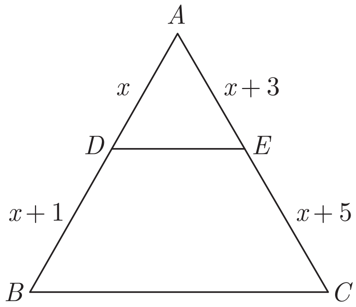
CHAPTER 6

Triangles

VERY SHORT ANSWER TYPE QUESTIONS

1. In $\Delta ABC, DE || BC$, find the value of x.



Ans : [Board Term-1, 2016, Set-O4YP6G7]

In the given figure $DE || BC$, thus

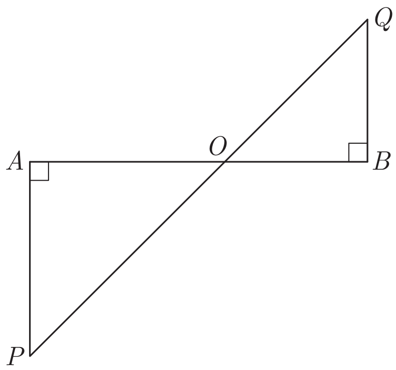
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

2. In the given figure, if $\angle A = 90^0, \angle B = 90^0$, $OB = 4.5\,cm$, $OA = 6\,cm$ and $AP = 4cm$, then find QB.



Ans : [Board Term-1, 2015, DDEE]

In ΔPAO and ΔQBO we have

$$\angle A = \angle B = 90^0$$

Vertically opposite angle,

$$\angle POA = \angle QOB$$

Thus $\Delta PAO \sim \Delta QBO$

$$\frac{OA}{OB} = \frac{PA}{QB}$$

$$\frac{6}{4.5} = \frac{4}{QB}$$

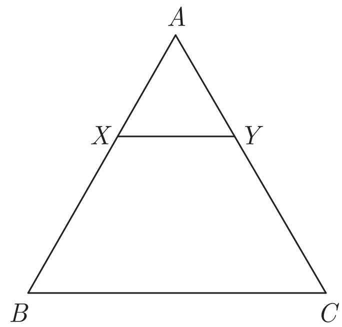
$$QB = \frac{4 \times 4.5}{6} = 3 \text{ cm}$$

Thus $QB = 3$ cm

3. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Ans : [Term-1, 2016, MV98HN3], [Term-1, 2015, CJTOQ]

As per question we have drawn figure given below.



In this figure we have

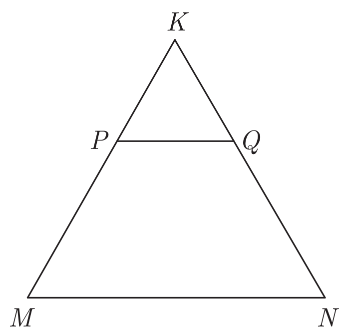
$$\frac{AX}{XB} = \frac{3}{4}, AY = 5, YC = 9$$

$$\frac{AX}{XB} = \frac{3}{4} \text{ and } \frac{AY}{YC} = \frac{5}{9}$$

Since $$\frac{AX}{XB} \neq \frac{AY}{YC}$$

Hence XY is not parallel to BC.

4. In the figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4cm$, then find KQ.



Ans :

In the given figure $PQ || MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN} \qquad \text{(By BPT)}$$

$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

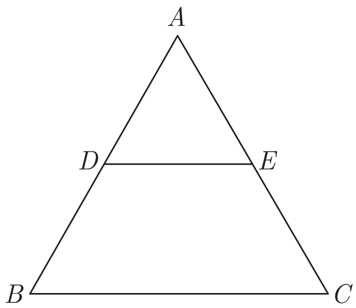
$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

$$4 \times 20.4 - 4KQ = 13KQ$$

$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$

5. In given figure $DE || BC$. If $AD = 3c, DB = 4cm$ and $AE = 6cm$, then find EC.



Ans : [Board Term-1, 2016, Set-ORDAWEZ]

In the given figure $DE \ || \ BC$, thus

$$\frac{AD}{AB} = \frac{AE}{EC}$$

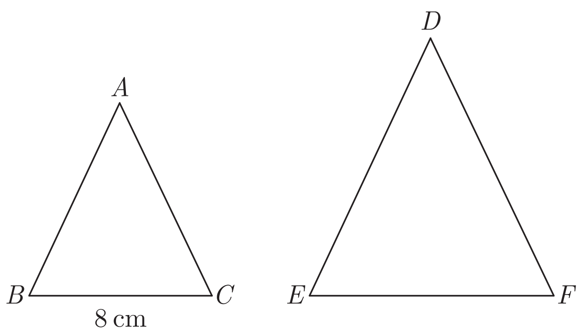
$$\frac{3}{4} = \frac{6}{EC}$$

$$EC = 8 \text{ cm}$$

6. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8cm$, then find EF.

Ans :

As per given condition we have drawn the figure below.



Here we have $2AB = DE$ and $BC = 8$ cm

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

7. Are two triangles with equal corresponding sides always similar?

Ans : [Board Term-1, 2015, Set-FHN8MGD]

Yes, Two triangles having equal corresponding sides are are congruent and all congruent Δs have equal angles, hence they are similar too.

8. If ratio of corresponding sides of two similar triangles is $5:6$, then find ratio of their areas.

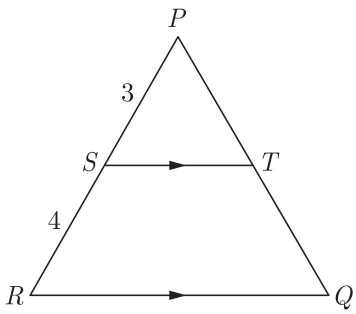
Ans : [Board Term-1, 2015, Set-WJQZQBN]

Let the triangles be ΔABC and ΔDEF

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

$$25:36$$

9. In the given figure, $ST || RQ, PS = 3$ cm and $SR = 4cm$. Find the ratio of the area of ΔPST to the area of ΔPRQ.



Ans : [Sample Question paper 2017]

We have $PS = 3$ cm, $SR = 4$ cm, and $ST || RQ$.

Now $PR = PS + SR$

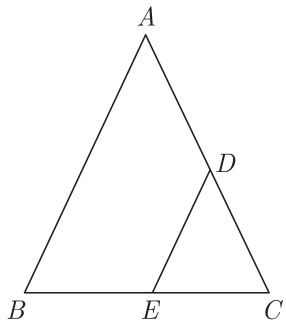
$$= 3 + 4 = 7cm$$

$$\frac{ar\Delta PST}{ar\Delta PQR} = \frac{PS^2}{PR^2} = \frac{3^2}{7^2} = \frac{9}{49}$$

Hence required ratio is $9:49$

SHORT ANSWER TYPE QUESTIONS - I

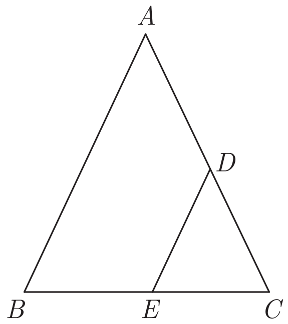
1. In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that $DE || AB$, $AD = 2x, DC = x + 3, BE = 2x - 1$ and $CE = x$. Then, find x.



OR

In the figure of $\Delta ABC, DE || AB$. $DE || AB$. If

$AD = 2x$, $DC = x+3$, $BE = 2x-1$ and $CE = x$, then find the value of x.



Ans : , [Term-1, 2016, GRKEGO], [Term-1, 2015, DDE-M]

We have $\dfrac{CD}{AD} = \dfrac{CE}{BE}$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Alternative Method :

In ABC, $DE \parallel AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$

$$\frac{CD}{CD+AD} = \frac{CE}{CE+BE}$$

$$\frac{x+3}{x+3+2x} = \frac{x}{2+2x-1}$$

$$\frac{x+3}{3x+3} = \frac{x}{3x-1}$$

$$(x+3)(3x-1) = x(3x+3)$$

$$3x^2 - x + 9x - 3 = 3x^2 + 3x$$

$$8x - 3 = 3x$$

$$8x - 3x = 3$$

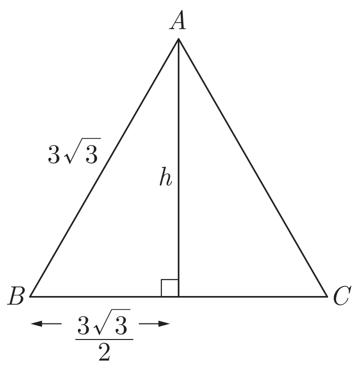
$$5x = 3$$

$$x = \frac{3}{5}$$

2. In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

Ans : [Board Term-1, 2016, Set-MV98HN3]

Let ΔABC be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



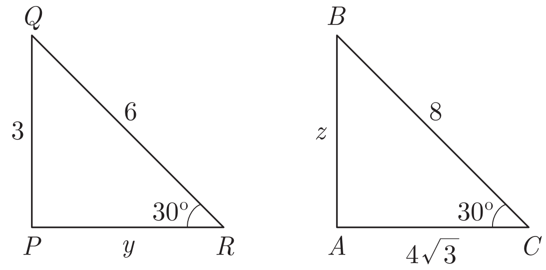
Now
$$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

$$27 = h^2 + \frac{27}{4}$$

$$h^2 = 27 - \frac{27}{4} = \frac{81}{4}$$

$$h = \frac{9}{2} = 4.5 \text{ cm}$$

3. In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y+z$.



Ans : [Board Term-1, Set-CJTOQ]

In the given figure $\Delta ABC \sim \Delta PQR$

Thus $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$

$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$z = \frac{8\times 3}{6} \text{ and } y = \frac{4\sqrt{3}\times 6}{8}$$

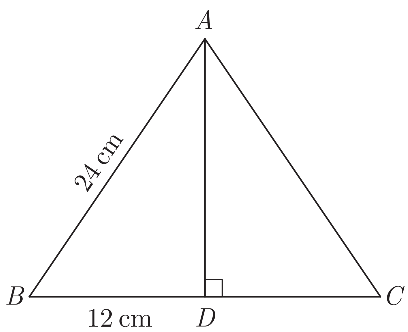
$$z = 4 \text{ and } y = 3\sqrt{3}$$

Thus $y + z = 3\sqrt{3} + 4$

4. In an equilateral triangle of side 24 cm, find the length of the altitude.

Ans : [Board Term-1, 2015, Set-DDE-E]

Let ΔABC be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $BD = \dfrac{BC}{2} = \dfrac{24}{2} = 12$ cm

$$AB = 24 \text{ cm}$$

$$AD = \sqrt{AB^2 - BD^2}$$
$$= \sqrt{(24)^2 - (12)^2}$$
$$= \sqrt{576 - 144}$$
$$= \sqrt{432} = 12\sqrt{3}$$

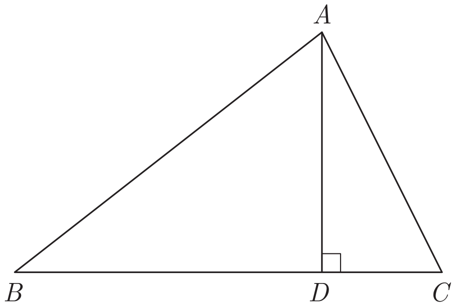
Thus $AD = 12\sqrt{3}$ cm

$\therefore$ The length of the altitude is $12\sqrt{3}$ cm.

5. In $\Delta ABC, AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

As per given condition we have drawn the figure below.



We have $AD^2 = BD \times CD$

$$\frac{AD}{CD} = \frac{BD}{AD}$$

Since $\angle D = 90^\circ$, by SAS we have

$$\Delta ADC \sim \Delta BDA$$

and $\angle BAD = \angle ACD;$

Since corresponding angles of similar triangles are equal

$$\angle DAC = \angle DBA$$

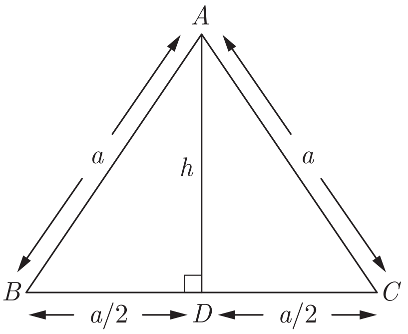
$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^\circ$$
$$2\angle BAD + 2\angle DAC = 180^\circ$$
$$\angle BAD + \angle DAC = 90^\circ$$
$$\angle A = 90^\circ$$

Thus ΔABC is right angled at A.

6. Find the altitude of an equilateral triangle when each of its side is 'a' cm.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

Let ΔABC be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



In ΔABD, $a^2 = \left(\frac{a}{2}\right)^2 + h^2$

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

Thus $h = \frac{\sqrt{3}a}{2}$

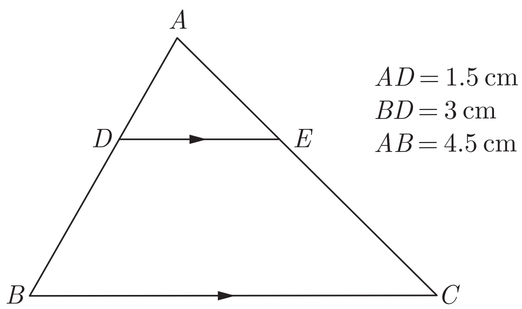
7. Let $\Delta ABC \sim \Delta DEF$. if ar $(\Delta ABC) = 100$ cm^2, $ar(DEF) = 196$ cm^2 and $DE = 7$, then find AB.

Ans : [Board Term-1, 2015, Set-DDE-M]

We have $\Delta ABC \sim \Delta DEF$, thus

$$\frac{ar(\Delta ABC)}{are(\Delta DEF)} = \frac{AB^2}{DE^2}$$
$$\frac{100}{196} = \frac{AB^2}{(7)^2}$$
$$\frac{100}{196} = \frac{AB^2}{49}$$
$$AB^2 = \frac{49 \times 100}{196}$$
$$AB^2 = 25$$
$$AB = 5 \text{ cm}$$

8. In the given figure, $DE || BC$. If $AD = 1.5$ cm $BD = 2AD$, then find $\frac{ar(\Delta ADE)}{ar(trapezium\ BCED)}$



Ans : [Board Term-1, 2013, set FFC]

We have $AD = 1.5$ cm, $BD = 3$

and $AB = AD + BD = 1.5 + 3.0 = 4.5cm$ cm

In triangle ADE and ABC, $\angle A$ is common and $DE || BC$

Thus $\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

(corresponding angles)

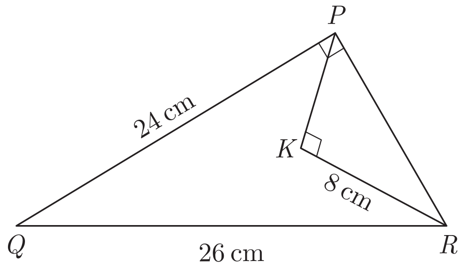
By AA similarity we have

$$\Delta ADE \sim \Delta ABC$$

Now $\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{(1.5)^2}{(4.5)^2} = \frac{1}{9}$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC) - ar(\Delta ADE)} = \frac{1}{9-1}$$
$$\frac{ar(\Delta ADE)}{ar(trapezium\ BCED)} = \frac{1}{8}$$

9. In the given triangle $PQR, \angle QPR = 90^\circ, PQ = 24$ cm and $QR = 26$ cm and in $\Delta PKR, \angle PKR = 90^o$ and $KR = 8$ cm, find PK.



Ans : [Board Term-1, 2012, Set-21]

In the given triangle we have

$$\angle QPR = 90^o$$

Thus $$QR^2 = QP^2 + PR^2$$

$$PR = \sqrt{26^2 - 24^2}$$

$$= \sqrt{100} = 10 \text{ cm}$$

Now $$\angle PKR = 90^o$$

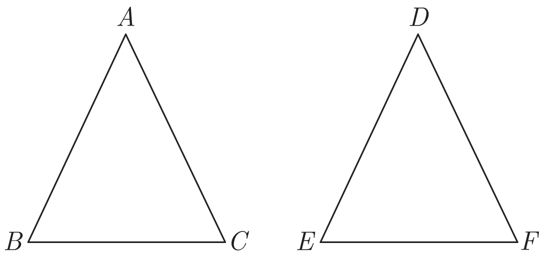
Thus $$PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$

$$= \sqrt{36} = 6 \text{ cm}$$

10. The sides AB and AC and the perimeter P_1 of ΔABC are respectively three times the corresponding sides DE and DF and the parameter P_2 of ΔDEF. Are the two triangles similar? If yes, find $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$

Ans : [Board Term-1, 2012, SEt-39]

As per given condition we have drawn the figure below.



In ΔABC and ΔDEF,

$$AB = 3DE$$

and $$AC = 3DF$$

$$\frac{AB}{DE} = 3; \frac{AC}{DF} = 3;$$

Since $P_1 = 3P_2$, $BC = 3EF$

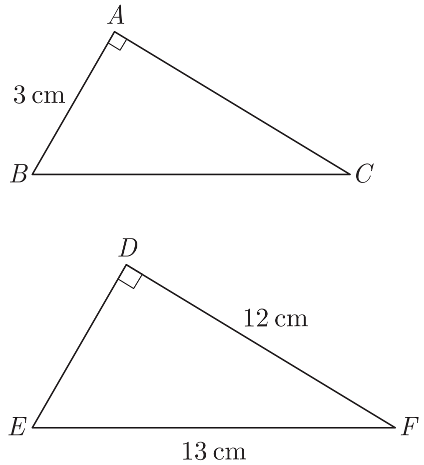
Thus $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$$

By SSS criterion we have

$$\Delta ABC \sim \Delta DEF$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = (3)^2 = 9$$

11. Given $\Delta ABC \sim \Delta DEF$, find $\frac{\Delta ABC}{\Delta DEF}$



Ans : [Board Term-1, 2016, Set-ORDAWEZ]

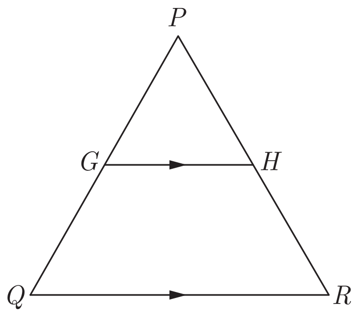
In ΔDEF, we have

$$DE = \sqrt{(13)^2 - (12)^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = 5$$

Thus $$\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

12. In the given figure, G is the mid-point of the side PQ of ΔPQR and $GH || QR$. Prove that H is the mid-point of the side PR or the triangle PQR.



Ans : [Board Term-1, 2012, Set-43]

Since G is the mid-point of PQ we have

$$PG = GQ$$

$$\frac{PG}{GQ} = 1$$

According to the question, $GH || QR$, thus

$$\frac{PG}{GQ} = \frac{PH}{HR} \quad \text{(By BPT)}$$

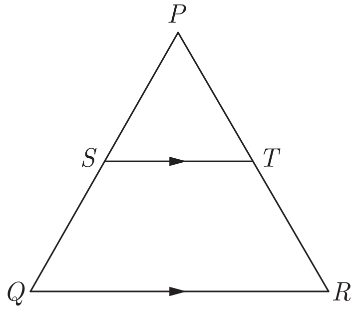
$$1 = \frac{PH}{HR}$$

$$PH = HR. \quad \text{Hence proved.}$$

Hence, H is the mid-point of PR.

13. In the given figure, in a triangle $PQR, ST || QR$ and

$\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT.



Ans : [Board Term-1, 2012, Set-64; Set-I 2011]

We have $\frac{PS}{SQ} = \frac{3}{5}$

$$\frac{PS}{PS+SQ} = \frac{3}{3+5}$$

$$\frac{PS}{PQ} = \frac{3}{8}$$

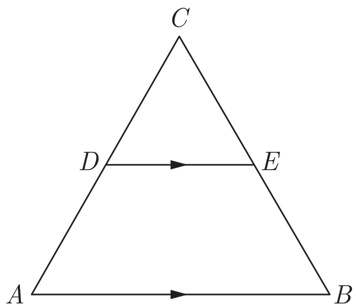
According to the question, $ST || QR$, thus

$$\frac{PS}{PQ} = \frac{PT}{PR} \quad \text{(By BPT)}$$

$$PT = \frac{PS}{PQ} \times PR$$

$$= \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

14. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE || AB$.



Ans : [Board Term-1, 2012, set-63]

In ΔCAB, we have

$$\angle A \ \angle B \quad (1)$$

By isoscales triangle property we have

$$AC = CB$$

But, we have been given

$$AD = BE \quad (2)$$

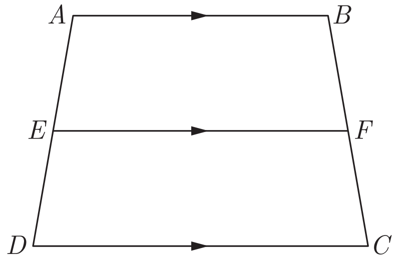
Dividing equation (2) by (1) we get,

$$\frac{CD}{AD} = \frac{CE}{BE}$$

By converse of BPT,

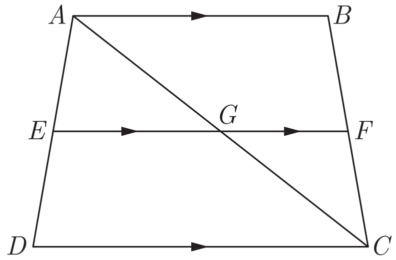
$DE || AB$. Hence Proved

15. In the given figure, if $ABCD$ is a trapezium in which $AB || CD || EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans : [Board Term-1, 2012, Set-25]

We draw, AC intersecting EF at G as shown below.



In $\Delta CAB, GF || AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \quad ...(1)$$

In ΔADC, $EG || DC$, thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \quad ...(2)$$

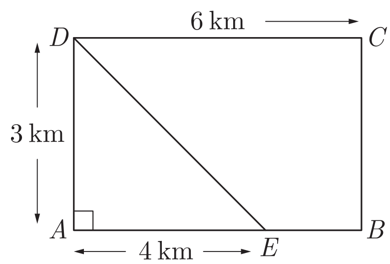
From equations (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC}.$$ Hence Proved.

16. In a rectangle $ABCD, E$ is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Ans : [Board Term-1, 2016, Set-LGRKEGO]

As per given condition we have drawn the figure below.



We have $AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4$ km

In right triangle ADE,

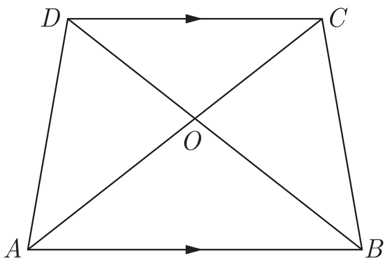
$$DE^2 = (3)^2 + (4)^2 = 25$$

Thus $DE = 5$ km

17. $ABCD$ is a trapezium in which $AB || CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans : [Board Term-1, 2012, Set-66]

As per given condition we have drawn the figure below.



In ΔAOB and ΔCOD, $AB \parallel CD$,

Thus $\angle OAB = \angle DCO$

and $\angle OBA = \angle ODC$ (Alternate angles)

By AA similarity we have

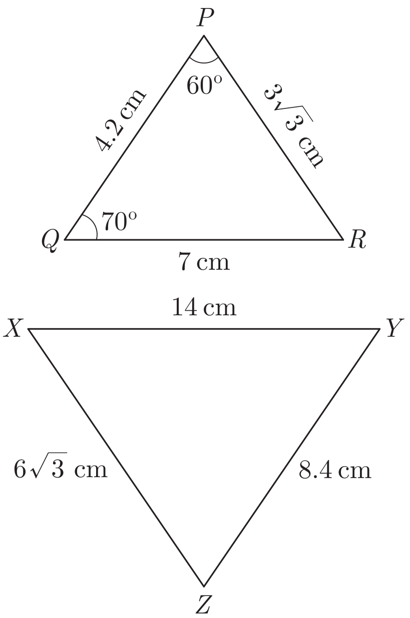
$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}.$$ Hence Proved

18. In the given figures, find the measure of $\angle X$.



Ans : [Board Term-1, 2012, Set-38]

From given figures,

$$\frac{PQ}{ZY} = \frac{4.2}{8.4} = \frac{1}{2},$$

$$\frac{PR}{ZX} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

and $$\frac{QR}{YX} = \frac{7}{14} = \frac{1}{2}$$

Thus $$\frac{QP}{ZY} = \frac{PR}{ZX} = \frac{QR}{YX}$$

By SSS criterion we have

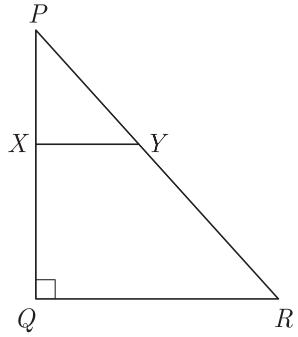
$$\Delta PQR \sim \Delta ZYX$$

Thus $\angle X = \angle R$

$$= 180^\circ - (60^\circ + 70^\circ) = 50^\circ$$

Thus $\angle X = 50^\circ$

19. In the given figure, PQR is a triangle right angled at Q and $XY \parallel QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1:2$. Calculate the length of PR and QR.



Ans : [Board Term-1, 2012, Set-44]

Since $XY \parallel OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY}$$

$$= \frac{4}{PR - 4}$$

$$PR - 4 = 8$$

$$PR = 12 \text{ cm}$$

In right ΔPQR we have

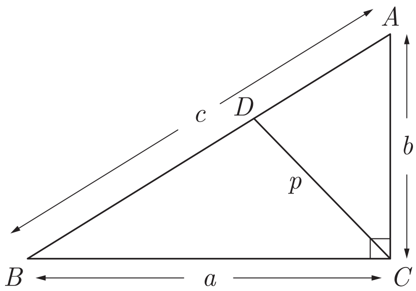
$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2$$

$$= 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

20. ABC is a right triangle right angled at C. Let $BC = a$, $CA = b$, $AB = c$ and p be the length of perpendicular from C to AB. Prove that $cp = ab$.



Ans : [Board Term-1, 2012, Set-65]

In the given figure $CD \perp AB$, and

$$CD = p$$

Area $$\Delta ABC = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times AB \times CD = \frac{1}{2} cp$$

Also,Area of $\Delta ABC = \frac{1}{2}\times BC \times AC = \frac{1}{2}ab$

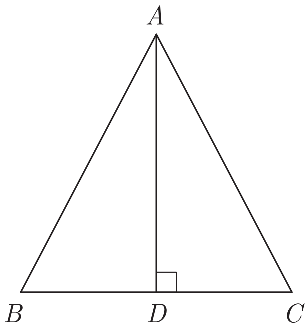
Thus $\frac{1}{2}cp = \frac{1}{2}ab$

$cp = ab$ Proved

21. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.

Ans : [Board Terom-1, 2012, Set-40]

In ΔABD, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

Since $AB = BC = CA$, we get

$$BC^2 = AD^2 + BD^2,$$

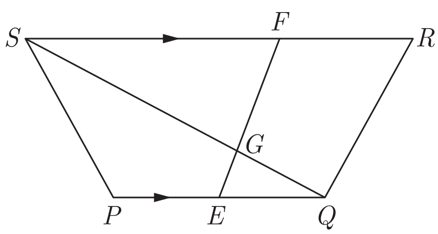
Since $\perp$ is the median in an equilateral Δ, $BC = 2BD$

$$(2BD)^2 = AD^2 + BD^2$$

$$4BD^2 - BD^2 = AD^2$$

$$3BD^2 = AD^2$$

22. In the figure, $PQRS$ is a trapezium in which $PQ || RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.



Ans : [Board Term-1, 2016, Set-O4YP6G7]

In ΔGEQ and ΔGFS

$\angle EGQ = \angle FGS$ (vert. opp. angles)

$\angle EQG = \angle FSG$ (alt. angles)

Thus by AA similarity we have

$$\Delta GEQ \sim GFS$$

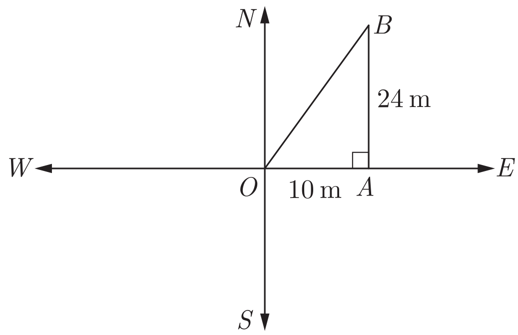
$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

$$EQ \times GS = GQ \times FS$$

23. A man steadily goes 10 m due east and then 24 m due north.
(1) Find the distance from the starting point.
(2) Which mathematical concept is used in this problem?

Ans :

(1) Let the initial position of the man be at O and his final position be B. Since the man goes to 10 m due east and then 24 m due north. Therefore, ΔAOB is a right triangle right angled at A such that $OA = 10$ m and $AB = 24$ m. We have shown this condition in figure below.



By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2$$

$$= (10)^2 + (24)^2$$

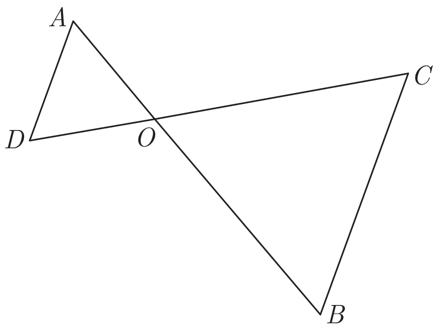
$$= 100 + 576 = 676$$

or, $OB = \sqrt{676} = 26$ m

Hence, the man is at a distance of 26 m from the starting point.

(2) Pythagoras Theorem

24. In the given figure, $OA \times OB = OC \times OD$, show that $\angle A = \angle C$ and $\angle B = \angle D$.



Ans : [Board Term-1, 2012, Set-71]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OB}{OC}$$

$$\angle AOD = \angle COB$$

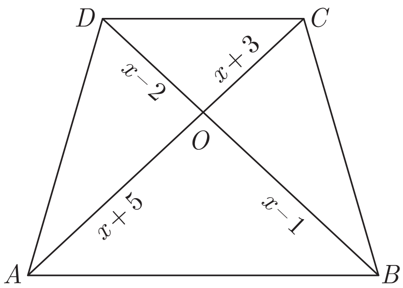
(Vertically opposite angles)

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

25. In the given figure, if $AB || DC$, find the value of x.



Ans : [Board Term-1, 2012, Set-35]

We know that diagonals of a trapezium divide each other proportionally. Therefore

$$\frac{OA}{OC} = \frac{BO}{OD}$$

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$$(x+5)(x-2) = (x-1)(x+3)$$

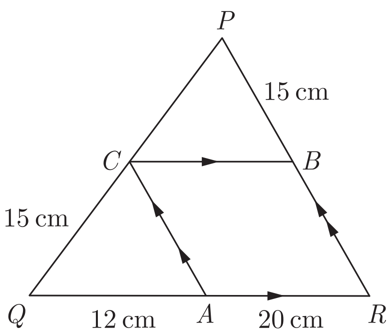
$$x^2 + 2x + 85x - 10 = x^2 + 3x - x - 3$$

$$3x - 2x = 10 - 3$$

$$x = 7$$

Thus $x = 7$.

26. In the given figure, $CB || QR$ and $CA || PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR.



Ans : [Board Term-1, 2012, Set-55]

In ΔPQR, $CA || PR$

By BPT similarity we have

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In ΔPQR, $CB || QR$

Thus $$\frac{PC}{CQ} = \frac{PR}{BR}$$

$$\frac{25}{15} = \frac{15}{BR}$$

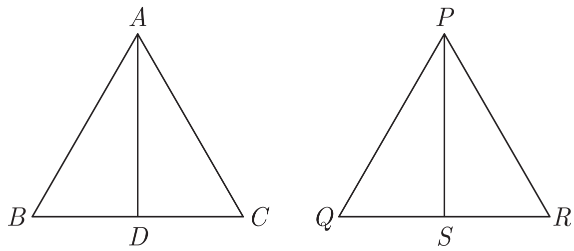
$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

SHORT ANSWER TYPE QUESTIONS - II

1. If $\Delta ABC \sim \Delta PQR$ and AD and PS are bisectors of corresponding angles A and P, then prove that $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$.

Ans : [Board Term-1, 2016, Set-MV98HN3]

As per given condition we have drawn the figure below.



Since $\Delta ABC \sim \Delta PQR$ we have

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad ...(1)$$

Now $$\angle A = \angle P$$

$$\frac{1}{2}\angle A = \frac{1}{2}\angle P$$

$$\angle BAD = \angle QPS$$

By AA similarity we have

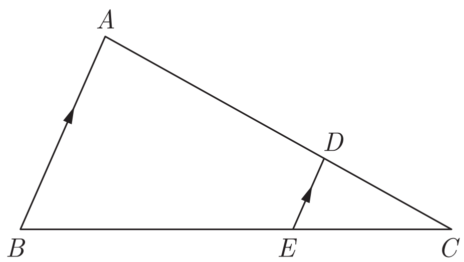
$$\Delta BAD \sim \Delta QPS$$

$$\frac{BA}{QP} = \frac{AD}{PS} \quad ...(2)$$

By equation (1) and (2), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$$ Hence proved

2. In given figure, D is a point on AC such that $AD = 2CD$, also $DE || AB$.



Find : $\frac{ar\Delta ACB}{ar\Delta DCE}$

Ans : [Board Term-1, 2015, Set-FHN8MGD]

In given figure we have

$$AD = 2CD$$

In ΔCDE and ΔCAB

$$\angle C = \angle C \quad \text{(Common)}$$

$$\angle CDE = \angle CAB$$

(Corresponding angles)

By AA similarity rule we get

$$\Delta CDE \sim \Delta CAB$$

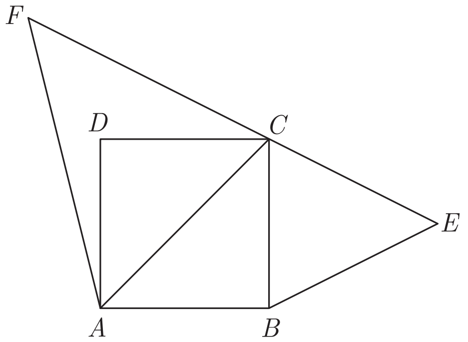
Now, $$\frac{ar(\Delta DCE)}{ar(\Delta ACB)} = \frac{CD^2}{CA^2} = \frac{CD^2}{(AD+DC)^2}$$

$$= \frac{CD^2}{(2DC+DC)^2} = \frac{CD^2}{(3CD)^2} = \frac{1}{9}$$

3. Prove that area of the equilateral triangle described on the side of a square is half of this area of the equilateral triangle described on its diagonal.

Ans : [Board Term-1, 2015, WJQZQBN]

As per given condition we have drawn the figure below.



Equilateral triangle are equiangular also and they are similar by AAA similarity criterion.

Thus $\Delta BCE \sim \Delta ACF$

Here ΔABC is a right triangle.

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

Here $AB = BC$ because of sides of a square,

$$AC^2 = 2BC^2$$

$$AC = \sqrt{2}\,BC$$

Now, $$\frac{ar\Delta ACF}{ar\Delta BCE} = \frac{AC^2}{DC^2} = \frac{(\sqrt{2}\,BC)^2}{BC^2} = 2$$

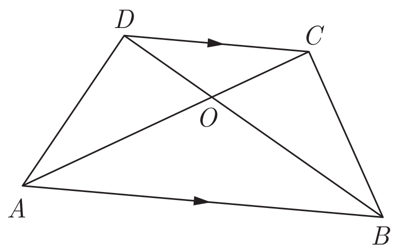
$$ar(\Delta ACF) = 2ar(\Delta BEC)$$

$$ar(\Delta BEC) = \frac{1}{2}ar(\Delta ACF)$$ Hence Proved.

4. In a trapezium $ABCD$, diagonals AC and BD intersect at O. I $AB = 3DC$, then find ratio of areas of triangles COD and AOB.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

As per given condition we have drawn the figure below.



because of AA similarity we have

$$\Delta AOB \sim \Delta COD$$

$$\frac{ar(\Delta COD)}{ar(\Delta AOB)} = \frac{CD^2}{AB^2}$$

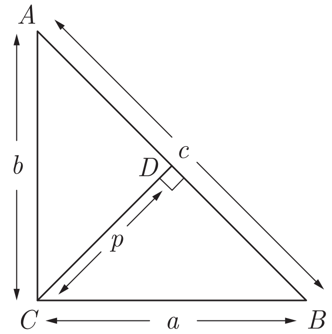
$$= \frac{CD^2}{(3CD)^2} = \frac{CD^2}{9CD^2} = \frac{1}{9}$$

ratio $= 1:9$

5. ΔABC is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle A, \angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

As per given condition we have drawn the figure below.



In ΔACB and ΔCDB

$$\angle ABC = \angle CDB = 90^0$$

$$\angle B = \angle B \quad \text{(common)}$$

Because of AA similarity we have

$$\Delta ABC \sim \Delta CDB$$

Now $$\frac{b}{p} = \frac{c}{a}$$

$$\frac{1}{p} = \frac{c}{ab}$$

$$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

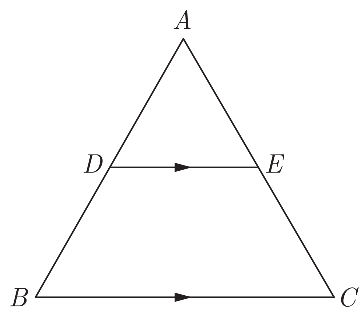
$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ Hence Proved

6. In $\Delta ABC, DE || BC$. If $AD = x+2, DB = 3x+16$, $AE = x$ and $EC = 3x+5$, them find x.

Ans : [Board Term-1, 2015, Set-DDE-E]

As per given condition we have drawn the figure below.



In the give figure

$$DE \parallel BC$$

By BPT we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x+2}{3x+16} = \frac{x}{x3+5}$$

$$(x+2)(3x+5) = x(3x+16)$$

$$3x^2+5x+6x+10 = 3x^2+16x$$

$$11x+10 = 16x$$

$$11x+10 = 10$$

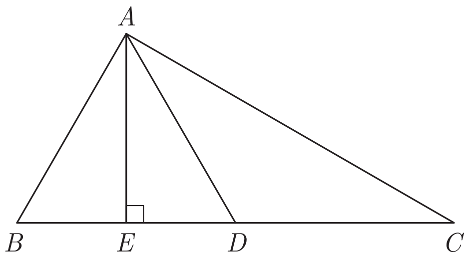
$$5x = 10$$

$$x = 2$$

7. If in ΔABC, AD is median and $AE \perp BC$, then prove that $AB^2+AC^2 = 2AD^2+\frac{1}{2}BC^2$.

Ans : [Board Term-1, 2015, Set-DEE-E]

As per given condition we have drawn the figure below.



In ΔABE, using Pythagoras theorem we have

$$AB^2 = AE^2+BE^2$$

$$= AD^2-DE^2+(BD-DE)^2$$

$$= AD^2-DE^2+BD^2+DE^2-2BD\times DE$$

$$= AD^2+BD^2-2BD\times DE \quad ...(1)$$

In ΔAEC,

$$AC^2 = AE^2+EC^2$$

$$= (AD^2-ED^2)+(ED+DC)^2$$

$$= AD^2-ED^2+ED^2+DC^2+2ED\times DC$$

$$= AD^2+CD^2+2ED\times CD$$

$$= AD^2+DC^2+2DC\times DE \quad ...(2)$$

Adding equation (1) and (2) we have

$$AB^2+AC^2 = 2(AD^2+BD^2) \quad (BD=DC)$$

$$= \left[2AD^2+2\left(\frac{1}{2}BC\right)^2\right] \quad \left(BD=\tfrac{1}{2}BC\right)$$

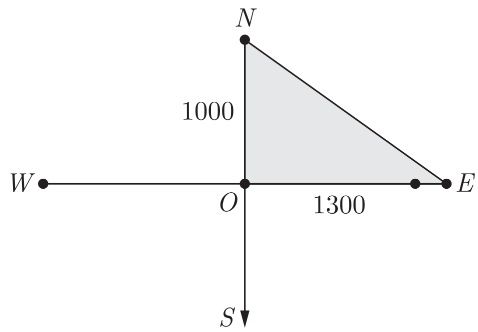
$$= 2AD^2+\frac{1}{2}BC^2$$

Hence Proves

8. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours.

Ans : [Board Term-1, 2015, Set-DDE-E]

As per given condition we have drawn the figure below.



Distance covered by first aeroplane due North after two hours $= 500\times 2 = 1{,}000$ km.
Distance covered by second aeroplane due East after two hours $= 650\times 2 = 1{,}300$ km.
Distance between two aeroplane after 2 hours

$$NE = \sqrt{ON^2+OE^2}$$

$$= \sqrt{(1000)^2+(1300)^2}$$

$$= \sqrt{1000000+1690000}$$

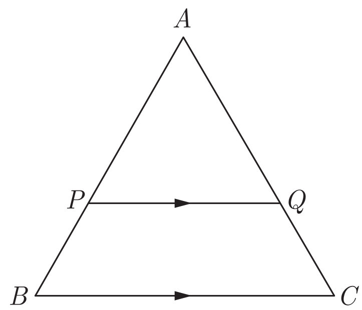
$$= \sqrt{2690000}$$

$$= 1640.12 \text{ km}$$

9. ABC is a triangle, PQ is the line segment intersecting AB in P and AC in Q such that $PQ \parallel BC$ and divides ΔABC into two parts, equal in area, find $BP:AB$,

Ans : [Board Term-1, 2012, LK-59]

As per given condition we have drawn the figure below.



Here, Since $PQ \parallel BC$ and PQ divides ΔABC into two equal parts, thus $\Delta APQ \sim \Delta ABC$

Now $$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$\frac{1}{2} = \frac{AP^2}{AB^2}$$

$$\frac{1}{\sqrt{2}} = \frac{AP}{AB}$$

$$\frac{1}{\sqrt{2}} = \frac{AB-BP}{AB} \quad (AB = AP+BP)$$

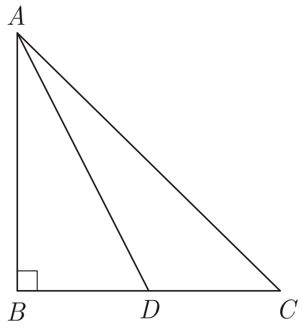
$$\frac{1}{\sqrt{2}} = 1-\frac{BP}{AB}$$

$$\frac{BP}{AB} = 1-\frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$BP:AB = (\sqrt{2}-1):\sqrt{2}$$

10. In the given figure, ABC is a right angled triangle, $\angle B = 90^\circ$. D is the mid-point of BC. Show that

$AC^2 = AD^2 + 3CD^2$.



Ans : [Board Term-1, 2016, Set-ORDAWEZ 2011, Set-60]

We have $BD = CD = \frac{BC}{2}$

$$BC = 2BD$$

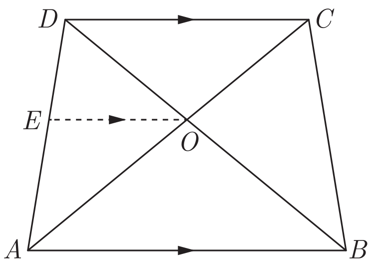
Using Pythagoras theorem in the right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= AB^2 + 4BD^2$$
$$= (AB^2 + BD^2) + 3BD^2$$
$$AC^2 = AD^2 + 3CD^2$$

11. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Ans : [Board Term-1, 2011, Set-39]

As per given condition we have drawn the figure below.



We have drawn $EO || AB$ on DA.

In quadrilateral $ABCD$,

$$\frac{AO}{BO} = \frac{CO}{DO} \quad \text{(Given)}$$

or, $$\frac{AO}{CO} = \frac{BO}{DO} \quad ...(1)$$

In ΔABD, $EO || AB$

By BPT we have

$$\frac{AE}{ED} = \frac{BO}{DO} \quad ...(2)$$

From equation (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In ΔADC, $\frac{AE}{ED} = \frac{AO}{CO}$

$EO || DC$ (Converse of BPT)

$EO || AB$ (Construction)

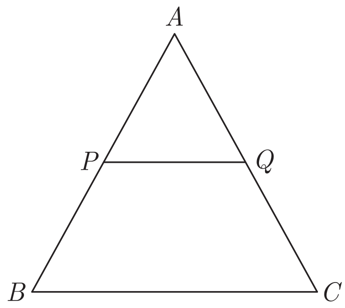
$AB || DC$

Thus in quadrilateral $ABCD$ we have

$$AB = DC$$

Thus $ABCD$ is a trapezium. Hence Proved

12. In the given figure, P and Q are the points on the sides AB and AC respectively of ΔABC, such that $AP = 3.5$cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC.



Ans : [Board Term-1, 2011, Set-40]

We have $\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3}$

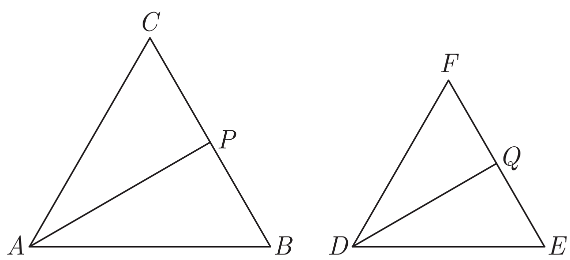
$$\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$$

In ΔABC, $\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common.

Thus due to SAS we have

$$\Delta APQ \sim \Delta ABC$$
$$\frac{AP}{AB} = \frac{PQ}{BC}$$
$$\frac{1}{3} = \frac{4.5}{BC}$$
$$BC = 13.5 \text{ cm.}$$

13. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



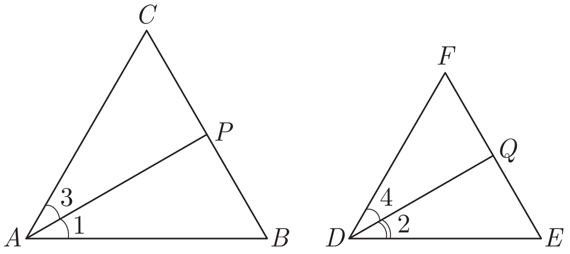
Prove that :

(1) $\frac{AP}{DQ} = \frac{AB}{DE}$

(2) $\Delta CAP \sim \Delta FDQ$.

Ans : [Board Term-1, 2016, Set-LGRKEGO]

As per given condition we have redrawn the figure below.



(1) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$

$$2\angle 1 = 2\angle 2$$

Also $\angle B = \angle E$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE} \qquad \text{Hence Proved}$$

(2) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D$$

and $\angle C = \angle F$

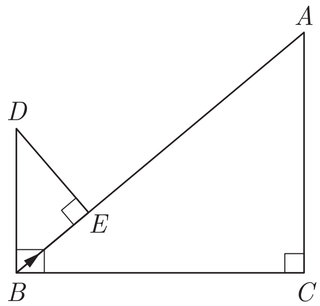
$$2\angle 3 = 2\angle 4$$

$$\angle 3 = \angle 4$$

By AA similarity we have

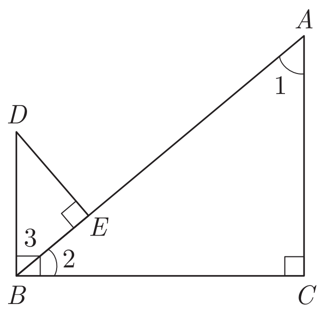
$$\Delta CAP \sim \Delta FDQ$$

14. In the given figure, $DB \perp BC, DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Ans : [Board Term-1, 2011, Set-40]

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In ΔABC,

$$\angle 1 + \angle 2 = 90^\circ \qquad [\angle C = 90^\circ]$$

But we have been given

$$\angle 2 + \angle 3 = 90^\circ$$

Hence $\angle 1 = \angle 3$

In ΔABC and ΔBDE,

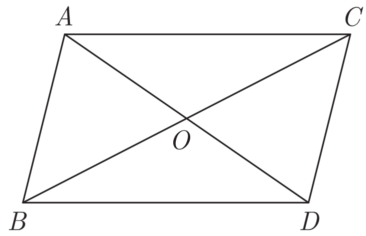
$$\angle 1 = \angle 3 \quad \text{(Proved)}$$

$$\angle ACB = \angle DEB = 90^\circ \quad \text{(Given)}$$

$$\Delta ABC \sim \Delta BDE \quad \text{(By } AA \text{ Similarity)}$$

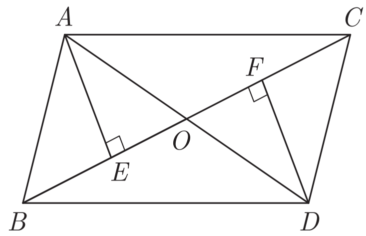
Thus $\frac{AC}{BC} = \frac{BE}{DE}$. Hence Proved

15. In the given figure, ΔABC and ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



Ans : [Term-1, 2011, Set-40], [Term-1, 2016, ORDAWEZ]

As per given condition we have redrawn the figure below. Here we have drawn $AE \perp BC$ and $DF \perp BC$.



In ΔAOE and ΔDOF,

$$\angle AOE = \angle DOF$$

(Vertically opposite angles)

$$\angle AEO = \angle DFO = 90^\circ \quad \text{(Construction)}$$

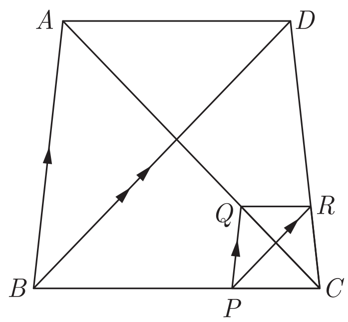
or, $\Delta AOE \sim \Delta DOF$ (By AA similarity)

Thus $\frac{AO}{DO} = \frac{AE}{DF}$ ...(1)

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$

$$= \frac{AE}{DF} = \frac{AO}{DO} \text{From equation (1)}$$

16. In the given figure, two triangles ABC and DBC lie on the same side of BC such that $PQ || BA$ and $PR || BD$. Prove that $QR || AD$.



Ans : [Boar term-1, 2011, Set-21]

In ΔABC, we have $PQ || AB$ and $PR || BD$

$$\frac{BP}{PC} = \frac{AQ}{QC} \quad \text{(by BPT) ...(1)}$$

Again in ΔBCD, we have

$$PR \,||\, BD$$

$$\frac{BP}{PC} = \frac{DR}{RC} \quad \text{(by BPT) ...(2)}$$

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

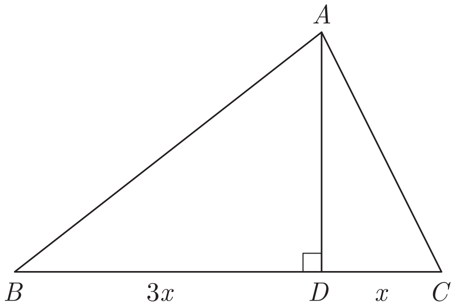
$$PR \,||\, AD \quad \text{(By converse of BPT)}$$

Hence proved

17. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that $DB = 3CD$. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.

Ans : [Board Term-1, 2011, Set-44, 60, 2012, 2016 Set-39, Set-NH3]

As per given condition we have drawn the figure below.



In ΔADB, we have

$$AB^2 = AD^2 + BD^2 \quad \text{...(1)}$$

(Pythagoras Theorem)

In ΔADC, $\quad AC^2 = AD^2 + CD^2 \quad$ ...(2)

(Pythagoras theorem)

Subtracting eqn. (2) from eqn. (1), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

$$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{BC^2}{2}$$

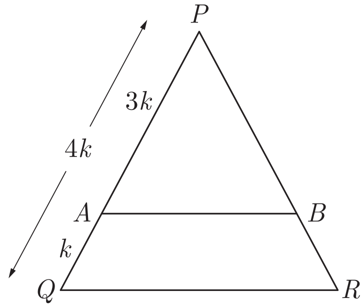
$$2(AB^2 - AC^2) = BC^2$$

$$2(AB)^2 = 2AC^2 + BC^2. \quad \text{Hence Proved}$$

18. In the given figure, $\frac{PA}{AQ} = \frac{BR}{BR} = 3$. If the area of ΔPQR is $32\,cm^2$, then find the area of the quadrilateral $AQRB$.

Ans : [Board Term-1, 2011, Set-44]

As per given condition we have drawn the figure below.



Since $\angle P$ common and $\frac{PA}{AQ} = \frac{PB}{BR}$, therefore

We have, $\Delta PQR \sim \Delta PAB$

$$\frac{area(\Delta PQR)}{area(\Delta PAB)} = \frac{PQ^2}{PA^2}$$

$$\frac{32}{are(\Delta PAB)} = \frac{(4k)^2}{(3k^2)} = \frac{16k^2}{9k^2}$$

$$\text{area}\Delta PAB = 18\,cm^2$$

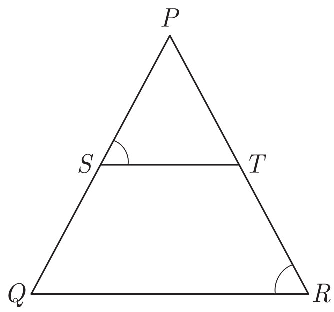
Thus area of quadrilateral $AQRB$,

$$= area\ of\ \Delta PQR - area\ of\ \Delta PAB$$

$$= 32 - 18$$

$$= 14\,cm^2$$

19. In the given figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.



Ans : [Board Term-1, 2011, Set-74]

We have $\quad \frac{PS}{SQ} = \frac{PT}{TR}$

and $\quad \angle PST = \angle PRQ$

By converse of BPT,

$$ST \,||\, QR$$

$$\angle PST = \angle PQR$$

(Corresponding angles)

and $\quad \angle PST = \angle PRQ$

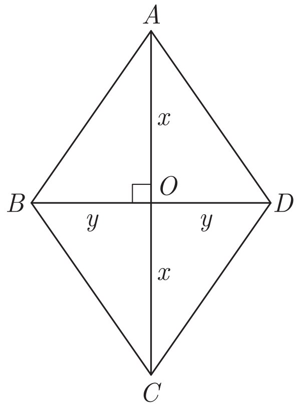
Thus $\quad \angle PQR = \angle PRQ$

So, ΔPQR is an isosceles triangle. Hence Proved

20. Prove that the sum of squares on the sides of a rhombus is equal to sum of squares of its diagonals.

Ans : [Board Term-1, 2011, Set-21]

Let, $ABCD$ is a rhombus and since diagonals of a rhombus bisect each other at $90°$.



Now $AO = OC \Rightarrow AO^2 = OC^2$

$BO = OD \Rightarrow BO^2 = OD^2$

and $\angle AOB = 90^0$

$AB^2 = OA^2 + BO^2$

Similarly, $AD^2 = x^2 + y^2 = BC^2 = CD^2$

$$AB^2 + BC^2 + CD^2 + DA^2 = 4AO^2 + 4DO^2$$

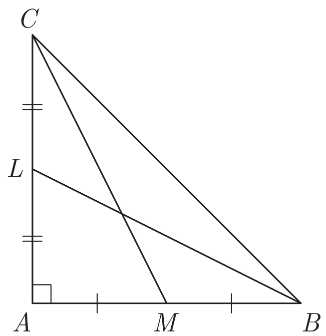
$$= (2AO)^2 + (2DO)^2$$

$$= (2x)^2 + (2y)^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Hence Proved

21. In the given figure, BL and CM are medians of ΔABC, fight angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



Ans : [Board Term-1, 2011, Set-74]

We have a right angled triangle ΔABC at A where BL and CM are medians.

In ΔABL, $BL^2 = AB^2 + AL^2$

$$= AB^2 + \left(\frac{AC}{2}\right)^2 \quad (BL \text{ is median})$$

In ΔACM, $CM^2 = AC^2 + AM^2$

$$= AC^2 + \left(\frac{AB}{2}\right)^2 (CM \text{ is median})$$

Now

$$BL^2 + CM^2 = AB^2 + AC^2 + \frac{AC^2}{4} + \frac{AB^2}{4}$$

$$4(BL^2 + CM^2) = 5AB^2 + 5AC^2$$

$$= 5(AB^2 + AC^2)$$

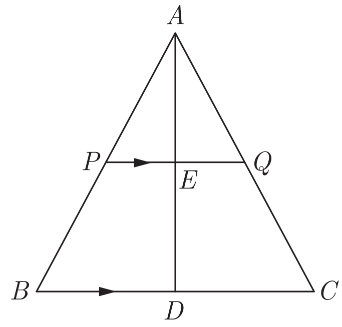
$$= 5BC^2.$$

Hence Proved

22. In a ΔABC, let P and Q be points on AB and AC respectively such that $PQ || BC$. Prove that the median AD bisects PQ.

Ans : [Board Term-1, 2011, Set-70]

As per given condition we have drawn the figure below.



The median AD intersects PQ at E.

We have, PQ || BE

or, $\angle ApE = \angle B$ and $\angle AQE = \angle C$

(Corresponding angles)

Thus in ΔAPE and ΔABD we have

$\angle APE = \angle ABD$

$\angle PAE = \angle BAD$

Thus $\Delta APE \sim \Delta ABD$

$$\frac{PE}{BD} = \frac{AE}{AD} \quad ...(1)$$

Similarly, $\Delta AQE \sim \Delta ACD$

or, $$\frac{QE}{CD} = \frac{AE}{AD} \quad ...(2)$$

From eqns. (1) and (2),

$$\frac{PE}{BD} = \frac{QE}{CD}$$

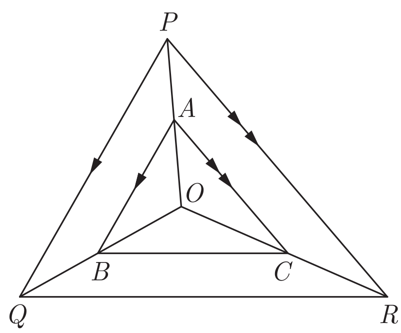
As $CD = BD$, we get

$$\frac{PE}{BD} = \frac{QE}{BD}$$

or, $PE = QE$

Hence, AD bisects PQ.

23. In the given figure A, B and C are points on OP, OQ and OR respectively such that $AB || PQ$ and $AC || PR$. Prove that $BC || QR$.



Ans : [Board Term-1, 2012, Set-66]

In ΔPOQ, $AB \parallel PQ$

By BPT $\frac{AO}{AP} = \frac{OB}{BQ}$ ...(1)

In ΔOPR, $AC \parallel PR$,

By BPT $\frac{OA}{AP} = \frac{OC}{CR}$ (2)

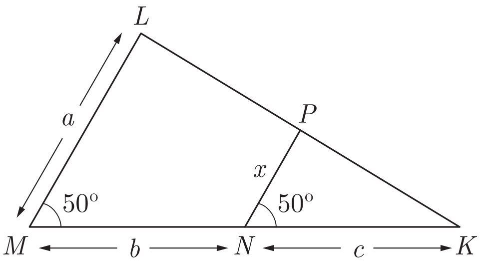
From equations (1) and (2), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

(By converse of BPT)

$BC \parallel QR$ Hence Proved

24. In the given figure, find the value of x in terms of a, b and c.



Ans : [Board Term-1, 2012, Set-52]

In triangles LMK and PNK,

$\angle M = \angle N = 50°$ (Given)

$\angle K = \angle K$ (Common)

Due to AA similarity,

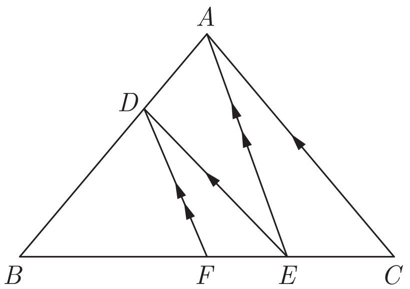
$$\Delta LMK \sim \Delta PNK$$

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c}$$

$$x = \frac{ac}{b+c}$$

25. In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BE}{FE} = \frac{BE}{EC}$.



Ans : [Board Term-1, 2012 Set-66]

In ΔABC, $DE \parallel AC$, (Given)

By BPT $\frac{BD}{DA} = \frac{BE}{EC}$ ...(1)

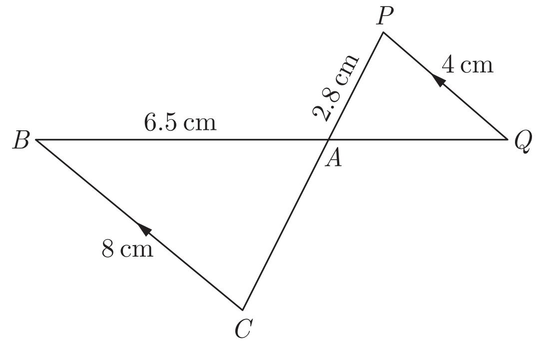
In ΔABE, $DF \parallel AE$, (Given)

By BPT $\frac{BD}{DA} = \frac{BF}{FE}$ ...(2)

From (1) and (2), we have

$\frac{BF}{FE} = \frac{BE}{EC}$. Hence proved

26. In the given figure, $BC \parallel PQ$ and $BC = 8 cm$, $PQ = 4 cm$, $BA = 6.5 \, cm$, $AP = 2.8 cm$. Find CA and AQ.



Ans : [Board Term-1, 2012, Set-66]

In ΔABC and ΔAPQ, $AB = 6.5$ cm, $BC = 8$ cm, $PQ = 4$ cm and $AP = 2.8$ cm

$BC \parallel PQ$ (Given)

Due to alternate angles

$$\angle CBA = \angle AQP$$

Due to vertically opposite angles,

$$\angle BAC = \angle PAQ$$

Due to AA similarity

$$\Delta ABC \sim \Delta AQP$$

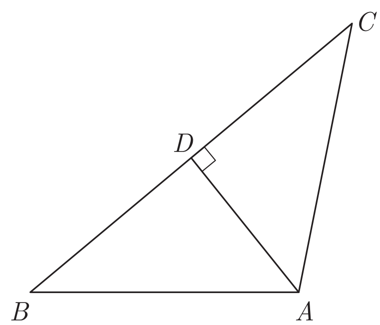
$$\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$$

$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{AC}{AP}$$

$$AQ = \frac{6.5}{2} = 3.25 \text{ cm}$$

$$AC = 2 \times 2.5 = 5.6 \text{ cm}$$

27. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.



In right angled ΔADC,

$AC^2 = AD^2 + CD^2$ ...(1)

In right ΔADB,

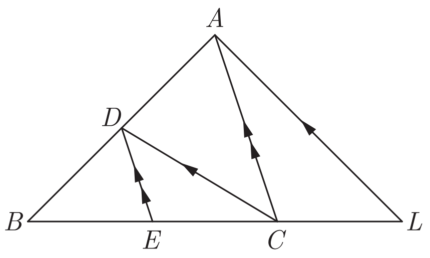
$$AB^2 = AD^2 + BD^2 \quad ...(2)$$

Subtracting eqn. (1) from eqn. (2)

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$AB^2 + CD^2 = AC^2 + BD^2. \quad \text{Hence proved.}$$

28. In the given figure, $CD || LA$ and $DE || AC$. Find the length of CL, if $BE = 4$ cm and $EC = 2$ cm.



Ans : [Board Term-1, 2012, Set-39]

In ΔABC,, $DE || AC$, $BE = 4$ cm and $EC = 2$cm

By BPT $\dfrac{BD}{DA} = \dfrac{BE}{EC}$ ...(1)

In ΔABL, $DC || AL$

By BPT $\dfrac{BD}{DA} = \dfrac{BC}{CL}$ ...(2)

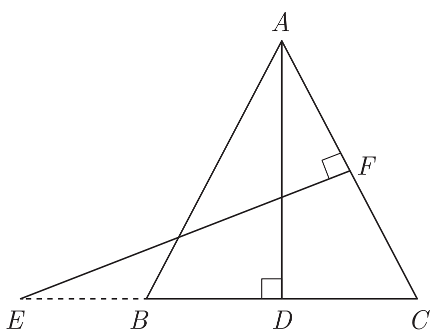
From equations (1) and (2),

$$\frac{BE}{EC} = \frac{BC}{CL}$$

$$\frac{4}{2} = \frac{6}{CL}$$

$$CL = 3 \text{ cm}$$

29. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC. Prove that ΔABD is similar to ΔCEF.



Ans : [Board Term-1, 2012, Set-60]

In ΔABD and ΔCEF, we have

$$AB = AC$$

Thus $\angle ABC = \angle ACB$

$$\angle ABD = \angle ECF$$

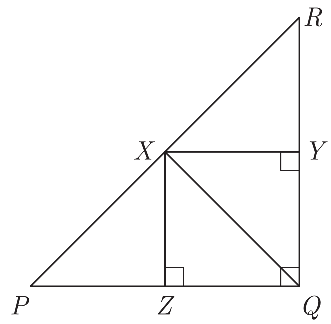
$$\angle ADB = \angle EFC \quad \text{(each } 90^\circ)$$

Due to AA similarity

$$\Delta ABD \sim \Delta ECF \quad \text{Hence proved}$$

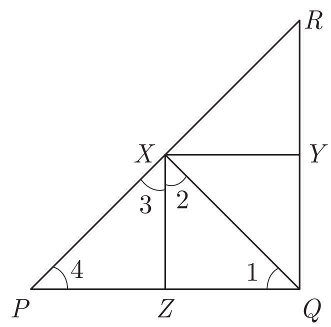
LONG ANSWER TYPE QUESTIONS

1. ΔPQR is right angled at Q. $Q.QX \perp PR, XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.



Ans : [Board Term-1, 2015, Set-MV98HN3]

We have redrawn the given figure as below.



It may be easily seen that $RQ \perp PQ$

and $XZ \perp PQ$ or $XZ || YQ$

Similarly $XY || ZQ$

Thus $XYQZ$ is a rectangle.

In ΔXZQ, $\angle 1 + \angle 2 = 90^\circ$ ...(1)

and in ΔPZX, $\angle 3 + \angle 4 = 90^\circ$ ...(2)

$XQ \perp PR$ or, $\angle 2 + \angle 3 = 90^\circ$ ...(3)

From eq. (1) and (3), $\angle 1 = \angle 3$

From eq. (2) and (3), $\angle 2 = \angle 4$

Due to AA similarity

$$\Delta PZX \sim \Delta XZQ$$

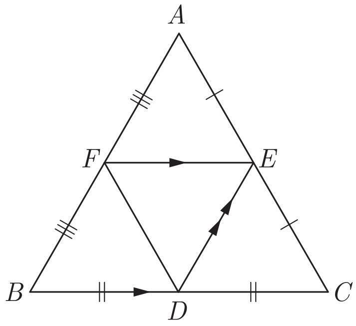
$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

$$XZ^2 = PZ \times ZQ \quad \text{Hence proved}$$

2. In ΔABC, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of $ar(\Delta DEF)$ to $ar(\Delta ABC)$

Ans : [Board Term-1, 2015, Set-DDE-M]

As per given condition we have drawn the figure below. Here F, E and D are the mid-points of AB, AC and BC respectively.



Hence, $FE || BC, DE || AB$ and $DF || AC$
By mid-point theorem,

If $\quad DE \parallel BA$ then $DE \parallel BF$

and if $\quad FE \parallel BC$ then $FE \parallel BD$

Therefore $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of Parallelogram divides it into two equal Areas.

Hence $\quad ar(\Delta BDF) = ar(\Delta DEF) \quad ...(1)$

Similarly $ar(\Delta CDE) = ar(\Delta DEF) \quad ...(2)$

$(\Delta AFE) = ar(\Delta DEF) \quad ...(3)$

$(\Delta DEF) = ar(\Delta DEF) \quad ...(4)$

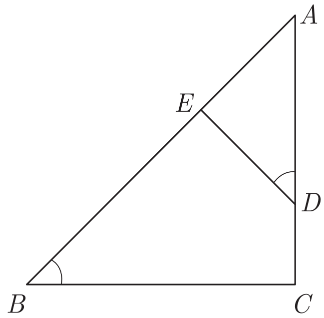
Adding equation (1), (2), (3) and (4), we have

$$ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF) = 4ar(\Delta DEF)$$

$$ar(\Delta ABC) = 4ar(\Delta DEF)$$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

3. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$.
Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE.



Ans : [Board Term-1, 2015, WJQZQBN]

In ΔADE and ΔABC, $\angle A$ is common

and $\quad \angle ADE = \angle ABC \quad$ (Given)

Due to AA similarity

$$\Delta ADE \sim \Delta ABC$$

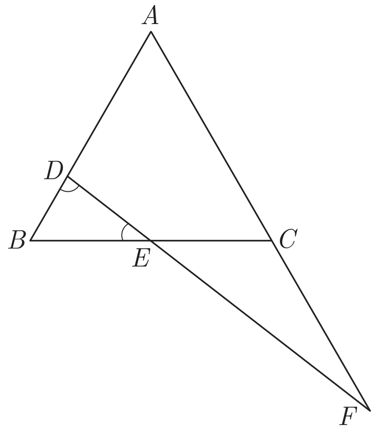
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{AD}{AE + BE} = \frac{DE}{BC}$$

$$\frac{7.6}{4.2 + 4.2} = \frac{DE}{8.4}$$

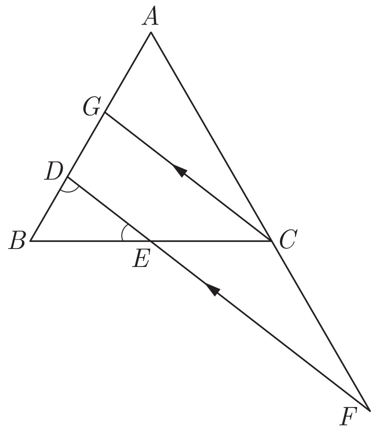
$$DE = \frac{7.6 \times 8.4}{11.4} = 5.6 \text{ cm}$$

4. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



Ans :

We have redrawn the given figure as below. Here $CG || FD$.



We have $\quad \angle BED = \angle BDE$

Since E is mid-point of BC,

or, $\quad BE = BD = EC \quad ...(1)$

In ΔBCG, $\quad DE \parallel FG$

$$\frac{BD}{DG} = \frac{BE}{EC} = 1 \quad \text{(from (1))}$$

$$BD = DG = EC = BE \quad \text{[using (1)]}$$

In ΔADF, $\quad CG \parallel FD$

$$\frac{AG}{GD} = \frac{AC}{CF} \quad \text{(By BPT)}$$

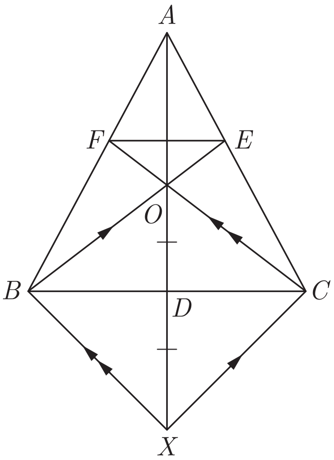
$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

, $$\frac{AD}{GD} = \frac{AF}{CF}$$

Thus $$\frac{AF}{CF} = \frac{AD}{BE} \quad \text{(using (1))}$$

5. In $\Delta ABC, AD$ is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such

that $OD = DX$ as shown in figure.
Prove that :
(1) $EF || BC$
(2) $AO : AX = AF : AB$



Ans : [board Term-1, 2015, Set-O4YP6G7]

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE || XC$ and $BX || CF$.
In ΔABX, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \quad ..(1)$$

In ΔAXC, $$\frac{AE}{EC} = \frac{AO}{OX} \quad ...(2)$$

From (1) and (2) we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

By converse of BPT we have

$$EF || BC$$

From (1) we get $\frac{OX}{OA} = \frac{FB}{AF}$

$$\frac{OX + OA}{OA} = \frac{FB + AF}{AF}$$

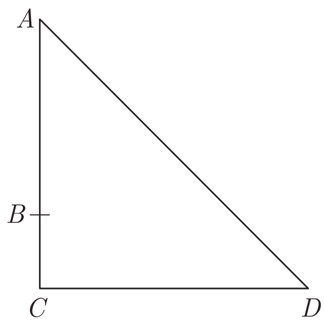
$$\frac{AX}{OA} = \frac{AB}{AF}$$

$$\frac{AO}{AX} = \frac{AF}{AB}$$

Thus $AO : AX = AF : AB$ Hence Proved

6. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x, BC = h$ and $CD = d$, then find x (in term of h and d).

Ans : [Board Term-1, 2015, Set-FHN8MGD]



We have $AB + AD = BC + CD$

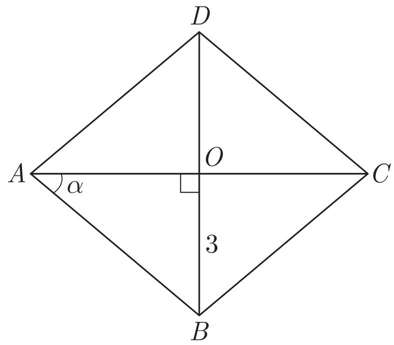
$$AD = BC + CD - AB$$
$$AD = h + d - x$$

In right angled triangle ΔACD,

$$AD^2 = AC^2 + DC^2$$
$$(h + d - x)^2 = (x + h)^2 + d^2$$
$$(h + d - x)^2 - (x + h)^2 = d^2$$
$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$
$$(d - 2x)(2h + d) = d^2$$
$$2hd + d^2 - 4hx - 2xd = d^2$$
$$2hd = 4hx + 2xd$$
$$= 2(2h + d)x$$

or, $$x = \frac{hd}{2h + d}$$

7. $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD.



Ans : [Board Term-1, 2013, Set FFC]

We have $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm

In ΔAOB, $\cos\alpha = \frac{2}{3} = \frac{AO}{AB}$

Let $OA = 2x$ then $AB = 3x$

Now in right angled triangle ΔAOB we have

$$AB^2 = AO^2 + OB^2$$
$$(3x)^2 = (2x)^2 + (3)^2$$
$$9x^2 = 4x^2 + 9$$
$$5x^2 = 9$$
$$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

Hence, $$OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}} \text{ cm}$$

and $$AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}} \text{ cm}$$

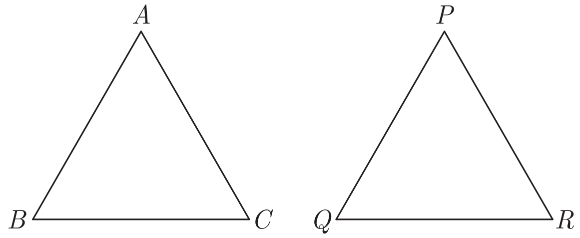
Diagonal $$BD = 2 \times OB = 2 \times 3 = 6 \text{ cm}$$

and $$AC = 2AO$$
$$= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}} \text{ cm}$$

8. If the area of two similar triangles are equal, prove that they are congruent.

Ans : [Board Term-1, 2012, Set-35]

As per given condition we have drawn the figure below.



We have $\Delta ABC \sim \Delta PQR$,

and $ar \Delta ABC = ar \Delta PQR$

Since $\Delta ABC \sim \Delta PQR$, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \qquad ...(1)$$

Since $ar(\Delta ABC) = ar(\Delta PQR)$ we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$$

From equation (1), we get

$$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} = 1$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1$$

$$AB = PQ,$$

$$BC = QR$$

and $CA = RA$

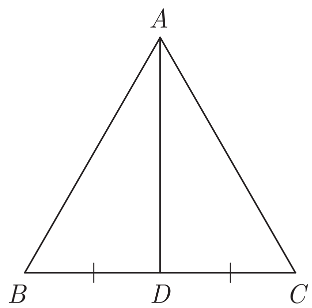
By SSS similarity we have

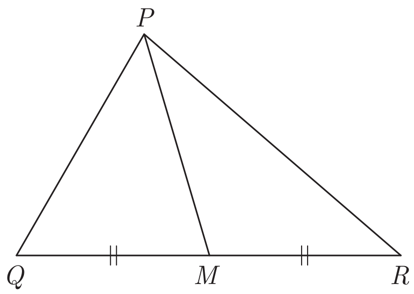
$$\Delta ABC \cong \Delta PQR$$

9. In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.
Prove that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1, 2013 FFC; 2012, SEt-48]

As per given condition we have drawn the figure below.





In ΔABC AD is the median, therefore

$$BC = 2BD$$

and in ΔPQR, PM is the median,

$$QR = 2QM$$

Given, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In ΔABC and ΔPQR,

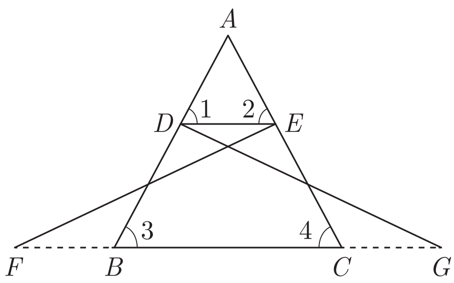
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus $\Delta ABC \sim \Delta PQR$. Hence Proved.

10. In the following figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \cong \Delta ABC$.



Ans : [Board Term-1, 2012, Set-21]

Since $\Delta FEC \cong \Delta GBD$

$$EC = BD \qquad ...(1)$$

Since $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \qquad ...(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$DE \parallel BC$, (Converse of BPT)

Due to corresponding angles we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 = \angle 4$$

Thus in ΔADE and ΔABC,

$$\angle A = \angle A$$

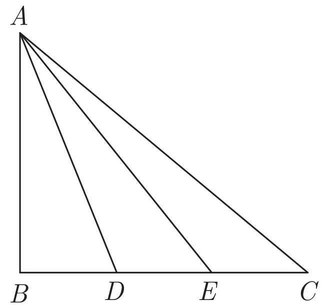
$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4$$

Sy by AAA criterion of similarity,

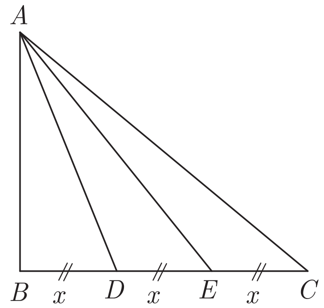
$\Delta ADE \sim \Delta ABC$ Hence proved

11. In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Ans : [Board Term-1, 2013, LK-59]

As per given condition we have drawn the figure below.



Since D and E trisect BC, let $BD = DE = EC$ be x.

Then $BE = 2x$ and $BC = 3x$

In ΔABE, $AE^2 = AB^2 + BE^2 = AB^2 + 4x^2$ ...(1)

In ΔABC, $AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$ ...(2)

In ΔADB, $AD^2 = AB^2 + BD^2 = AB^2 + x^2$ ...(3)

Multiplying (2) by 3 and (3) by 5 and adding we have

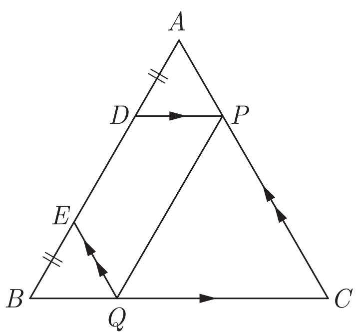
$$3AC^2 + 5AD^2 = 3(AB^2 + 9x^2) + (AB^2 + x^2)$$
$$= 3AB^2 + 27x^2 + 5AB^2 + 5x^2$$
$$= 8AB^2 + 32x^2$$
$$= 8(AB^2 + 4x^2) = 8AE^2$$

Thus $3AC^2 + 5AD^2 = 8AE^2$ Hence Proved

12. Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP \,||\, BC$ and $EQ \,||\, AC$, then prove that $PQ \,||\, AB$.

Ans : [Board Term-1, 2012, Set-44]

As per given condition we have drawn the figure below.



In ΔABC, $DP \,||\, BC$

By BPT we have $\frac{AD}{DB} = \frac{AP}{PC}$, ...(1)

Similarly, in ΔABC, $EQ \,||\, AC$

$$\frac{BQ}{QC} = \frac{BE}{EA} \quad ...(2)$$

From figure, $EA = AD + DE$

$= BE + ED$ $(\ BE = AD)$

$= BD$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \quad ...(3)$$

From (1) and (3), we get

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

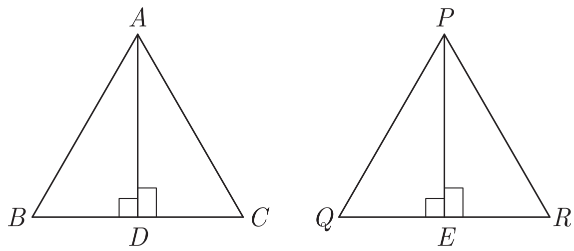
By converse of BPT,

$PQ \,||\, AB$ Hence Proved

13. Prove that ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Ans : [Board Term-1, 2015, FHN8MGD, Sample Paper 2017]

As per given condition we have drawn the figure below. Here $\Delta ABC \sim \Delta PQR$



We have drawn $AD \perp BC$ and $PE \perp QR$

Since $\Delta ABC \sim \Delta PQR$, due to corresponding sides of similar triangles

$$\frac{AB}{PQ} \ \frac{AC}{QR} = \frac{AC}{PR} \quad ...(1)$$

$$\angle B = \angle Q$$

In ΔADB and ΔPEQ,

$\angle B = \angle Q$ (Proved)

$\angle ADB = \angle PEQ$ [each 90°]

$\Delta ADB \sim \Delta PEQ$ (AA Similarity)

Corresponding sides of similar triangle,

$$\frac{AD}{PE} = \frac{AB}{PQ} \quad ...(2)$$

From eq. (1) and eq. (2),

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PE} \quad ...(3)$$

Now, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PE}$

$$= \left(\frac{BC}{QR}\right) \times \left(\frac{AD}{PE}\right)$$

$$= \frac{BC}{QR} \times \frac{BC}{QR}$$

From equation (3) we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} \qquad ...(4)$$

From equation (3) and equation (4) we have

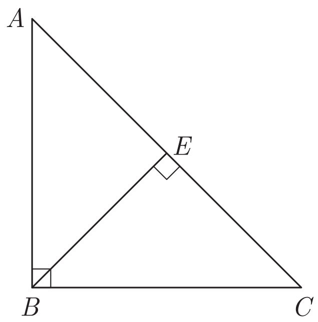
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

14. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus $ABCD$, $4AB^2 = AC^2 + BD^2$.

Ans : [Board Term-1, 2015 CJTOQ, [Sample Paper 2017]

(1) As per given condition we have drawn the figure below. Here $AB \perp BC$.

We have drawn $BE \perp AC$



In ΔAEB and ΔABC $\angle A$ common and

$$\angle E = \angle B \qquad \text{(each } 90^\circ)$$

By AA similarity we have

$$\Delta AEB \sim \Delta ABC$$

$$\frac{AE}{AB} = \frac{AB}{AC}$$

$$AB^2 = AE \times AC$$

Now, In ΔCEB and ΔCBA, $\angle C$ common and

$$\angle E = \angle B \qquad \text{(each } 90^\circ)$$

By AA similarity we have

$$\Delta AEB \sim \Delta CBA$$

$$\frac{CE}{BC} = \frac{BC}{AC}$$

$$BC^2 = CE \times AC \qquad ...(2)$$

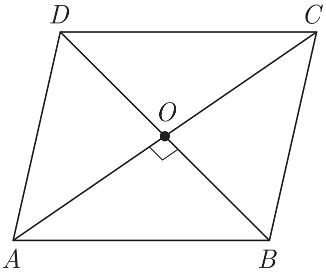
Adding eqns. (1) and (2) we have

$$AB^2 + BC^2 = AE \times AC + CE \times AC$$

$$AB^2 + BC^2 = AC \times AC$$

$$AB^2 + BC^2 = AC^2 \qquad \text{Hence proved}$$

(2) As per given condition we have drawn the figure below. Here $ABCD$ is a rhombus.



We have drawn diagonal AC and BD.

$$AO = OC = \frac{1}{2}AC$$

and

$$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

Since diagonal of rhombus bisect each other at right angle,

$$\angle AOB = 90^\circ$$

$$AB^2 = OA^2 + OB^2$$

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$= \frac{AC^2}{4} + \frac{BD^2}{4}$$

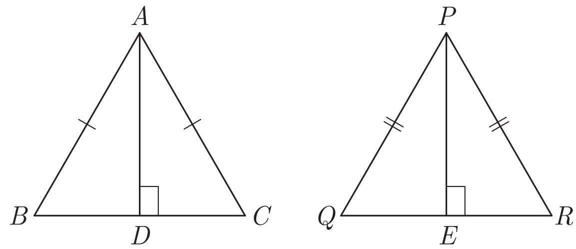
or

$$4AB^2 = AC^2 + BD^2 \qquad \text{Hence proved}$$

15. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio $16:25$, then find the ratio of their altitudes drawn from vertex to the opposite side.

Ans : [Board Term-1, 2015, Set CJTOQ]

As per given condition we have drawn the figure below.



Here

$$\angle A = \angle P$$

$$\angle B = \angle C, \angle Q = \angle R$$

Let $\angle A = \angle P$ be x.

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

$$x^2 + \angle B + \angle B = 180^\circ \qquad (\angle B = \angle C)$$

$$2\angle B = 180^\circ - x$$

$$\angle B = \frac{180^\circ - x}{2} \qquad ...(1)$$

Now, in ΔPQR

$$\angle P + \angle Q + \angle R = 180^\circ \qquad (\angle Q = \angle R)$$

$$x^2 + \angle Q + \angle Q = 180^\circ$$

$$2\angle Q = 180^\circ - x$$

$$\angle Q = \frac{180^\circ - x}{2} \qquad ...(2)$$

In ΔABC and ΔPQR,

$$\angle A = \angle P \quad \text{[Given]}$$

$$\angle B = \angle Q \quad \text{[From eq. (1) and (2)]}$$

Due to AA similarity,

$$\Delta ABC \sim \Delta PQR$$

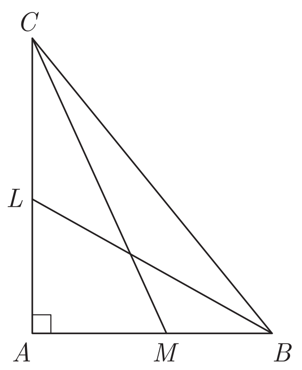
Now $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PE^2}$$

$$\frac{16}{25} = \frac{AD^2}{PE^2}$$

$$\frac{4}{5} = \frac{AD}{PE}$$

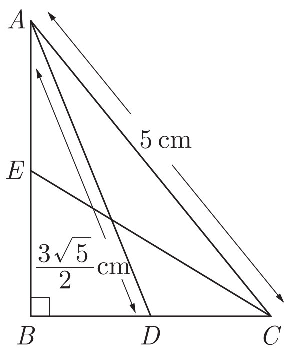
$$\frac{AD}{PE} = \frac{4}{5}$$

16. In the figure, ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



Ans : [Board Term-1, 2013 FFC]

We have redrawn the given figure as below.



Here in ΔABC, $\angle B = 90°$, AD and CE are two medians.

By Pythagoras theorem we get

$$AC^2 = AB^2 + BC^2 = (5)^2 = 25 \quad ...(1)$$

In ΔABD, $$AD^2 = AB^2 + BD^2$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\frac{45}{4} = AB^2 + \frac{BC^2}{4} \quad ...(2)$$

In ΔEBC, $$CE^2 = BC^2 + \frac{AB^2}{4} \quad ..(3)$$

Subtracting equation (2) from equation (1),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

$$BC^2 = \frac{55}{3} \quad ...(4)$$

From equation (2) we have

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

$$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$

From equation (3) we get

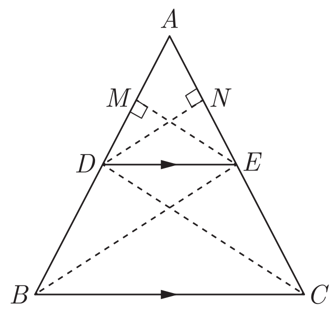
$$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4} = \frac{240}{12} = 20$$

Thus $$CE = \sqrt{20} = 2\sqrt{5} \text{ cm.}$$

17. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Ans : [Board Term-1, 2012 FFC, 2012 Set 15]

A triangle ABC is given in which $DE || BC$. We have drawn $DN \perp AE$ and $EM \perp AD$ as shown below. We have joined BE and CD.



In ΔADE,

$$\text{area }(\Delta ADE) = \frac{1}{2} \times AE \times DN \quad ...(1)$$

In ΔDEC,

$$\text{area }(\Delta DCE) = \frac{1}{2} \times CE \times DN \quad ...(2)$$

Dividing eqn. (1) by eqn. (2),

$$\frac{area(\Delta ADE)}{area(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

or, $$\frac{area(\Delta ADE)}{area(\Delta DEC)} = \frac{AE}{CE} \quad ...(3)$$

Now in ΔADE,

$$area(\Delta ADE) = \frac{1}{2} \times AD \times EM \quad ...(4)$$

and in ΔDEB,

$$area(\Delta DEB) = \frac{1}{2} \times EM \times BD \quad ...(5)$$

Dividing eqn. (4) by eqn. (5),

$$\frac{area(\Delta ADE)}{area(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

or, $$\frac{area(\Delta ADE)}{area(\Delta DEB)} \frac{AD}{BD} \quad ...(6)$$

Since ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$$\text{area }(\Delta DEB) = area(\Delta DEC)$$

From equation (3) we have

$$\frac{area(\Delta ADE)}{area(\Delta DEB)} = \frac{AE}{CE} \quad ...(7)$$

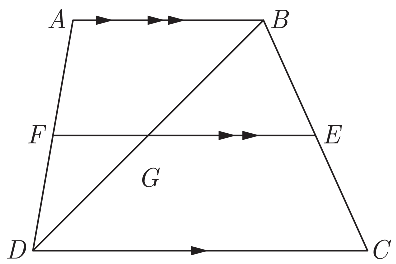
From equations (6) and (7) we get

$$\frac{AE}{CE} = \frac{AD}{BD}. \quad \text{Hence proved.}$$

18. In a trapezium $ABCD, AB || DC$ and $DC = 2AB$. $EF = AB$, where E and F lies on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$ diagonal DB intersects EF at G. Prove that, $7EF = 11AB$.

Ans : [Board Term-1, 2012, Set-65]

As per given condition we have drawn the figure below.



In trapezium $ABCD$,

$$AB || DC \text{ and } DC = 2AB.$$

Also, $$\frac{BE}{EC} = \frac{4}{3}$$

In trapezium $ABCD$,

$$EF || AB || CD$$

$$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In ΔBGE and ΔBDC, $\angle B$ is common and

Due to corresponding angles,

$$\angle BEG = \angle BCD$$

Due to AA similarity we get

$$\Delta BGE \sim \Delta BDC$$

, $$\frac{EG}{CD} = \frac{BE}{BC} \quad ...(1)$$

As, $$\frac{BE}{EC} = \frac{4}{3}$$

$$\frac{BE}{BE+EC} = \frac{4}{4+3} = \frac{4}{7}$$

$$\frac{BE}{BC} = \frac{4}{7} \quad ...(2)$$

From (1) and (2) we have

$$\frac{EG}{CD} = \frac{4}{7}$$

, $$EG = \frac{4}{7}CD \quad ...(3)$$

Similarly, $$\Delta DGF \sim \Delta DBA$$

$$\frac{DF}{DA} = \frac{FG}{AB}$$

$$\frac{FG}{AB} = \frac{3}{7}$$

$$FG = \frac{3}{7}AB \quad ...(4)$$

$$\left[\frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA}\right]$$

Adding equation (3) and (4) we have

$$EG + FG = \frac{4}{7}DC + \frac{3}{7}AB$$

oe $$EF = \frac{4}{7} \times (2AB) + \frac{3}{7}AB$$

$$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{77}{7}AB$$

$$7EF = 11AB. \quad \text{Hence proved.}$$

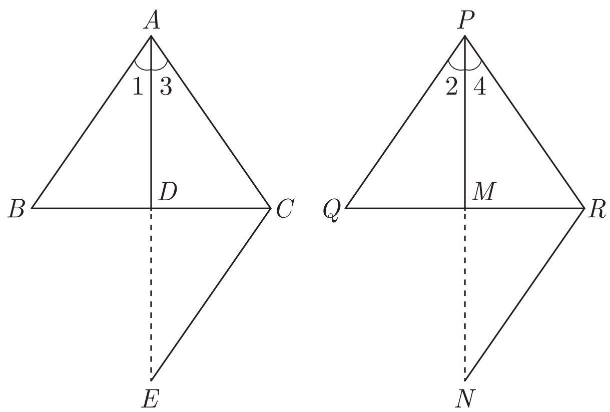
19. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1, 2012, Set-62]

It is given that in ΔABC and ΔPQR, AD and PM are their medians,

such that $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. Join CE As per given condition we have drawn the figure below. and RN.



In ΔABD and ΔEDC,

$$AD = DE \quad \text{(By construction)}$$

$$\angle ADB = \angle EDC \quad \text{(VOA)}$$

$$BD = DC \quad (AD \text{ is a median})$$

By SAS congruency

$$\Delta ABD \cong \Delta EDC$$

$$AB = CE \quad \text{(By CPCT)}$$

Similarly, $$PQ = RN \text{ and } \angle A = \angle 2$$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PE} \quad \text{(Given)}$$

, $$\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\Delta AEC \sim \Delta PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

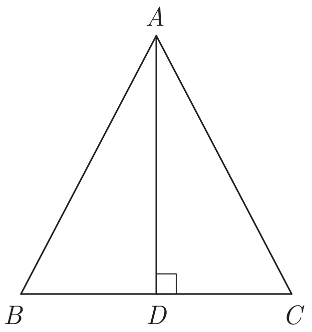
By SSS similarity, we have

$$\Delta ABC \sim \Delta PQR$$ Hence Proved

20. In $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$. Prove that $5AB^2 = 5AC^2 + BC^2$.

Ans : [Board Term-1, 2015 Set DDE-E]

It is given in a triangle $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$.
As per given condition we have drawn the figure below.



Since $2DB = 3CD$

$$\frac{DB}{CD} = \frac{3}{2}$$

Let DB be $3x$, then CD will be $2x$ so $BC = 5x$

Since $CD = 90°$ in ΔADB, we have

$$AB^2 = AD^2 + DB^2 = AD^2 + (3x)^2$$

$$= AD^2 + 9x^2$$

or, $5AB^2 = 5AD^2 + 45x^2$

$$5AD^2 = 5AB^2 - 45x^2 \quad ...(1)$$

and $AC^2 = AD^2 + CD^2 = AD^2 + (2x)^2$

$$= AD^2 + 4x^2$$

or, $5AC^2 = 5AD^2 + 20x^2$

$$5AD^2 = 5AC^2 - 20x^2 \quad ...(2)$$

Comparing eq. (1) and eq. (2) we have

$$5AB^2 - 45x^2 = 5AC^2 - 20x^2$$

$$5AB^2 = 5AC^2 - 20x^2 + 45x^2$$

$$= 5AC^2 + 25x^2$$

$$= 5AC^2 + (5x)^2$$

$$= 5AC^2 + BC^2 \quad [BC = 5x]$$

Therefore $5AB^2 = 5AC^2 + BC^2$ Hence proved

HOTS QUESTIONS

1. In a right triangle ABC, right angled at C. P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio $2:1$.

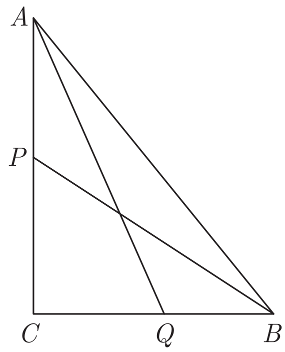
Prove that : $9AQ^2 = 9AC^2 + 4BC^2$

$$9BP^2 = 9BC^2 + 4AC^2$$

$$9(AQ^2 + BP^2) = 13AB^2$$

Ans :

As per given condition we have drawn the figure below.



Since P divides AC in the ratio $2:1$

$$CP = \frac{2}{3}AC$$

$$QC = \frac{2}{3}BC$$

$$AQ^2 = QC^2 + AC^2$$

$$= \frac{4}{9}BC^2 + AC^2$$

or, $9AQ^2 = 4BC^2 + 9AC^2 \quad ...(1)$

Similarly, we get

$$9BP^2 = BC^2 + 4A^2 \quad ...(2)$$

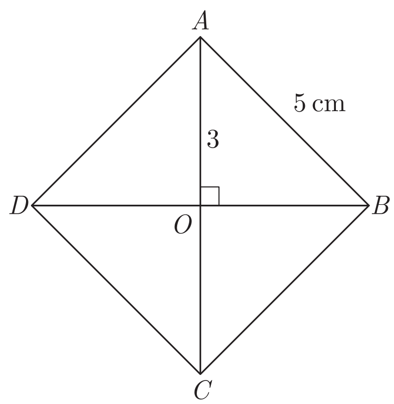
Adding equation (1) and (2), we get

$$9(AQ^2 + BP^2) = 13AB^2$$

2. Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Ans :

As per given condition we have drawn the figure below.



We have $AB = BC = CD = AD = 5$ cm and $AC = 6$ cm

Since $AO = OC$, $AO = 3$ cm

Here ΔAOB is right angled triangle as diagonals of rhombus intersect at right angle.

By Pythagoras theorem,

$$OB = 4 \text{ cm.}$$

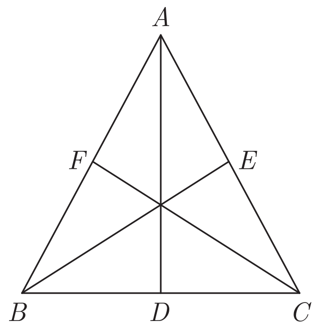
Since $DO = OB, BD = 8$ cm, length of the other diagonal $= 2(BO)$ where $BO = 4$ cm

Hence $BD = 2 \times BO = 2 \times 4 = 8$ cm

3. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans :

As per given condition we have drawn the figure below.



In triangle sum of squares of any two sides is equal to twice the square of half of the third side, together with twice the square of median bisecting it.
If AD is the median,

$$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$$

$$2(AB^2 + AC^2) = 4AD^2 + BC^2 \qquad ...(1)$$

Similarly by taking BE and CF as medians,

$$2(AB^2 + AC^2) = 4BE^2 + AC^2 \qquad ...(2)$$

and $\quad 2(AC^2 + BC^2) = 4CF^2 + AB^2 \qquad ...(3)$

Adding, (1), (2) and (iii), we get

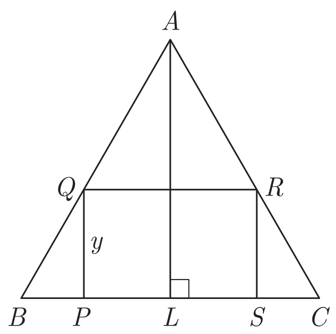
$$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$$

Hence proved

4. ABC is an isosceles triangle in which $AB = AC = 10cm$. $BC = 12cm$. $PQRS$ is a rectangle inside the isosceles triangle. Given $PQ = SR = ycm, PS = PR = 2x$. Prove that $x = 6 - \frac{3y}{4}$.

Ans :

As per given condition we have drawn the figure below.



Here we have drawn $AL \perp BC$.
Since it is isosceles triangle, AL is median of BC,

$$BL = LC = 6 \text{ cm.}$$

In right ΔALB, by Pythagoras theorem,

$$AL^2 = AB^2 - BL^2$$

$$= 10^2 - 6^2 = 64 = 8^2$$

Thus $AL = 8$ cm.

In ΔBPQ and ΔBLA,

$$\angle B = \angle C \qquad \text{(Isosceles triangle)}$$

$$\angle BPQ = \angle BLA = 90^\circ$$

Thus by AA similarity we get

$$\Delta BPQ \sim \angle BLA$$

$$\frac{PB}{PQ} = \frac{BL}{AL}$$

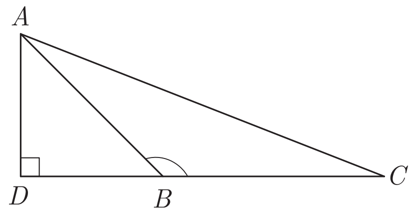
$$\frac{6-x}{y} = \frac{6}{8}$$

$$x = 6 - \frac{3y}{4} \qquad \text{Hence proved.}$$

5. If ΔABC is an obtuse angled triangle, obtuse angled at B and if $AD \perp CB$. Prove that :
$AC^2 \; AB^2 + BC^2 + 2BC \times BD$

Ans :

As per given condition we have drawn the figure below.



In ΔADB, By Pythagoras theorem

$$AB^2 = AD^2 + BD^2 \qquad ...(1)$$

In ΔADC, By Pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$

$$= AD^2 + (BC + BD)^2$$

$$= AD^2 + BC^2 + 2BC \times BD + BD^2$$

$$= (AD^2 + BD^2) + 2BC \times BD$$

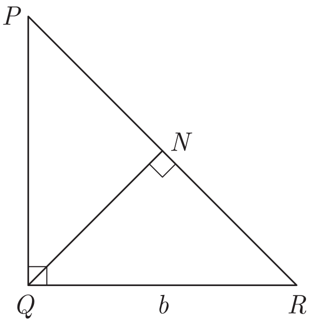
Substituting $(AD^2 + BD^2) = AB^2$ we have

$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

6. If A be the area of a right triangle and b be one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

Ans :

As per given condition we have drawn the figure below.



Let $QR = b$, then we have

$$A = ar(\Delta PQR)$$
$$= \frac{1}{2} \times b \times PQ$$

or, $$PQ = \frac{2 \cdot A}{b} \quad ...(1)$$

Due to AA similarity we have

$$\Delta PNQ \sim \Delta PQR$$
$$\frac{PQ}{PR} = \frac{NQ}{QR} \quad ...(2)$$

From ΔPQR

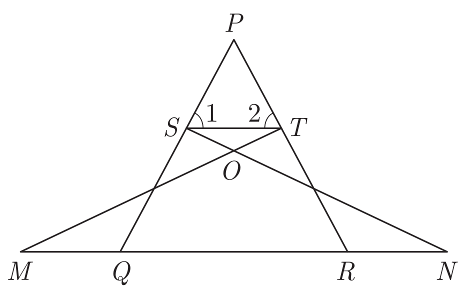
$$PQ^2 + QR^2 = PR^2$$
$$\frac{4A^2}{b^2} + b^2 = PR^2$$
$$PR = \sqrt{\frac{4A^2 + b^4}{b^2}} = \frac{\sqrt{4A^2 + b^4}}{b}$$

Equation (2) becomes

$$\frac{2A}{b \times PR} = \frac{NQ}{b}$$
$$NQ = \frac{2A}{PR}$$

Altitude $$NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}}$$ Hence Proved.

7. In given figure $\angle 1 = \angle 2$ and $\Delta NSQ \sim \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRO$.



Ans : [Sample Question Paper 2017]

Given, $\Delta NSQ \cong \Delta MTR$

By CPCT we have

$$\angle SQN = \angle TRM$$

From angle sum property we have

$$\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$$
$$\angle 1 + \angle 2 = \angle PQR + \angle PRQ$$

Since $\because \angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$2\angle 1 = 2\angle PQR$$
$$\angle 1 = \angle PQR$$

Also $\angle 2 = \angle QPR$ common

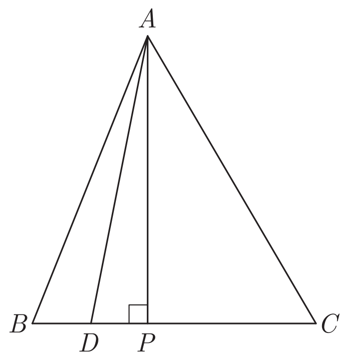
Thus by AAA Similarity

$$\Delta PTS \sim \Delta PRQ$$

8. In an equilateral triangle ABC, D is a point on the side BC such the $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Ans : [Sample Question Paper 2017]

As per given condition we have drawn the figure below. Here we have drawn $AP \perp BC$



Here $AB = BC = CA$ and $BD = \frac{1}{3}BC$.

In ΔADP,

$$AD^2 = AP^2 + DP^2$$
$$= AP^2 + (BP - BD)^2$$
$$= AP^2 + BP^2 + BD^2 + 2BP \cdot BD$$

From ΔAPB using $AP^2 + BP^2 = AB^2$ we have

$$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$$
$$= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} = \frac{7}{9}AB^2$$
$$9AD^2 = 7AB^2$$ Hence Proved