|-3(8) + 5(-6) + -5(2-4)|$$
$$= 22 \text{ sq. units}$$

Area of triangle BCD

$$\Delta_{BCD} = \frac{1}{2}|5(-2) + 7(-8) - 5(10)|$$
$$= 58 \text{ sq. units}$$
$$\text{Area}_{ABCD} = \Delta_{ABD} + \Delta_{BCD}$$
$$= 22 + 58 = 80 \text{ sq. units}$$

3. If $A(-4,8), B(-3,-4), C(0,-5)$ and $D(5,6)$ are the vertices of a quadrilateral $ABCD$, find its area.

Ans : [Delhi CBSE Board, 2015 Set I, III]

We have $A(-4,8)$, $B(-3,-4)$, $C(0,5)$ and $D(5,6)$

Area of quadrilateral

$$= \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3)$$
$$+(x_4y_1 - x_1y_4)]$$
$$\text{Area} = \frac{1}{2}[\{-4 \times (-4) - (-3)(8)\} + \{(-3)(-5) - 0$$
$$\times (-4)\} + \{0 \times 6 - 5(-5)\} + [\{5 \times 8 - (-4)(6)\}]$$
$$= \frac{1}{2}[16 + 24 + 15 - 0 + 0 + 25 + 40 + 24]$$
$$= \frac{1}{2}[40 + 15 + 25 + 40 + 24] = \frac{1}{2} \times 144 = 72 \text{ sq. units}$$

4. If $P(-5,-3), Q(-4,-6), R(2,-3)$ and $S(1,2)$ are the vertices of a quadrilateral $PQRS$, find its area.

Ans : [Delhi CBSE Board, 2015 Set II]

We have $P(-5,-3)$, $Q(-4,-6)$, $R(2,-3)$ and $S(1,2)$

Area of quadrilateral

$$= \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3)$$
$$+(x_4y_1 - x_1y_4)]$$

Area

$$= \frac{1}{2}[-5(-6) - (-4)(-3) + (-4)(-3) - 2(-6)$$
$$+(2)(2) - 1 \times (-3) + 1 \times (-3) - (-5)(2)]$$
$$= \frac{1}{2}[30 - 12 + 12 + 12 + 4 + 3 - 3 + 10]$$
$$= \frac{1}{2}[30 + 12 + 4 + 10] = \frac{1}{2}[56] = 28 \text{ sq. units}$$

5. Find the values of k so that the area of the triangle with vertices $(1,-1),(-4,2k)$ and $(-k,-5)$ is 24 sq. units.

Ans : [Outside Delhi CBSE Board, 2015, Set I]

We have $(1,-1),(-4,2k)$ and $(-k,-5)$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$24 = \frac{1}{2}[1(2k+5) - 4(-5+1) - k(-1-2k)]$$
$$48 = 2k + 5 + 16 + k + 2k^2$$
$$2k^2 + 3k - 27 = 0$$
$$(k-3)(2k+9) = 0$$
$$k = 3, -\frac{9}{2}$$

6. Find the values of k so that the area of the triangle with vertices $(k+1,1), (4,-3)$ and $(7,-k)$ is 6 sq. units.

Ans : [Outside Delhi CBSE Board, 2015, Set I]

We have $(k+1,1), (4,-3)$ and $(7,-k)$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$6 = \frac{1}{2}[(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$$
$$12 = [k^2 - 2k - 3 - 4k - 4 + 28]$$
$$12 = k^2 - 6k + 21$$
$$k^2 - 6k + 9 = 0$$
$$(k-3)(k-3) = 0$$
$$k = 3,3$$

7. Find the values of k for which the points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear.

Ans : [Outside Delhi CBSE Board, 2015, Set III]

If three points are collinear, then area covered by given points must be zero.

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[(k+1)(2k+3-5k) + 3k(5k-2k) + +(5k-1)(2k-2k-3) = 0$$

$$-3k^2 + 3k - 3k + 3 + 9k^2 - 15k + 3 = 0 = 0$$

$$6k^2 - 15k + 6 = 0$$

$$2k^2 - 5k + 2 = 0$$

$$(k-2)(2k-1) = 0$$

Thus $k = 2$ or $k = \frac{1}{2}$

8. The vertices of quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ and $D(1,-4)$. Prove that $ABCD$ is a rhombus.

Ans : [Board Term-2, 2015]

The vertices of the quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)\,D(1,-4)$.

Now $AB = \sqrt{(8-5)^2 + (3+1)^2}$

$= \sqrt{3^2 + 4^2} = 5$ units

$BC = \sqrt{(8-4)^2 + (3-0)^2}$

$= \sqrt{4^2 + 3^2} = 5$ units

$CD = \sqrt{(4-1)^2 + (0+4)^2}$

$= \sqrt{(3)^2 + (4)^2} = 5$ units

$AD = \sqrt{(5-1)^2 + (-1+4)^2}$

$= \sqrt{(4)^2 + (3)^2} = 5$ units

Diagonal, $AC = \sqrt{(5-4)^2 + (-1-0)^2}$

$= \sqrt{1^2 + 1^2} = \sqrt{2}$ units

Diagonal $BD = \sqrt{(8-1)^2 + (3+4)^2}$

$= \sqrt{(7)^2 + (7)^2} = 7\sqrt{2}$ units

As the length of all the sides are equal but the length of the diagonals are not equal. Thus $ABCD$ is not square but a rhombus.

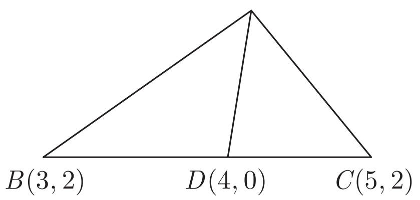
9. $A(4,-6), B(3,-2)$ and $C(5,2)$ are the vertices of a ΔABC and AD is its median. Prove that the median AD divides ΔABC into two triangles or equal areas.

Ans : [CBSE O.D. 2014]

Since AD is the median of ΔABC from vertex A, we have

$$D(x,y) = \left(\frac{3+5}{2} + \frac{-2+2}{2}\right) = (4,0)$$

As per question statement triangle is shown below.



Area of ΔADB,

$$\Delta_{ADB} = \frac{1}{2} \times (4(0+2) + (-2+6) + 3(-6-0))$$

$$= \frac{1}{2} \times (8 + 16 + -18)$$

$$= \frac{1}{2} \times 3 = 3 \text{ square units} \quad (1)$$

Area of ΔACB

$$\Delta_{ACB} = \frac{1}{2} \times (4(0-2) + 4(2+6) + 5(-6-0))$$

$$= \frac{1}{2} \times (-8 + 32 - 30)$$

$$= \frac{1}{2} \times -6 = -3$$

Since area can not be negative, we take positive value.

Thus $\Delta_{ACB} = 3$ square units (2)

From (1) and (2) we seen that $\Delta_{ADB} = \Delta_{ACB}$. It is verified that median of ΔABC divides it into two triangles of equal areas.

10. The co-ordinates of vertices of ΔABC are $A(0,0), B(0,2)$ and $C(2,0)$. Prove that ΔABC is an isosceles triangle. Also find its area.

Ans : [Board Term-2, 2014]

Using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ we have

$$AB = \sqrt{(0-0)^2 + (0-2)^2} = \sqrt{4} = 2$$

$$AC = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Clearly, $AB = AC \neq BC$

Thus ΔABC is an isosceles Triangle

Now, $AB^2 + AC^2 = 2^2 + 2^2 = 4 + 4 = 8$

also, $BC^2 = (2\sqrt{2})^2 = 8$

$$AB^2 + AC^2 = BC^2$$

Thus ΔABC is an isosceles right angled triangle.

Now, area of ΔABC

$$\Delta_{ABC} = \frac{1}{2} base \times height$$

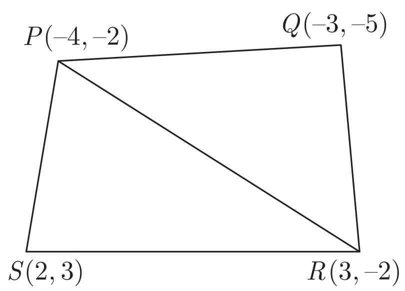
$$= \frac{1}{2} \times 2 \times 2$$

$$= 2 \text{ sq. units.}$$

11. Find the area of the quadrilateral $PQRS$. The co-ordinates of whose vertices are $P(-4,-2), Q(-3,-5)$, $R(3,-2)$ and $5(2,3)$.

Ans : [Outside Delhi Set-II, 2017]

As per question quadrilateral $PQRS$ is shown below.



Area $\square_{PQRS} = \Delta_{PQR} + \Delta_{PRS}$

Area ΔPQR

$$\Delta_{PQR} = \frac{1}{2}[x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_2(y_1 - y_2)]$$

$$= \frac{1}{2}[-4(-2-(-5)) + 3(-5-(-2)) + -3(-2-(-2))]$$

$$= \frac{1}{2}[-4\times 3 + 3\times -3 + 3\times 0]$$

$$= \frac{1}{2}\times(12+9) = \frac{21}{2} \text{ sq. units}$$

Area ΔPRS

$$\Delta_{PRS} = \frac{1}{2}[-4(-2-3) + 3(3+2) + 2(-2+2)]$$

$$= \frac{1}{2}[-4\times -5 + 3\times 5 + 0]$$

$$= \frac{1}{2}\times(20+15) = \frac{35}{2} \text{ sq. units}$$

Area $\square_{PQRS} = \frac{21}{2} + \frac{35}{2} = 28$ sq. units

12. If the co-ordinates of two points are $A(3,4), B(5,-2)$ and a point $P(x,5)$ is such that $PA = PB$ then find the area of ΔPAB.

Ans : [Outside Delhi Compt. Set-I, 2017]

Since $PA = PB$

$$PA^2 = PB^2$$

Using distance formula we have

$$(x-3)^2 + (5-4)^2 = (x-5)^2 + (5+2)^2$$

$$x^2 - 6x + 9 + 1 = x^2 - 10x + 25 + 49$$

$$10x - 6x = 74 - 10$$

$$x = 16$$

Thus area ΔPAB

$$\Delta_{PAB} = \frac{1}{2}[16(4+2) + 3(-2-5) + 5(5-4)]$$

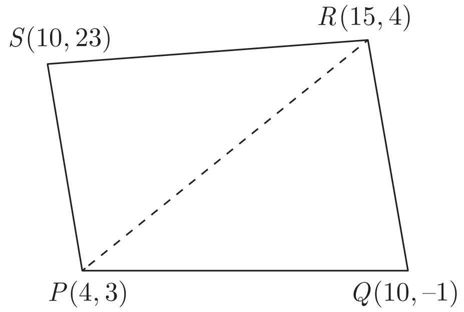
$$= \frac{1}{2}[96 - 21 + 5] = 40$$

Hence, area of triangle is 40 sq. units

13. Find the area of a quadrilateral $PQRS$ whose vertices are $P(4,3), Q(10,-1), R(15,4)$ and $S(10,23)$.

Ans : [Delhi Compt. Set III 2017]

As per question quadrilateral $PQRS$ is shown below.



Area $\square_{PQRS} = \Delta_{PQR} + \Delta_{PRS}$

Area ΔPQR

$$\Delta_{PQR} = \frac{1}{2}[x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_2(y_1 - y_2)]$$

$$= \frac{1}{2}[4(-5) + 10(1) + 15(4)]$$

$$= \frac{1}{2}\times 50 = 25 \text{ sq. units}$$

Area ΔPRS

$$\Delta_{PRS} = \frac{1}{2}[4(-19) + 15(20) + 10(-1)]$$

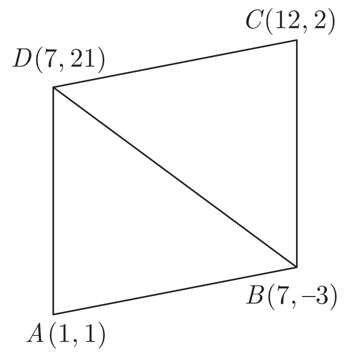
$$= \frac{1}{2}\times 214 = 107 \text{ sq. units}$$

Area $\square_{PQRS} = 25 + 107 = 132$ sq. unit

14. Find the area of a quadrilateral $ABCD$, whose vertices are $A(1,1), B(7,-3), C(12,2)$ and $D(7,21)$.

Ans : [Delhi Compt. Set I 2017]

As per question quadrilateral $ABCD$ is shown below.



Area of quadrilateral $ABCD$

$$\square_{ABCD} = \Delta_{ABD} + \Delta_{BCD}$$

Area ΔABD,

$$\Delta_{ABD} = \frac{1}{2}[1(-3-21) + 7(21-1) + 7(1+3)]$$

$$= \frac{1}{2}[-24 + 7\times 20 + 7\times 4]$$

$$= \frac{1}{2}[-24 + 140 + 28]$$

$$= \frac{1}{2}\times 144 = 72 \text{ sq. units}$$

Area ΔBCD,

$$\Delta_{BCD} = \frac{1}{2}[7(2-21) + 12(21+3) + 7(-3-2)]$$

$$= \frac{1}{2}[7\times -19 + 12\times 24 + 7\times -5]$$

$$= \frac{1}{2}[-133 + 288 - 35]$$

$$= \frac{1}{2}[288 - 168]$$

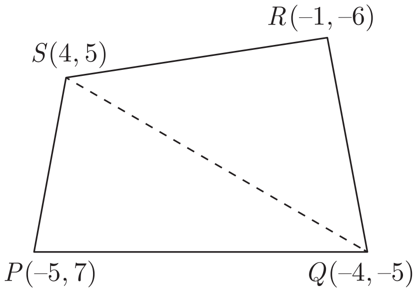
$$= \frac{1}{2}\times 120 = 60 \text{ sq. units}$$

Area $\square_{ABCD} = 72 + 60 = 132$ sq. units.

15. Find the area of a quadrilateral $PQRS$ whose vertices area $P(-5,7), R(-1,-6)$ and $S(4,5)$

Ans : [Delhi Compt. Set II, 2017]

As per question quadrilateral $PQRS$ is shown below.



Area $\square_{PQRS} = \Delta_{PQR} + \Delta_{QRS}$

Area ΔPQR

$$\Delta_{PQR} = \frac{1}{2}[x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_2(y_1 - y_2)]$$

$$= \frac{1}{2}[-5(-5-5) + -4(5-7) + 4(7+5)]$$

$$= \frac{1}{2}[50 + 8 + 48]$$

$$= \frac{1}{2} \times 106 = 53 \text{ sq. units.}$$

Area ΔQRS

$$\Delta_{QRS} = \frac{1}{2}[-4(-6-5) + -1(5+5) + 4(-5+6)]$$

$$= \frac{1}{2}[44 + (-10) + 4]$$

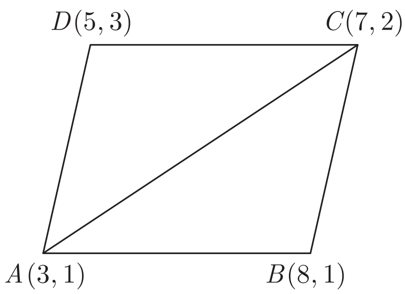
$$= \frac{1}{2} \times 38 = 19 \text{ sq. units}$$

Area $\square_{PQRS} = 53 + 19 = 72$ sq. units

16. Find the area of the quadrilateral whose vertices are $A(3,1), B(8,1), C(7,2)$ and $D(5,3)$

Ans : [Delhi Compt. Set II 2017]

As per question quadrilateral $ABCD$ is shown below.



Area of quadrilateral $ABCD$

$$\square_{ABCD} = \Delta_{ABC} + \Delta_{ACD}$$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_2(y_1 - y_2)]$$

Area ΔABC

$$\Delta_{ABC} = \frac{1}{2}[3(1-2) + 8(2-1) + 7(1-1)]$$

$$= \frac{1}{2}(3 \times -1 + 8 \times 1 + 7 \times 0)$$

$$= \frac{1}{2}[-3 + 8] = \frac{5}{2} \text{ sq. units.}$$

Area ΔACD

$$\Delta_{ACD} = \frac{1}{2}[3(2-3) + 7(3-1) + 5(1-2)]$$

$$= \frac{1}{2}[3 \times -1 + 7 \times 2 + 5 \times -1]$$

$$= \frac{1}{2}[-3 + 14 - 5]$$

$$= 3 \text{ units}$$

Area $\square_{ABCD} = \frac{5}{2} + 3 = \frac{11}{2}$ sq. units.

HOTS QUESTIONS

1. Find the ratio is which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of the point of division.

Ans : [CBSE O.D. 2014]

We have $A(3,-3)$ and $B(-2,7)$

At any point on x-axis y-coordinate is always zero.

So, let the point be $(x,0)$ that divides line segment AB in ratio $k:1$.

Now $$(x,0) = \left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$$

$$\frac{7k-3}{k+1} = 0$$

$$7k - 3 = 0 \Rightarrow k = \frac{3}{7}$$

The line is divided in the ratio of 3 : 7

Now $$\frac{-2k+3}{k+1} = x$$

$$\frac{-2 \times \frac{3}{7} + 3}{\frac{3}{7} + 1} = x$$

$$\frac{-6+21}{3+7} = x$$

$$\frac{15}{10} = x$$

$$x = \frac{3}{2}$$

The coordinates of the point is $\left(\frac{3}{2}, 0\right)$.

2. Determine the ratio in which the straight line $x - y - 2 = 0$ divides the line segment joining $(3, -1)$ and $(8, 9)$.

Ans : [Board Term-2, 2012 Set (44)]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $k:1$.

Now
$$x_1 = \frac{8k+3}{k+1}$$
$$y_1 = \frac{9k-1}{k+1}$$

Since point $P(x_1, y_1)$ lies on line $x - y - 2 = 0$, so co-ordinates of P must satisfy the equation of line.

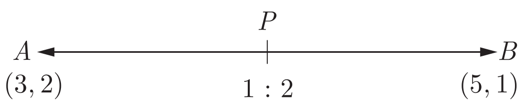
Thus
$$\frac{8k+3}{k+1} - \frac{9k-1}{k+1} - 2 = 0$$
$$8k + 3 - 9k + 1 - 2k - 2 = 0$$
$$-3k + 2 = 0$$
$$k = \frac{2}{3}$$

So, line $x - y - 2 = 0$ divides AB in the ratio $2:3$

3. The line segment joining the points $A(3, 2)$ and $B(5, 1)$ is divided at the point P in the ratio $1:2$ and P lies on the line $3x - 18y + k = 0$. Find the value of k.

Ans : [Board Term-2, 2012 Set (I)]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $1:2$.



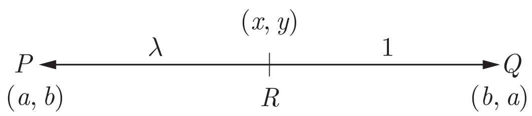
$$x_1 = \frac{mx_2 + nx_1}{m+n} = \frac{1 \times 5 + 2 \times 3}{1+2} = \frac{11}{3}$$
$$y_2 = \frac{my_2 + ny_1}{m+n} = \frac{1 \times 2 + 2 \times 2}{1+2} = \frac{5}{3}$$

Since point $P(x_1, y_1)$ lies on line ,$3x - 18y + k = 0$, so co-ordinates of P must satisfy the equation of line.
$$3 \times \frac{11}{3} - 18 \times \frac{5}{3} + k = 0$$
$$k = 19$$

4. If $R(x, y)$ is a point on the line segment joining the points $P(a, b)$ and $Q(b, a)$, then prove that $x + y = a + b$.

Ans : [Board Term-2, 2012 Set (28)]

As per question line is shown below.



Let point $R(x, y)$ divides the line joining P and Q in the ratio $k:1$, then we have
$$x = \frac{kb+a}{k+1}$$

and
$$y = \frac{ka+b}{k+1}$$

Adding,
$$x + y = \frac{kb + a + ka + b}{k+1}$$
$$= \frac{k(a+b) + (a+b)}{k+1}$$
$$= \frac{(k+1)(a+b)}{k+1} = a + b$$
$$x + y = a + b$$

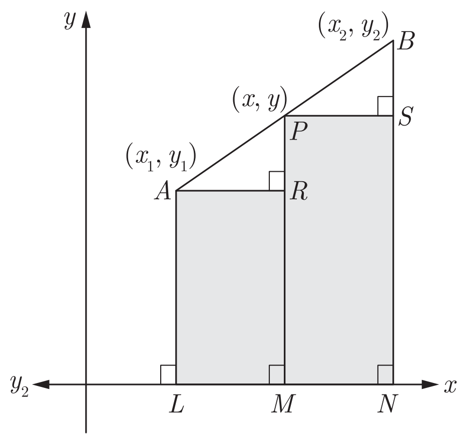
Hence Proved

5. (i) Derive section formula.
(ii) In what ratio does $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$

Ans : [KVS 2014]

(i) Section Formula : Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points. Let $P(x, y)$ be a point on line, joining A and B, such that P divides it in the ratio $m_1 : m_2$.

Now
$$(x, y) = \left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right)$$



Proof : Let AB be a line segment joining the points. $A(x_1, y_1), B(x_2, y_2)$.
Let P divides AB in the ratio $m_1 : m_2$. Let P have co-ordinates (x, y).
Draw $AL, PM, PN, \perp$ to x-axis
It is clear form figure, that
$$AR = LM = OM - OL = x - x_1$$
$$PR = PM - RM = y - y_1.$$

also,
$$PS = ON - OM = x_2 - x$$
$$BS = BN - SN = y_2 - y$$

Now $\Delta APR \sim \Delta PBS$ $[AAA]$

Thus
$$\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$$

and
$$\frac{AR}{PS} = \frac{AP}{PB}$$
$$\frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$$
$$m_2x - m_2x_1 = m_1x_2 - m_1x$$
$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

Now
$$\frac{PR}{BS} = \frac{AP}{PB}$$

$$\frac{y-y_2}{y_2-y} = \frac{m_1}{m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Thus co-ordinates of P are $\left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$

(ii) Assume that $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$ in ratio $k:1$

Using section formula for x co-ordinate we have

$$-4 = \frac{k(3)-6}{k+1}$$

$$-4k-4 = 3k-6 \Rightarrow k = \tfrac{2}{7}$$

6. If the points $A(0,1), B(6,3)$ and $C(x,5)$ are the vertices of a triangle, find the value of x such that area of $\Delta ABC = 10$

Ans : [CBSE S.A.2 2016 HODM4OL]

We have $A(0,1), B(6,3)$ and $C(x,5)$
Since area of the triangle ABC is 10, we have

$$\tfrac{1}{2}[0(3-5)+6(5-1)+x(1-3)] = 10$$

$$\tfrac{1}{2}[0+24-2x] = 10$$

Here area may be negative also. So we have to consider the negative area also.
For positive area

$$24-2x = 20 \Rightarrow x = 2$$

For negative area,

$$24-2x = -20 \Rightarrow x = 22$$

7. The co-ordinates of the points A, B and C are $(6,3), (-3,5)$ and $(4,-2)$ respectively. $P(x,y)$ is any points in the plane. Show that $\frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \left|\frac{x+y-2}{7}\right|$

Ans : [Foreign Set I, 2016]

We have $A(6,3), B(-3,5), C(4,-2)$ and $P(x,y)$

Area of ΔPBC,

$$\text{ar}(\Delta PBC) = \tfrac{1}{2}|x(7)+3(2+y)+4(y-5)|$$

$$= \tfrac{1}{2}|7x+7y-14|$$

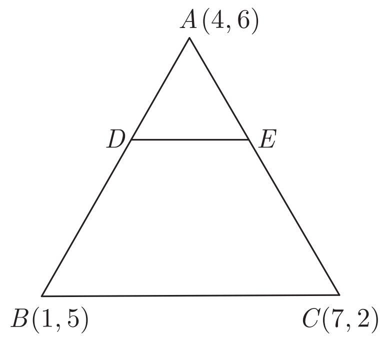
Area of ΔABC,

$$\text{ar}(\Delta ABC) = \tfrac{1}{2}|6\times 7-3(-5)+4(3-5)| = \frac{49}{2}$$

Thus
$$\frac{\text{ar}(\Delta PBC)}{\text{ar}(\Delta ABC)} = \frac{\tfrac{1}{2}(7x+7y-14)}{\tfrac{49}{2}}$$

$$= \frac{7(x+y-2)}{49} = \left|\frac{x+y-2}{7}\right|$$

8. In the given figure, the vertices of ΔABC are $A(4,6)$, $B(1,5)$ and $C(7,2)$. A line-segment DE is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.



Ans : [O.D. Set I, II, III, 2016]

Area of a triangle having vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$\Delta = \tfrac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

Thus area of triangle ABC is,

$$\Delta_{ABC} = \tfrac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$$

$$= \tfrac{1}{2}[12+(-4)+7] = \frac{15}{2} \text{ sq units}$$

In ΔADE and ΔABC, we have

$$\frac{AD}{AB} = \frac{AE}{EC} = \frac{1}{3}$$

and $\angle DAE = \angle BAC$

Hence $\Delta DAE \sim \Delta ABC$

Now $\frac{\Delta_{ADE}}{\Delta_{ABC}} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$$\frac{\Delta_{ADE}}{\frac{15}{2}} = \frac{1}{9}$$

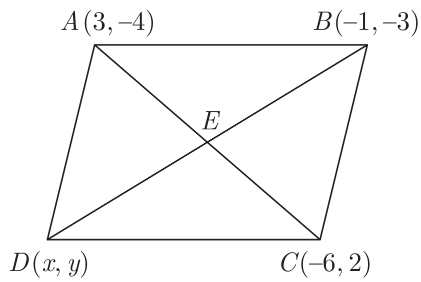
Area $\Delta_{ADE} = \frac{1.5}{2\times 9} = \frac{5}{6}$ Sq. units

Area $\Delta_{ADE} : \Delta_{ABC} = \frac{5}{6} : \frac{15}{2} = 1:9$

9. The three vertices of a parallelogram $ABCD$ are $A(3,-4), B(-1,-3)$ and $C(-6,2)$. Find the co-ordinates of vertex D and find the area of $ABCD$.

Ans : [Board Term-2, 2013]

Let 4th vertices of parallelogram be $D(x,y)$. As per question the parallelogram is shown below.



Diagonals of a parallelogram bisect each other. Here E is mid-point of AC and BD.
From bisection of AC we have

$$E = \left(\frac{3-6}{2}, \frac{-4+2}{2}\right) = \left(\frac{-3}{2}, 1\right) \quad (1)$$

From bisection of BD we have

$$E = \left(\frac{x-1}{2}, \frac{y-3}{2}\right) \quad (2)$$

From (1) and (2) we have

$$\frac{x-1}{2} = -\frac{3}{2} \Rightarrow x = -3+1 \Rightarrow x = -2$$

and $$\frac{y-3}{2} = -1 \Rightarrow y-3 = -2 \Rightarrow y = 1$$

Thus fourth vertex D is $(-2, 1)$

Area of ΔABC

$$\Delta_{ABC} = \frac{1}{2}[x_1(y_2-y_1)+x_2(y_3-y_1)+x_2(y_1-y_2)]$$

$$= \frac{1}{2}[3(-3-2)-1(2+4)-6(-4+3)]$$

$$= \frac{1}{2}[-15-6+6]$$

$$= \frac{1}{2}\times(-15) = -\frac{15}{2} = \frac{15}{2} \text{ sq. units}$$

Since diagonal divides parallelogram into two equal parts, So Area of parallelogram $ABCD$

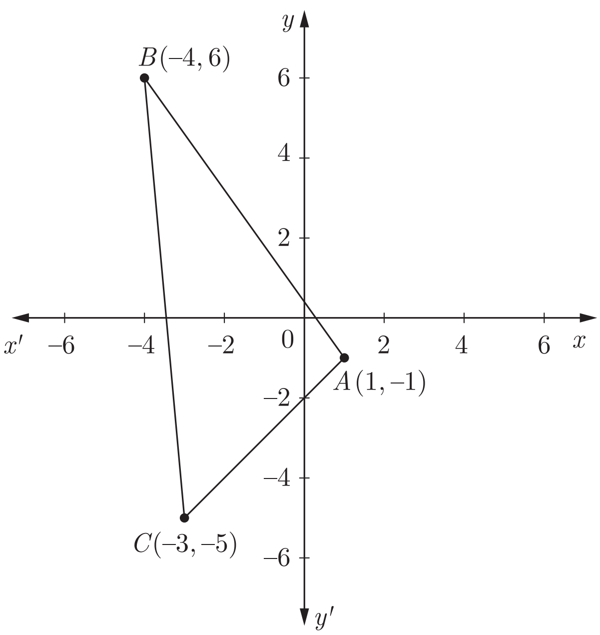
$$\square_{ABCD} = 2\times\Delta_{ABC}$$

$$= 2\times\frac{15}{2} = 15 \text{ sq. units}$$

10. The co-ordinates of vertices of ΔABC are $A(1,-1)$, $B(-4,6)$ and $C(-3,-5)$. Draw the figure and prove that ΔABC a scalene triangle. Find its area also.

Ans : [Board Term-2, 2014]

As per question diagram is shown below.



The co-ordinates of the vertices of ΔABC are $A(1,-1)$, $B(-4,6)$ and $C(-3,-5)$ respectively

Now $$AB = \sqrt{(1+4)^2+(-1-6)^2}$$

$$= \sqrt{25+49} = \sqrt{74} = \sqrt{74}$$

$$BC = \sqrt{(-4+3)^2+(6+5)^2}$$

$$= \sqrt{1+121} = \sqrt{122} = \sqrt{122}$$

$$AC = \sqrt{(1+3)^2-(-1+5)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

Since $AB \neq BC \neq AC$ triangle ΔABC is scalene.

Now, area of ΔABC,

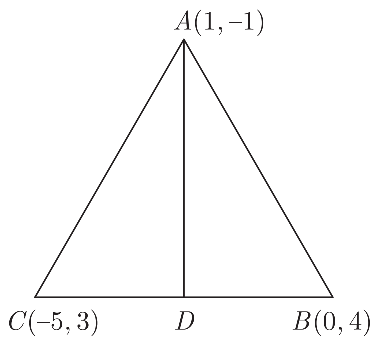
$$= \frac{1}{2}[1(6+5)+(-4)(-5+1)+(-3)(-1-6)]$$

$$= \frac{1}{2}[11+16+21] = 24 \text{ sq. units}$$

11. $(1,-1), (0,4)$ and $(-5,3)$ are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1,-1)$ the mid-point of the opposite side.

Ans : [Board Term-2, 2015]

Let the vertices of ΔABC be $A(1,-1)$, $B(0,4)$ and $C(-5,3)$. Let $D(x,y)$ be mid point of BC. Now the triangle is shown below.



Using distance formula, we get

$$AB = \sqrt{(1-0)^2+(-1-4)^2} = \sqrt{1+5^2} = \sqrt{26}$$

$$BC = \sqrt{(-5,0)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(-5-1)^2+(3+1)^2} = \sqrt{36+16} = 2\sqrt{13}$$

Since $AB = BC \neq AC$, triangle ΔABC is isosceles.

Now, using mid-section formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$

$$y = \frac{3+4}{2} = \frac{7}{2}$$

$$D(x,y) = \left(-\frac{5}{2}, \frac{7}{2}\right)$$

Length of median AD

$$AD = \sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}+1\right)^2}$$

$$= \sqrt{\left(\frac{-7}{2}\right)+\left(\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2}\ unit^2$$

Thus length of median AD is $\frac{\sqrt{130}}{2}$ units.

12. If $a \neq b \neq 0$, prove that the points $(a, a^2), (b, b^2), (0, 0)$ will not be collinear.

Ans : [Delhi Set I, II, III 2017]

If three points are collinear, then area covered by given points must be zero.

$$\text{area} = \frac{1}{2}[a(b^2-0)+b(0-a^2)+0(a^2-b2)]$$

$$= \frac{1}{2}[ab^2 - a^2b + 0]$$

$$= \frac{1}{2}[ab(b-a)] \neq 0 \text{ as } a \neq b \neq 0$$

Hence, the given points are not collinear.

13. If the points $A(k+1, 2k), B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear, then find the value of k.

Ans : [Delhi Set I, II, III, 2017]

14. If the points $A(k+1, 2k), B(3k, 2k+2)$ and $C(5k-1, 5k)$ are collinear, then find the value of k.

Ans : [Outside Delhi, Set-II, 2017]

If three points are collinear, then area covered by given points must be zero.

Thus area

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_2(y_1-y_2) = 0$$

Here $x_1 = k+1, x_2 = 3k, x_3 = 5k-1$

$y_1 = 2k, y_2 = 2k+3, y_3 = 5k.$

$$(k+1)(2k+3-5k)+3k(5k-2k)+$$
$$+(5k-1)(2k-2k-3) = 0$$
$$(k+1)(3-3k)+3k(3k)+(5k-1)(-3) = 0$$
$$3(1+k)(1-k)+3(k)(3k)-3(5k-1) = 0$$
$$3[1-k^2+3k^2-5k+1] = 0$$
$$2k^2-5k+2 = 0$$
$$2k^2-4k-k+2 = 0$$
$$2k(k-2)-1(k-2) = 0$$
$$(2k-1)(k-2) = 0$$

Thus $k = 2$ and $\frac{1}{2}$.

15. Thus $k = 2$ and $\frac{1}{2}$.The points $A(4, -2), B(7, 2), C(0, 9)$ and $D(-3, 5)$ from a parallelogram. Find the length of altitude of the parallelogram on the base AB.

Ans : [Sample Question Paper 2017]

Let the height of parallelogram taking AB as based be h.

Now
$$AB = \sqrt{(7-4)^2+(2+2)^2}$$
$$= \sqrt{3^2+4^2} = \sqrt{9+16}$$
$$= 5 \text{ units}$$

Area of $\Delta\, ABC$

$$\Delta_{ABC} = \frac{1}{2}[x_1(y_2-y_1)+x_2(y_3-y_1)+x_2(y_1-y_2)]$$

$$= \frac{1}{2}[4(2-9)+7(9+2)+0(2-2)]$$

$$= \frac{1}{2}\times 49 = \frac{49}{2} \text{ sq. units}$$

Now, $\frac{1}{2}\times AB \times h = \frac{49}{2}$

$$\frac{1}{2}\times 5\times h = 49$$

$$h = \frac{49}{5} = 9.8 \text{ units.}$$

16. Point $(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$. Find the values of y. Hence find the radius of the circle.

Ans : [Delhi CBSE, Term-2, 2014]

Since, $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$, OA and OB are the radius of circle and are equal. Thus

$$OA = OB$$
$$\sqrt{(-1-2)^2+(y+3y)^2} = \sqrt{(5-2)^2+(7+3y)^2}$$
$$9+16y^2 = 9y^2+42y+58$$
$$y^2-6y-7 = 0$$
$$(y+1)(y-7) = 0$$
$$y = -1, 7$$

When $y = -1$, centre is $O(2, -3y) = (2, 3)$ and radius

$$OB = \left|\sqrt{(5-2)^2+(7-3)^2}\right|$$
$$= \sqrt{9+16} = 5 \text{ unit}$$

When $y = 7$, centre is $O(2, -3y) = (2, -21)$ and radius

$$OB = \left|\sqrt{(2-5)^2+(-21-7)^2}\right|$$
$$= \left|\sqrt{9+784}\right| = \sqrt{793} \text{ unit}$$