# CHAPTER 5

# Arithmetic Progression

## TOPIC 1 : To find $n^{th}$ Term of the Arithmetic Progression

### VERY SHORT ANSWER TYPE QUESTIONS

**1.** Is $-150$ a term of the A.P. 11, 8, 5, 2, .......?

**Ans :** [CBSE S.A2 2016 Set-HODM40L]

Let the first term of an A.P. be $a$ and common difference be $d$.

We have $a = 11,\ d = -3,\ a_n = -150$

Now
$$\begin{aligned} a_n &= a+(n-1)d \\ -150 &= 11+(n-1)(-3) \\ -150 &= 11-3n+3 \\ 3n &= 164 \end{aligned}$$

or,
$$n = \frac{164}{3} = 54.66$$

Since, 54.66 is not a whole number, $-150$ is not a term of the given A.P.



**2.** Which of the term of $A.P. 5, 2, -1, ......$ is $-49$?

**Ans :** [CBSE Marking Scheme, 2012]

Let the first term of an A.P. be $a$ and common difference $d$.

We have $a = 5,\ d = -3$

Now
$$a_n = a+(n-1)d$$

Substituting all values we have
$$\begin{aligned} -49 &= 5+(n-1)(-3) \\ -49 &= 5-3n+3 \\ 3n &= 49+5+3 \\ n &= \frac{57}{3} = 19^{th} \text{ term.} \end{aligned}$$

**3.** Find the first four terms of an A.P. Whose first term is $-2$ and common difference is $-2$.

**Ans :** [Board Term-2, 2012 Set (17)]

We have
$$\begin{aligned} a_1 &= -2, \\ a_2 = a_1+d &= -2+(-2) = -4 \\ a_3 = a_1+d &= -4+(-2) = -6 \\ a_4 = a_3+d &\ -6+(-2) = -8 \end{aligned}$$

Hence first four terms are $-2, -4, -6, -8$

**4.** Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, ....$

**Ans :** [Board Sample paper, 2016]

Let the first term of an A.P. be $a$ and common difference be $d$.

Given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}$ or $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2}...$

where, $a = \sqrt{2}, d = \sqrt{2}, n = 10$

Now
$$\begin{aligned} a_n &= a+(n-1)d \\ a_{10} &= \sqrt{2}+(10-1)\sqrt{2} \\ &= \sqrt{2}+9\sqrt{2} \\ &= 10\sqrt{2} \end{aligned}$$

Therefore tenth term of the given sequence $\sqrt{200}$.

**5.** Find the next term of the series $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ....

**Ans :** [Board Term-2, 2012 Set (22)]

Let the first term of an A.P. be $a$ and common difference $d$.

Here,
$$\begin{aligned} a &= \sqrt{2},\ a+d = \sqrt{8} = 2\sqrt{2} \\ d &= 2\sqrt{2}-\sqrt{2} = \sqrt{2} \\ \text{Next term} &= \sqrt{32}+\sqrt{2} \\ &= 4\sqrt{2}+\sqrt{2} \\ &= 5\sqrt{2} \\ &= \sqrt{50} \end{aligned}$$

**6.** Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ....$ an A.P.? Give reason.

**Ans :** [Board Term-2, 2015]

Let common difference be $d$ then we have
$$\begin{aligned} d = a_2-a_1 &= \sqrt{6}-\sqrt{3} = \sqrt{3}(\sqrt{2}-1) \\ d = a_3-a_2 &= \sqrt{9}-\sqrt{6} = 3-\sqrt{6} \\ d = a_4-a_3 &= \sqrt{12}-\sqrt{9} = 2\sqrt{3}-3 \end{aligned}$$

As common difference are not equal, the given series is not in A.P.

**7.** What is the next term of an A.P. $\sqrt{7},\ \sqrt{28},\ \sqrt{63}, ....$?

**Ans :** [Foreign Set I, II, III, 2014]

Let the first term of an A.P. be $a$ and common difference be $d$.

Here, $a = \sqrt{7},\ a+d = \sqrt{28}$

$$\begin{aligned} d &= \sqrt{28} - \sqrt{7} = 2\sqrt{7} - \sqrt{7} \\ &= 7 \\ \text{Next term } &= \sqrt{63} + \sqrt{7} \\ &= 3\sqrt{7} + \sqrt{7} = 4\sqrt{7} \\ &= \sqrt{7 \times 16} \\ &= \sqrt{112} \end{aligned}$$

**8.** If the common difference of an A.P. is $-6$, find $a_{16} - a_{12}$.

**Ans :** [KVS 2014]

Let the first term of an A.P. be $a$ and common difference be $d$.

Now $\quad d = -6$

$$\begin{aligned} a_{16} &= a + (16-1)(-6) = a - 90 \\ a_{12} &= a + (12-1)(-6) = a - 66 \\ a_{16} - a_{12} &= (a-90) - (a-66) = a - 90 - n + 66 \\ &= -24 \end{aligned}$$

**9.** For what value of $k$ will the consecutive terms $2k+1$, $3k+3$ and $5k-1$ from an A.P.?

**Ans :** [Foreign Set I, II, III, 2016]

If $x, y$ and $z$ are in AP the we have

$$y - x = z - y$$

Thus if $2k+1$, $3k+3$, $5k-1$ are in A.P. then

$$\begin{aligned} (5k-1) - 3k + 3 &= (3k+3) - (2k+1) \\ 5k - 1 - 3k - 3 &\ 3k + 3 - 2k - 1 \\ 2k - 4 &= k + 2 \\ 2k - k &= 4 + 2 \\ k &= 6 \end{aligned}$$

**10.** Find the $25^{th}$ term of the A.P. $-5, \frac{-5}{2}, \frac{5}{2}, .....$

**Ans :** [Foreign Set I, II, III, 2015]

Let the first term of an A.P. be $a$ and common difference be $d$.

Here, $\quad a = -5, d = -\frac{5}{2} - (-5) = \frac{5}{2}$

$$\begin{aligned} a_n &= a + (n-1)d \\ a_{25} &= 5 + (25-1) \times \left(\frac{5}{2}\right) \\ &= -5 + 60 \\ &= 55 \end{aligned}$$

**11.** The first three terms of an A.P. are $3y-1$, $3y+5$ and $5y+1$ respectively then find $y$.

**Ans :** [Delhi CBSE Term-2, 2015]

If $x, y$ and $z$ are in AP then we have

$$y - x = z - y$$

Therefore if $3y-1, 3y+5$ and $5y+1$ in A.P.

$$\begin{aligned} (3y+5) - (3y-1) &= (5y+1) - (3y+5) \\ 3y + 5 - 3y + 1 &= 5y + 1 - 3y - 5 \\ 6 &= 2y - 4 \\ 2y &= 6 + 4 \\ y &= \frac{10}{2} = 5 \end{aligned}$$

**12.** For what value of $k$ will $k+9$, $2k-1$ and $2k+7$ are the consecutive terms of an A.P.

**Ans :** [Outsidde Delhi Set II, 2016]

If $x, y$ and $z$ are consecutive terms of an A.P. then we have

$$y - x = z - y$$

Thus if $k+9, 2k-1$, and $2k+7$ are consecutive terms of an A.P. then we have

$$\begin{aligned} (2k-1) - (k+9) &= (2k+7) - (2k-1) \\ 2k - 1 - k - 9 &= 2k + 7 - 2k + 1 \\ k - 10 &= 8 \\ k &= 10 + 8 = 18 \end{aligned}$$

**13.** What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?

**Ans :** 2016

Let the first term of an A.P. be $a$ and common difference be $d$.

$$\begin{aligned} a_{21} - a_7 &= 84 \\ a + (21-1)d - [a + (7-1)d] &= 84 \\ a + 20d - a - 6d &= 84 \\ 14d &= 84 \\ d &= 6 \end{aligned}$$

**14.** In the A.P. $2, x, 26$ find the value of $x$.

**Ans :** [Board Term-2, 2012(13)]

If $x, y$ and $z$ are in AP then we have

$$y - x = z - y$$

Since $2, x$ and 26 are in A.P. we have

$$\begin{aligned} x - 2 &= 26 - x \\ 2x &= 26 + 2 \\ x &= \frac{28}{2} = 14 \end{aligned}$$

**15.** For what value of $k; k+2$, $4k-6$, $3k-2$ are three consecutive terms of an A.P.

**Ans :** [Board, Term-2, Delhi 2014], [Board Term-2, 2012 Set (1)]

If $x, y$ and $z$ are three consecutive terms of an A.P. then we have

$$y - x = z - y$$

Since $k+2$, $4k-6$ and $3k-2$ are three consecutive terms of an AP, we obtain

$$\begin{aligned} (4k-6) - (k+2) &= (3k-2) - (4k-6) \\ 4k - 6 - k - 2 &= 3k - 2 - 4k + 6 \\ 3k - 8 &= -k + 4 \\ 4k &= 4 + 8 \\ k &= \frac{12}{4} = 3 \end{aligned}$$

**16.** If $18, a, b, -3$ are in AP, then find $a+b$.

**Ans :** [Board Term-2, 2012 Set (34)]

If $18, a, b, -3$ are in AP, then,

$$\begin{aligned} a - 18 &= -3 - b \\ a + b &= -3 + 18 \end{aligned}$$

$$a + b = 15$$

**17.** Find the common difference of the A.P. $\frac{1}{3q}$, $\frac{1-6q}{3q}$, $\frac{1-12q}{3q}$, .....

**Ans :** [Board Term-2, Delhi 2013]

Let common difference be $d$ then we have

$$d = \frac{1-6q}{3q} - \frac{1}{3q}$$

$$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

**18.** Find the first four terms of an A.P. whose first term is $3x+y$ and common difference is $x-y$.

**Ans :** [Board Term-2, 2012 Set(25)]

Let the first term of an A.P. be $a$ and common difference be $d$.

Now $a_1 = 3x+y$

$$a_2 = a_1 + d = 3x + y + x - y = 4x$$

$$a_3 = a_2 + d = 4x + x - y = 5x - y$$

$$a_4 = a_3 + d = 5x - y + x - y$$

$$= 6x - 2y$$

So, the four terms are $3x+y$, $4x, 5x-y$ and $6x-2y$.



**19.** Find the $37^{th}$ term of the A.P. $\sqrt{x}$, $3\sqrt{x}$, $5\sqrt{x}$

**Ans :** [Board Term-2, 2012 Set (50)]

Let the $n$th term of an A.P. be $a_n$ and common difference be $d$.

Here, $a_1 = \sqrt{x}$

$$a_2 = 3\sqrt{x}$$

$$d = a_2 - a_1 = 3\sqrt{x} - \sqrt{x} = 2\sqrt{x}$$

$$a_n = a + (n-1)d$$

$$a_{37} = \sqrt{x} + (37-1)2\sqrt{x}$$

$$= \sqrt{x} + 36 \times 2\sqrt{x}$$

$$= 73\sqrt{x}$$

**20.** For an A.P., if $a_{25} - a_{20} = 45$, then find the value of $d$.

**Ans :** [Board Term-2, 2011, Set B1]

Let the first term of an A.P. be $a$ and common difference be $d$.

Now $a_{25} - a_{20} = \{a + (25-1)d\} - \{a + (20-1)d\}$

$$45 = a + 24d - a - 19d$$

$$45 = 5d$$

$$d\ \frac{45}{5} = 9$$

## SHORT ANSWER TYPE QUESTIONS - I

**1.** Find, 100 is a term of the A.P. 25, 28, 31, ...... or not.

**Ans :** [Board Term-2, 2012(12)]

Let the first term of an A.P. be $a$, common difference be $d$ and number of terms be $n$.

Let $a_n = 100$

Here $a = 25, d = 28 - 25 = 31 - 28 = 3$

Now $a_n = a + (n-1)d$,

$$100 = 25 + (n-1) \times 3$$

$$100 - 25 = 75 = (n-1) \times 3$$

$$25 = n - 1$$

$$n = 26$$

Hence, 100 is a term of the given A.P.

**2.** Is 184 a term of the sequence 3, 7, 11, .......?

**Ans :** [Board Term-2, 2012(44)]

Let the first term of an A.P. be $a$, common difference be $d$ and number of terms be $n$.

Let $a_n = 184$

Here, $a = 3, d = 7 - 3 = 11 - 7 = 4$

Now $a_n = a + (n-1)d$,

$$184 = 3 + (n-1)4$$

$$\frac{181}{4} = n - 1$$

$$45.25 = n - 1$$

$$46.25 = n$$

Since 46.25 is not an whole number, thus 184 is not a term of given A.P.

**3.** Find the $7^{th}$ term from the end of A.P. 7, 10, 13, .... 184.

**Ans :** [Delhi Set 2014]
[Board Term-2, 1012 Set (34)]

Let us write A.P. in reverse order i.e., 184, ..... 13, 10, 7

Let the first term of an A.P. be $a$ and common difference be $d$.

Now $d = 7 - 10 = -3$

$$a = 184, n = 7$$

$7^{th}$ term from the end,

$$a_7 = a + 6d$$

$$a_7 = 184 + 6(-3)$$

$$= 184 - 18 = 166.$$

Hence, 166 is the $7^{th}$ term from the end.

**4.** Which term of an A.P. 150, 147, 144, ..... is its first negative term?

**Ans :** [KVS 2014]

Let the first term of an A.P. be $a$, common difference be $d$ and $n$th term be $a_n$.

For first negative term $a_n < 0$

$$a + (n-1)d < 0$$

$$150+(n-1)(-3) < 0$$
$$150-3n+3 < 0$$
$$-3n < -153$$
$$n > 51$$

Therefore, the first negative term is $52^{nd}$ term.

**5.** In a certain A.P. $32^{th}$ term is twice the $12^{th}$ term. Prove that $70^{th}$ term is twice the $31^{st}$ term.

**Ans :** [Board Term-2, 2015, 2012, Set-28]

Let the first term of an A.P. be $a$, common difference be $d$ and $n$th term be $a_n$.

Now we have $\quad a_{32} = 2a_{12}$

$$a+31d = 2(a+11d)$$
$$a+31d = 2a+22d$$
$$a = 9d$$
$$a_{70} = a+69d$$
$$= 9d+69d = 78d$$
$$a_{31} = a+30d$$
$$= 9d+30d = 39d$$
$$a_{70} = 2a_{31}$$

Hence Proved.

**6.** The $8^{th}$ term of an A.P. is zero. Prove that its $38^{th}$ term is triple of its $18^{th}$ term.

**Ans :** [Board Term-2, 2012(28)]

Let the first term of an A.P. be $a$, common difference be $d$ and $n$th term be $a_n$.
We have, $a_8 = 0$ or, $a+7d = 0$ or, $a = -7d$

Now $\quad a_{38} = a+37d$

$$a_{38} = -7d+37d = 30d$$
$$a_{18} = a+17d$$
$$= -7d+17d = 10d$$
$$a_{38} = 30d = 3\times 10d = 3\times a_{18}$$
$$a_{38} = 3a_{18}$$

Hence Proved

**7.** If five times the fifth term of an A.P. is equal to eight times its eighth term, show that its $13^{th}$ term is zero.

**Ans :** [Board Term-2, 2012(13)]

Let the first term of an A.P. be $a$, common difference be $d$ and $n$th term be $a_n$.

Now $\quad 5a_5 = 8a_8$

$$5(a+4d) = 8(a+7d)$$
$$5a+20d = 8a+56d$$
$$3a+36d = 0$$
$$3(a+12d) = 0$$
$$a+12d = 0$$
$$a_{13} = 0$$

Hence Proved

**8.** The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

**Ans :** [Foreign Set II, 2015]

Let the first term be $a$ and common difference be $d$.

$$a+4d = 20 \qquad ...(1)$$
$$a+6d+a+10d = 64$$
$$a+8d = 32 \qquad ...(2)$$

Solving equations (1) and (2), we have

$$d = 3$$

**9.** The ninth term of an A.P. is $-32$ and the sum of its eleventh and thirteenth term is $-94$. Find the common difference of the A.P.

**Ans :** [Foreign Set III, 2015]

Let the first term be $a$ and common difference be $d$.

Now $\quad a+8d = a_9$

$$a+8d = -32 \qquad ...(1)$$

and $\quad a_{11} - a_{13} = -94$

$$a+10d+a+12d = -94$$
$$a+11d = -47 \qquad ...(2)$$

Solving equation (1) and (2), we have

$$d = -5$$

**10.** The seventeenth term of an A.P. exceeds its $10^{th}$ term by 7. Find the common difference.

**Ans :** [Board Term-2, 2015, 14]

Let the first term be $a$ and common difference be $d$.

Now $\quad a_{17} = a_{10}+7$

$$a+16d = a+9d+7$$
$$16d-9d = 7$$
$$7d = 7$$
$$d = 1$$

Thus common difference is 1.

**11.** The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find the common difference.

**Ans :** [Foreign set I, 2015]

Let the first term be $a$ and common difference be $d$.

Now $\quad a_4 = 11$

$$a+3d = 11 \qquad ...(1)$$

and $\quad a_5 + a_7 = 34$

$$a+4d+a+6d = 34$$
$$, \quad 2a+10d = 34$$
$$, \quad a+5d = 17 \qquad ...(2)$$

Solving equations (1) and (2) we have

$$d = 3$$

**12.** Find the middle term of the A.P. 213, 205, 197, .... 37.

**Ans :** [Board Term-2, Delhi 2015 (Set II)]

Let the first term of an A.P. be $a$, common difference be $d$ and number of terms be $m$.
Here, $a = 213, d = 205-213 = -8, a_m = 37$

$$a_m = a+(m-1)d$$
$$37 = 213+(m-1)(-8)$$
$$37-213 = -8(m-1)$$
$$m-1 = \frac{-176}{-8} = 22$$

$$m = 22 + 1 = 23$$

The middle term will be $= \frac{23+1}{2} = 12^{th}$

$$a_{12} = a + (12-1)d$$
$$= 213 + (12-1)(-8)$$
$$= 213 - 88 = 125$$

Middle term will be 125.



**13.** Find the middle term of the A.P. 6, 13, 20, .... 216.

**Ans :** [board Term-2, Delhi 2015 (Set I, III)]

Let the first term of an A.P. be $a$, common difference be $d$ and number of terms be $m$.

Here, $a = 6, a_m = 216, d = 13 - 6 = 7$

$$a_m = a + (m-1)d$$
$$216 = 6 + (m-1)(7)$$
$$216 - 6 = 7(m-1)$$
$$m - 1 = \frac{210}{7} = 30$$
$$m = 30 + 1 = 31$$

The middle term will be $= \frac{31+1}{2} = 16^{th}$

$$a_{16} = a + (16-1)d$$
$$= 6 + (16-1)(7)$$
$$= 6 + 15 \times 7$$
$$= 6 + 105 = 111$$

Middle term will be 111.

**14.** If the $2^{nd}$ term of an A.P. is 8 and the $5^{th}$ term is 17, find its $19^{th}$ term.

**Ans :** [board Term-2, 2016 Set HoDM40L]

Let the first term be $a$ and common difference be $d$.

Now $a_2 = a + d$

$$8 = a + d \quad (1)$$

and $a_5 = a + 4d$

$$17 = a + 4d \quad (2)$$

Solving (1) and (2), we have

$$a = 5, d = 3,$$
$$a_{19} = a + 18d$$
$$= 5 + 54 = 59$$

**15.** If the number $x+3, 2x+1$ and $x-7$ are in A.P. find the value of $x$.

**Ans :** [Board Term-2 2012(5)]

If $x, y$ and $z$ are three consecutive terms of an A.P. then we have

$$y - x = z - y$$
$$(2x+1) - (x+3) = (x-7) - (2x+1)$$
$$2x + 1 - x - 3 = x - 7 - 2x - 1$$
$$x - 2 = -x - 8$$
$$2x = -6$$
$$x = -3$$

**16.** Find the values of $a, b$ and $c$, such that the numbers $a, 10, b, c, 31$ are in A.P.

**Ans :** [Board Term-2, 2012 (21)]

Let the first term be $a$ and common difference be $d$. Since $a, 10, b, c, 31$ are in A.P.

Now $a + d = 10 \quad (1)$

$$a + 4d = a_5$$
$$a + 4d = 31 \quad (2)$$

Solving (1) and (2) we have

$$d = 7 \text{ and } a = 3$$

Now $a = 3, b = 3 + 14 = 17, c = 3 + 21 = 24$

Thus $a = 3, b = 17, c = 24$.

**17.** For A.P. show that $a_p + a_{p+2q} = 2a_{p+q}$.

**Ans :** [Board Term-2, 2012(1)]

Let the first term be $a$ and the common difference be $d$. Let $a_n$ be the $n$th term.

$$a_p = a + (p-1)d$$
$$a_{p+2q} = a + (p+2q-1)d$$
$$a_p + a_{p+2q} = a + (p-1)d + a + (p+2q-1)d$$
$$= a + pd - d + a + pd + 2qd - d$$
$$= 2a + 2pd + 2qd - 2d$$

or $a_p + a_{p+2q} = 2[a + (p+q-1)d]$ ...(1)

But $2a_{p+q} = 2[a + (p+q-1)d]$ ...(2)

From (1) and (2), we get $a_p + a_{p+2q} = 2a_{p+q}$

**18.** The sum of first terms of an A.P. is give by $S_n = 2n^2 + 8n$. Find the sixteenth term of the A.P.

**Ans :** [Sample Question Paper 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

Now $S_n = 2n^2 + 3n$

$$S_1 = 2 \times 1^2 + 3 \times 1 = 2 + 3 = 5$$

Since $S_1 = a_1$,

$$a_1 = 5$$
$$S_2 = 2 \times 2^2 + 3 \times 2 = 8 + 6 = 14$$
$$a_1 + a_2 = 14$$
$$a_2 = 14 - a_1 = 14 - 5 = 9$$
$$d = a_2 - a_1 = 9 - 5 = 4$$
$$a_{16} = a + (16-1)d$$
$$= 5 + 15 \times 4 = 65$$

**19.** The $4^{th}$ term of an A.P. is zero. Prove that the $25^{th}$ term of the A.P. is three times its $11^{th}$ term.

**Ans :** [Outside Delhi Set, II 2016]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have, $a_4 = 0$

$$a + 3d = 0 \quad [a + (n-1)d = a_n]$$
$$3d = -a$$
$$-3d = a \quad ...(1)$$

Now, $a_{25} = a + 24d = -3d + 24d = 21d$ ...(2)

$$a_{11} = a+10d = -3d+10d = 7d \qquad ...(3)$$

From eqn (2) and eq (3) we have

$$a_{25} = 3a_{11} \qquad \text{Hence Proved.}$$



## SHORT ANSWER TYPE QUESTIONS - I

**1.** Find the $20^{th}$ term of an A.P. whose $3^{rd}$ term is 7 and the seventh term exceeds three times the $3^{rd}$ term by 2. Also find its $n^{th}$ term $(a_n)$.

**Ans :** [Board Term-2, 2012 (31)]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have
$$a_3 = a+2d = 7 \qquad (1)$$
$$a_7 = 3a_3+2$$
$$a+6d = 3\times 7+2 = 23 \qquad (2)$$

Solving (1) and (2) we have
$$4d = 16 \Rightarrow d = 4$$
$$a+8 = 7 \Rightarrow a = -1$$
$$a_{20} = a+19d = -1+19\times 4 = 75$$
$$a_1 = a+(n-1)d$$
$$= -1+4n-4$$
$$= 4n-5.$$

Hence $n^{th}$ term is $4n-5$

**2.** If $7^{th}$ term of an A.P. is $\frac{1}{9}$ and $9^{th}$ term is $\frac{1}{7}$, find $63^{rd}$ term.

**Ans :** [Board Term-2, Delhi, 2014]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have
$$a_7 = \frac{1}{9} \Rightarrow a+6d = \frac{1}{9} \qquad (1)$$
$$a_9 = \frac{1}{7} \Rightarrow a+8d = \frac{1}{7} \qquad (2)$$

Subtracting equation (1) from (2) we get
$$2d = \frac{1}{7}-\frac{1}{9} = \frac{2}{63} = \frac{1}{63}$$

Substituting the value of $d$ in (2) we get
$$a+8\times\frac{1}{63} = \frac{1}{7}$$
$$a = \frac{1}{7}-\frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus
$$a_{63} = a+(63-1)d$$
$$= \frac{1}{63}+62\times\frac{1}{63} = \frac{1+62}{63}$$
$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$

**3.** The ninth term of an A.P. is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

**Ans :** [Board Sample Paper, 2016]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

Now
$$a_9 = 7a_2$$
$$a+8d = 7(a+d)$$
$$a+8d = 7a+7d$$
$$-6a+d = 0 \qquad (1)$$

and
$$a_{12} = 5a_3+2$$
$$a+11d = 5(a+2d)+2$$
$$a+11d = 5a+10d+2$$
$$-4a+d = 2 \qquad ...(2)$$

Subtracting (2) from (1), we get
$$-2a = -2$$
$$a = 1$$

Substituting this value of $a$ in (1) we get
$$-6+d = 0$$
$$d = 6$$

Hence first term is 1 and common difference is 6.

**4.** Determine an A.P. whose third term is 9 and when fifth term is subtracted from $8^{th}$ term, we get 6.

**Ans :** [Board Term-2, 2015]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have
$$a_3 = 9$$
$$a+2d = 9 \qquad ...(1)$$

and
$$a_8-a_5 = 6$$
$$(a+7d)-(a+4d) = 6$$
$$3d = 6$$
$$d = 2$$

Substituting this value of $d$ in (1), we get
$$a+2(2) = 9$$
$$a = 5$$

So, A.P. is 5, 7, 9, 11, ...

**5.** Divide 56 in four parts in A.P. such that the ratio of the product of their extremes ($1^{st}$ and $4^{rd}$) to the product of means ($2^{nd}$ and $3^{rd}$) is 5:6.

**Ans :** [Foreign Set I, 2016]

Let the four numbers be $a-3d, a-d, a+d, a+3d$
Now $a-3d+a-d+a+d+a+3d = 56$
$$4a = 56 \Rightarrow a = 14$$

Hence numbers are $14-3d, 14-d, 14+d, 14+3d$

Now, according to question,
$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$
$$\frac{196-9d^2}{196-d^2} = \frac{5}{6}$$
$$6(196-9d^2) = 5(196-d^2)$$
$$6\times 196-54d^2 = 5\times 196-5d^2$$
$$(6-5)\times 196 = 49d^2$$
$$d^2 = \frac{196}{49} = 4$$

$$d = \pm 2$$

Thus numbers are
$$a-3d = 14-3\times 2 = 8$$
$$a-d = 14-2 = 12$$
$$a+d = 14+2 = 16$$
$$a+3d = 14+3\times 2 = 20$$

Thus required AP is $8,12,16,20$.

6. The $p^{th}$, $q^{th}$ and $r^{th}$ terms of an A.P. are $a$, $b$ and $c$ respectively, Show that $a(q-r)+b(r-p)+c(p-q)=0$.

**Ans :** [Foreign Set II, 2016]

Let the first term be $A$ and the common difference be $D$.

$$a = A+(p-1)D$$
$$b = A+(q-1)D$$
$$c = A+(r-1)D$$

Now $$a(q-r) = [A+(p-1)D][q-r]$$
$$b(r-p) = [A+(q-1)D][r-p]$$
and $$c[p-q] = [A+(r-1)D][p-q]$$

$$a(q-r)+b(r-p)+c(p-q)$$
$$= [A+(p-1)D][q-r]+$$
$$+[A+(q-1)D][r-p]+$$
$$+[A+(r-1)D][p-q]+$$
$$= A[p-q+q-p+q-r]+$$
$$+D(p-1)(q-r)+$$
$$+D(q-1)(r-p)+$$
$$+D(r-1)(p-q)$$
$$= A[0]+$$
$$+D[p(q-r)-(q-r)]$$
$$+D[q(r-p)-(r-p)]$$
$$+D[r(p-q)-(p-q)]$$
$$= D[p(q-r)+q(r-p)+r(p-q)]+$$
$$-D[(q-r)+(r-p)+(p-q)]$$
$$= D[pq-pr+qr-qp+rp-rq]+0$$
$$= D[0] = 0$$

7. The sum of $n$ terms of an A.P. is $3n^2+5n$. Find the A.P. Hence find its $15^{th}$ term.

**Ans :** [Board Term-2, 2013], [Board Term-2, 2012 Set (38, 39)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Now $$S_n = 3n^2+5m$$
$$S_{n-1} = 3(n-1)^2+5(n-1)$$
$$= 3(n^2+1-2n)+5n-5$$
$$= 3n^2+3-6n+5n-5$$
$$= 3n^2-n-2$$
$$a_n = S_n - S_{n-1}$$
$$= 3n^2+5n-(3n^2-n-2)$$
$$= 6n+2$$

Thus A.P. is 8, 14, 20, .......

Now $$a_{15} = a+14d = 8+14(6) = 92$$

8. The digit of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less then the original number. Find the number.

**Ans :** [Outside Delhi Set II, 2016]

Let these digit of 3 digit no be $-a-d, a, a+d$
Since their sum is 15,

$$a-d+a+a+d = 15$$
$$3a = 15 \Rightarrow a = 5$$

Required 3 digit no $= 100(a-d)+10a+a+d$
$$= 100a-100d+10a+a+d$$
$$= 111a-99d$$

No obtained by reversing digit
$$= 100(a+d)+100+a-d$$
$$= 100a+100d+10a+a-d$$
$$= 111a+99d$$

According the question,
$$111a+99d = 111a-99d-594$$
$$2\times 99d = 594 \Rightarrow d = -8$$

Thus number is $111a-99d = 111\times 5-99\times -3$
$$= 555+297 = 852$$

9. For what value of $n$, are the $n^{th}$ terms of two A.Ps 63, 65, 67, ... and 3, 10, 17, .... equal?

**Ans :**

Let $a, d$ and $A, D$ be the $1^{st}$ term and common difference of the 2 APs respectively.
$n$ is same

For 1st AP, $a = 63$, $d = 2$

For 2nd AP, $A = 3, D = 7$

Since $n$th term is same,

$$an = An$$
$$a+(n-1)d = A+(n-1)D$$
$$63+(n-1)2 = 3+(n-1)7$$
$$63+2n-2 = 3+7n-7$$
$$61+2n = 7n-4$$
$$65 = 5n \Rightarrow n = 13$$

When $n$ is 13, the $n^{th}$ terms are equal i.e., $a_{13} = A_{13}$

## LONG ANSWER TYPE QUESTIONS

1. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers.

**Ans :** [delhi Set III, 2016]

Let the three numbers in A.P. be $a-d, a, a+d$.

$$a-d+a+a+d = 12$$
$$3a = 12$$
$$a = 4$$

Also, $(4-d)^3+4^3+(4+d)^3 = 288$

$$64-48d+12d^2-d^3+64+64+48d+12d^2+d^3$$
$$= 288$$
$$24d^2+192 = 288$$

$$d^2 = 4$$
$$d = \pm 2$$

The numbers are 2, 4, 6 or 6, 4,2

**2.** Find the value of $a, b$ and $c$ such that the numbers $a, 7, b, 23$ and $c$ are in A.P.

**Ans :** [Board Term-2, 2015]

Let the common difference be $d$.
Since $a, 7, b, 23$ and $c$ are in AP, we have

$$a + d = 7 \quad ..(1)$$
$$a + 3d = 23 \quad ...(2)$$

Form (1) and (2), we get

$$a = -1, d = 8$$
$$b = a + 2d = -1 + 2 \times 8 = -1 + 16 = 15$$
$$c = a + 4d = -1 + 4 \times 8 = -1 + 32 = 31$$

Thus $a = -1, b = 15, c = 31$



## VERY SHORT ANSWER TYPE QUESTIONS

**1.** Find the sum of first ten multiple of 5.

**Ans :** [Board Term-2, Delhi, 2014]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here, $a = 5, n = 10, d = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2 \times 5 + (10-1)5]$$
$$= 5[10 + 9 \times 5]$$
$$= 5[10 + 45]$$
$$= 5 \times 55 = 275$$

Hence the sum of first ten multiple of 5 is 275.

**2.** Find the sum of first five multiples of 2.

**Ans :** [Board Term-2, 2012 st (05)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$the term be $S_n$
Here, $a = 2, d = 2, n = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_5 = \frac{5}{2}[2 \times 2 + (5-1)2]$$
$$= \frac{5}{2}[4 + 4 \times 2] = \frac{5}{2}[4 + 8]$$
$$= \frac{5}{2} \times 12 = 5 \times 6 = 30$$

**3.** Find the sum of first 16 terms of the A.P. 10, 6, 2, .....

**Ans :** [Board Term-2, 2012, Set (32)

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here, $a = 10, d = 6 - 1 = -4, n = 16$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$$
$$= 8[20 + 15 \times (-4)]$$
$$= 8[20 - 60]$$
$$= 8 \times (-40)$$
$$= -320$$

**4.** What is the sum of five positive integer divisible by 6.

**Ans :** [Board Term-2, 2012 Set (23)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$the term be $S_n$
Here, $a = 6, d = 6, n = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_5 = \frac{5}{2}[2 \times 6 + (5-1)(6)]$$
$$= \frac{5}{2}[12 + 4 \times 6]$$
$$= \frac{5}{2}[12 + 24] = \frac{5}{2}[36]$$
$$= 5 \times 18 = 90$$

**5.** If the sum of $n$ terms of an A.P. is $2n^2 + 5n$, then find the $4^{th}$ term.

**Ans :** [Board Term-2, 2012, Set (12)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Now, $S_n = 2n^2 + 5n$

$n^{th}$ term of A.P.

$$a_n = S_n - S_{n-1}$$
$$a_n = (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$$
$$= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$$
$$= 2n^2 + 5n - 2n^2 - n + 3$$
$$= 4n + 3$$

Thus $4^{th}$ term $a_4 = 4 \times 4 + 3 = 19$

**6.** If the sum of first $k$ terms of an A.P. is $3k^2 - k$ and its common difference is 6. What is the first term?

**Ans :** [Board Term-2, 2012, Set (44)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Let the sum of $k$ terms of A.P. is $S_n = 3k^2 - k$

We have $S_k = 3k^2 - k$
Now $k^{th}$ term of A.P.

$$a_k = S_n - S_{n-1}$$
$$a_k = (3k^2 - k) - [3(k-1)^2 - (k-1)]$$
$$= 3k^2 - k - [3k^2 - 6k + 3 - k + 1]$$
$$= 3k^2 - k - 3k^2 + 7k - 4$$
$$= 6k - 4$$

First term $a = 6 \times 1 - 4 = 2$

**7.** Which term of the A.P. $8, 14, 20, 26, ......$ will be 72 more than its $41^{st}$ term.

**Ans :** [Board Outside Delhi Set-II, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have $a = 8, d = 6$.

Since $n^{th}$ term is 72 more than $41^{st}$ term. we get

$$\begin{aligned} a_n &= a_{41} + 72 \\ 8 + (n-1)6 &= 8 + 40 \times 6 + 72 \\ 6n - 6 &= 240 + 72 \\ 6n &= 312 + 6 = 318 \\ n &= 53 \end{aligned}$$

**8.** If the $n^{th}$ term of an A.P. $-1, 4, 9, 14, .....$ is 129. Find the value of $n$.

**Ans :** [Board Outside Delhi Compt. Set I, II, III 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have $a = -1$ and $d = 4 - (-1) = 5$

$$\begin{aligned} -1 + (n-1) \times 5 &= a_n \\ -1 + 5n - 5 &= 129 \\ 5n &= 135 \\ n &= 27 \end{aligned}$$

Hence $27^{th}$ term is 129.

**9.** Write the $n^{th}$ term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, .....$

**Ans :** [Board Outside Delhi Compt. Set-I, II, III 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have $$a = \frac{1}{m}$$

$$d = \frac{1+m}{m} - \frac{1}{m} = 1$$

$$a_n = \frac{1}{m} + (n-1)1$$

Hence , $$a_n = \frac{1}{m} + n - 1$$

**10.** What is the common difference of an A.P. which $a_{21} - a_7 = 84$.

**Ans :** [Board Outside Delhi Set I, II, III, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have
$$\begin{aligned} a_{21} - a_7 &= 84 \\ a + 20d - a - 6d &= 84 \\ 14d &= 84 \\ d &= \frac{84}{14} = 6 \end{aligned}$$

Hence common difference is 6.

**11.** Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4} ....$ is the first negative.

**Ans :** [Board Outside Delhi Set I, II, III 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have $a = 20$ and $d = -\frac{3}{4}$

Let the $n^{th}$ term be first negative term, then

$$\begin{aligned} a + (n-1)d &< 0 \\ 20 + (n-1)\left(-\frac{3}{4}\right) &< 0 \\ 20 - \frac{3}{4}n + \frac{3}{4} &< 0 \\ 3n &> 83 \\ n &> \frac{83}{3} = 27\frac{2}{3} \end{aligned}$$

Hence $28^{th}$ term is first negative.

## SHORT ANSWER TYPE QUESTIONS - I

**1.** How many terms of the A.P. $65, 60, 55, ....$ be taken so that their sum is zero?

**Ans :** [Delhi Set III, 2016]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have $a = 65, d = -5, S_n = 0$

Now $$S_n = \frac{n}{2}[2a + (n-1)d]$$

Let sum of $n$ term be zero, then we have

$$\begin{aligned} \frac{n}{2}[130 + (n-1)(-5)] &= 0 \\ \frac{n}{2}[130 + 5n + 5] &= 0 \\ 135n - 5n^2 &= 0 \\ n(135 - 5n) &= 0 \\ 5n &= 135 \\ n &= 27 \end{aligned}$$

**2.** How many terms of the A.P. $18, 16, 14 .....$ be taken so that their sum is zero?

**Ans :** [Delhi Set I, 2016]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here $a = 18, d = -2, S_n = 0$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Let sum of $n$ term be zero, then we have

$$\begin{aligned} \frac{n}{2}[36 + (n-1)(-2)] &= 0 \\ n(38 - 2n) &= 0 \\ n &= 19 \end{aligned}$$

**3.** How many terms of the A.P. $27, 24, 21 ....$ should be taken so that their sum is zero?

**Ans :** [Delhi Set II, 2016]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here $$a = 27, d = -3, S_n = 0$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Let sum of $n$ term be zero, then we have

$$\frac{n}{2}[54 + (n-1)(-3)] = 0$$

$$n(-3n+57) = 0$$
$$n = 19$$

**4.** In an A.P., if $S_3 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where $S_n$ donotes the sum of first $n$ terms.

**Ans :** [Outside Delhi CBSE Board, Term-2, 2015, Set I, II, III]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_5 + S_7 = 167$$
$$\frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$$
$$24a+62d = 334$$
$$12a+31d = 167 \qquad ...(1)$$
$$S_{10} = 235$$
$$5(2a+9d) = 235$$
$$2a+9d = 47 \qquad (2)$$

Solving (1) and (2), we get

$$a = 1, d = 5$$

Thus AP is 1, 6, 11....

**5.** Find the sum of sixteen terms of an A.P. $-1, -5, -9, ......$

**Ans :** [Board Term-2, 2012 Set (8)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here, $a_1 = -1, a_2 = -5$ and $d = -4$

Now $\qquad S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{16} = \frac{16}{6}[2\times(-1)+(16-1)(-4)]$$
$$= 8[-2-60] = 8(-62)$$
$$= -496$$

**6.** If the $n^{th}$ term of an A.P. is $7-3n$, find the sum of twenty five terms.

**Ans :** [Board Term-2, 2012 Set (16)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here $n = 25, a_n = 7-3n$
Taking $n = 1,2,3,....$ we have

$$a_1 = 7-3\times 1 = 4$$
$$a_2 = 7-3\times 2 = 1$$
$$a_3 = 7-3\times 3 = -2$$

Thus required AP is $4, 1, -2, .....$

Here, $a = 4, d = 1-4 = -3$

Now, $\qquad S_n = \frac{n}{2}[2a+(n-1)d]$

$$= \frac{25}{2}[2\times 4+(25-1)(-3)]$$
$$= \frac{25}{2}[8+24(-3)]$$
$$= \frac{25}{2}(8-72) = -800$$

**7.** If the $1^{st}$ term of a series is 7 and $13^{th}$ term is 35. Find the sum of 13 terms of the swquence.

**Ans :** [Board Term-2, 2012, Set (36)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

Here $a = 7, a_{13} = 35$

$$a_n = a+(n-1)d$$
$$a_{13} = a+12d$$
$$35 = 7+12d \Rightarrow d = \frac{7}{3}$$

Now $\qquad S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{13} = \frac{13}{2}\left[2\times 7+12\times\left(\frac{7}{3}\right)\right]$$
$$= \frac{13}{2}[14+28]$$
$$= \frac{13}{2}\times 42 = 273$$

**8.** If the $n^{th}$ term of a squence is $3-2n$. Find the sum of fifteen terms.

**Ans :** [Board Term-2, 2012 Set (38)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here, $a_n = 3-2n$

Taking $n = 1$, $\qquad a_1 = 3-2 = 1$

15th term, $\qquad a_{15} = 3-2\times 15 = 3-30 = -27$

Now $\qquad S_n = \frac{n}{2}(a+1)$

$$S_{15} = \frac{15}{2}[1+(-27)]$$
$$= \frac{15}{2}[-26]$$
$$= 15\times(-13) = -195$$

**9.** If $S_n$ denotes the sum of $n$ terms of an A.P. whose common difference is $d$ and first term is $a$, find $S_n - 2S_{n-1} + S_{n-2}$.

**Ans :** [Board Term-2, 2011 (A1)]

We have $\qquad a_n = S_n - S_{n-1}$

$$a_{n-1} = S_{n-1} - S_{n-2}$$
$$S_n - 2S_{n-1} + S_{n-2} = S_n - S_{n-1} - S_{n-1} + S_{n-2}$$
$$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$$
$$= a_n - a_{n-1} = d$$

**10.** The sum of first $n$ terms of an A.P. is $5n - n^2$. Find the $n^{th}$ term of the A.P.

**Ans :** [Foreign Set I, II, III, 2014]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

We have, $\qquad S_n = 5n - n^2$

Now, $n^{th}$term of A.P.

$$a_n = S_n - S_{n-1}$$
$$= (5n-n^2) - \left[5(n-1)-(n-1)^2\right]$$

$$= 5n - n^2 - [5n - 5 - (n^2 + 1 - 2n)]$$
$$= 5n - n^2 - (5n - 5 - n^2 - 1 + 2n)$$
$$= 5n - n^2 - n + 6 + n^2$$
$$= - 2n + 6$$
$$a_n = - 2(n - 3)$$

Thus $n^{th}$ term is $= - 2(n - 3)$

**11.** The first and last term of an A.P. are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

**Ans :** [Board Term-2, 2012 Set (19)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.
We have $a = 5, a_n = 45$

Now $45 = 5 + (n - 1)d$

$(n - 1)d = 40$ ...(1)

Given, $S_n = 400$

Now $S_n = \frac{n}{2}(a + l)$

$$400 = \frac{n}{2}(5 + 45)$$
$$800 = 50n$$
$$n = 16$$

Substituting this value of $n$ in (1) we have

$$(n - 1)d = 40$$
$$15d = 40$$
$$d = \frac{40}{15} = \frac{8}{3}$$

**12.** If the sum of the first 7 terms of an A.P. is 49 and that of the first 17 terms is 289, find the sum of its first $n$ terms.

**Ans :** [Board Foreign Set-II, 2012]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Now $S_7 = \frac{7}{2}(2a + 6d) = 49$

$a + 3d = 7$ ...(1)

and $S_{17} = \frac{17}{2}(2a + 16d) = 289$

$$a + 8d = 17$$

Subtracting (1) from (2), we get

$$5d = 10 \Rightarrow d = 2$$

Substituting this value of $d$ in (1) we have

$$a = 1$$

Now $S_n = \frac{n}{2}[2 \times 1(n - 1)2]$

$$= \frac{n}{2}[2 + 2n - 2] = n^2$$

Hence, sum of $n$ terms is $n^2$.

**13.** How many terms of the A.P. $-6, \frac{-11}{2}, -5, -\frac{9}{2}....$ are needed to give their sum zero.

**Ans :** [Board outside Delhi compt. Set-III, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

We have $a = - 6, d = - \frac{11}{2} - (- 6) = \frac{1}{2}$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Let sum of $n$ term be zero, then we have

$$\frac{n}{2}\left[2 \times - 6 + (n - 1)\frac{1}{2}\right] = 0$$
$$\frac{n}{2}\left[- 12 + \frac{n}{2} - \frac{1}{2}\right] = 0$$
$$\frac{n}{2}\left[\frac{n}{2} - \frac{25}{2}\right] = 0$$
$$n^2 - 25n = 0$$
$$n(n - 25) = 0$$
$$n = 25$$

Hence 25 terms are needed.

**14.** Which term of the A.P. $3, 12, 21, 30, .....$ will be 90 more than its $50^{th}$ term.

**Ans :** [Board Compt. Set-III 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

We have $a = 3, d = 9$

Now $a_n = a + (n - 1)d$

$$a_{50} = 3 + 49 \times 9 = 444$$

Now, $a_n - a_{50} = 90$

$$3 + (n - 1)9 - 444 = 90$$
$$(n - 1)9 = 90 + 441$$
$$(n - 1) = \frac{531}{9} = 49$$
$$n = 49 + 1 = 60$$

**15.** The $10^{th}$ term of an A.P. is $-4$and its $22^{nd}$ term is $(-16)$. Find its $38^{th}$ term.

**Ans :** [Board Delhi compt. Set-I, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

$a_{10} = a + 9d = - 4$ (1)

and $a_{22} = a + 21d = - 16$ (2)

Subtracting (2) from (1) we have

$$12d = - 12 \Rightarrow d = - 16$$

Substituting this value of $d$ in (1) we get

$$a = 5$$

Thus $a_{38} = 5 + 37 \times - 1 = - 32$

Hence, $a_{38} = - 32$

**16.** Find how many integers between 200 and 500 are divisible by 8.

**Ans :** [Board Delhi compt. Set-I, II, III, 2017]

Number divisible by 8 are 208, 2016, 224, .... 496.
Which is an A.P.
Let the first term be $a$, common difference be $d$ and

$n$th term be $a_n$.
We have $aa = 208, d = 8$ and $a_n = 496$

Now $\quad a + (n-1)d = a_n$

$$208 + (n-1)d = 496$$
$$(n-1)8 = 496 - 208$$
$$n - 1 = \frac{288}{8} = 36$$
$$n = 36 + 1 = 37$$

Hence, required numbers divisible by 8 is 37.

**17.** The fifth term of an A.P. is 26 and its $10^{th}$ term is 51. Find the A.P.

**Ans :** [Outside Delhi Compt. set-II, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

$$a_5 = a + 4d = 26 \quad ...(1)$$
$$a_{10} = a + 9d = 51 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5d = 25$$
$$d = 5$$

Substituting this value of $d$ in (1) we get

$$a = 6$$

Hence, the AP is 6, 11, 17, ....

**18.** Find the A.P. whose third term is 5 and seventh term is 9.

**Ans :** [Board Outside Delhi Compt. Set-I, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

Now $\quad a_3 = a + 2d = 5 \quad ...(1)$

and $\quad a_7 = a + 6d = 9 \quad ...(2)$

Subtracting (2) from (1) we have

$$4d = 4 \Rightarrow d = 1$$

Substituting this value of $d$ in (1) we get

$$a = 3$$

Hence AP is 3, 4, 5, 6, ......

**19.** Find whether $-150$ is a term of the A.P. $11, 8, 5, 2, ....$

**Ans :** [Board Delhi Compt. Set-I, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.
Let the $n^{th}$ term of given A.P. $11, 8, 5, 2, ....$ be $-150$
Hence $a = 11$, $d = 8 - 11 = -3$ and $a_n = -150$

$$a + (n-1)d = a_n$$
$$11 + (n-1)(-3) = -150$$
$$(n-1)(-3) = -161$$
$$(n-1) = \frac{-161}{-3} = 53\frac{2}{3}$$

which is not a whole number. Hence $-150$ is not a term of given A.P.

**20.** If seven times the $7^{th}$ term of an A.P. is equal to eleven times the $11^{th}$ term, then what will be its $18^{th}$ term.

**Ans :** [Board Foreign Set-I, II, III, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

$$7a_7 = 11a_{11}$$

Now $\quad 7(a+6d) = 11(a+10d)$

$$7a + 42d = 11a + 110d$$
$$11a - 7a = 42d - 110d$$
$$, \quad 4a = -68d$$
$$4a + 68d = 0$$
$$4(a + 17d) = 0$$
$$a + 17d = 0$$

Hence, $\quad a_{18} = 0$



**21.** In an A.P. of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

**Ans :** [Board Foreign SET-III, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

$$S_{10} = 210$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\frac{10}{2}(2a + 9d) = 42$$
$$10a + 45d = 42 \quad (1)$$
$$S_{50} = \frac{50}{2}[2a + (50-1)d]$$
$$S_{35} = \frac{35}{2}[2a + (35-1)d]$$
$$a_{36} = a + 35d$$
$$a_{50} = a + 49d$$

Sum of last 15 terms

$$= \frac{n}{2}(a_{36} + a_{50})$$
$$2565 = \frac{15}{2}(a + 35d + a + 49d)$$
$$171 = \frac{1}{2}(2a + 84d)$$
$$a + 42d = 171 \quad (2)$$

Solving (1) and (2) we get

$$a = 3 \text{ and } d = 4$$

Hence, AP is 3, 7, 11, .....

## SHORT ANSWER TYPE QUESTIONS - II

**1.** In an A.P. the sum of first $n$ terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find the $25^{th}$ term.

**Ans :** [Board Sample Paper, 2016]

We have $\quad S_n = \frac{3n^2 + 13n}{2}$

$$a_n = S_n - S_{n-1}$$

$$a_{25} = S_{25} - S_{24}$$
$$= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2}$$
$$= \frac{1}{2}\{3(25^2 - 24^2) + 13(25 - 24)\}$$
$$= \frac{1}{3}(3 \times 49 + 13) = 80$$

**2.** The sum of first $n$ terms of three arithmetic progressions are $S_1, S_2$ and $S_3$ respectively. The first term of each A.P. is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

**Ans :** [O.D. Set III, 2016]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

We have $S_1 = 1 + 2 + 3 + ..... n$

$S_2 = 1 + 3 + 5 + .....$ up to $n$ terms

$S_3 = 1 + 4 + 7 + .....$ upto $n$ terms

Now $S_1 = \frac{n(n+1)}{2}$

$$S_2 = \frac{n}{2}[2 \times 1 + (n-1)2] = \frac{n}{2}[2n] = n^2$$

and $S_3 = \frac{n}{2}[2 \times 1 + (n-1)3] = \frac{n(3n-1)}{2}$

Now, $S_1 + S_3 = \frac{n(n+1)}{2} + \frac{n(3n-1)}{2}$

$$= \frac{n[n+1+3n-1]}{2}$$
$$= \frac{n[4n]}{2}$$
$$= 2n^2 = 2s_2$$

Hence Proved

**3.** If $S_n$ denotes, the sum of the first $n$ terms of an A.P. prove that $S_{12} = 3(S_8 - S_4)$.

**Ans :** [Delhi CBSE Board, 2015, Set I]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = 6[2a + 11d] = 12a + 66d$$
$$S_8 = 4[2a + 7d] = 8a + 28d$$
$$S_4 = 2[2a + 3d] = 4a + 6d$$
$$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$$
$$= 3[4a + 22d] = 12a + 66d$$
$$= 6[2a + 11d] = S_{12}$$

Hence Proved

**4.** The $14^{th}$ term of an A.P. is twice its $8^{th}$ term. If the $6^{th}$ term is $-8$, then find the sum of its first 20 terms.

**Ans :** [Outside Delhi CBSE Board, 2015, Set I]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

Here, $a_{14} = 2a_8$ and $a_6 = -8$

Now $a + 13d = 2(a + 7d)$

$$a + 13d = 2a + 14d$$

$a = -d$ ...(1)

and $a_6 = -8$

$a + 5d = -8$ ...(2)

Solving (1) and (2), we get

$$a = 2, d = -2$$

Now $S_{20} = \frac{20}{2}[2 \times 2 + (20-1)(-2)]$

$$= 10[4 + 19 \times (-2)]$$
$$= 10(4 - 38)$$
$$= 10 \times (-34) = -340$$

**5.** If the ratio of the sums of first $n$ terms of two A.P.'s is $(7n+1):(4n+27)$, find the ratio of their $m^{th}$ terms.

**Ans :** [O.D. Set I, 2016]

Let $a$, and $A$ be the first term and $d$ and $D$ be the common difference of two AP's, then we have

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} = \frac{7n+1}{4n+27}$$
$$= \frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7n+1}{4n+27}$$

$$\frac{a + (\frac{n-1}{2})d}{A + (\frac{n-1}{2})D} = \frac{7n+1}{4n+27}$$

Putting $\frac{n-1}{2} = m - 1$ or $n = 2m - 1$ we get

$$\frac{a + (m-1)d}{A + (m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$

**6.** If the sum of the first $n$ terms of an A.P. is $\frac{1}{2}[3n^2 + 7n]$, then find its $n^{th}$ term. Hence write its $20^{th}$ term.

**Ans :** [Delhi CBSE Board Term-2, 2015, set II]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

Sum of $n$ term $S_n = \frac{1}{2}[3n^2 + 7n]$

Sum of 1 term $S_1 = \frac{1}{2}[3 \times (1)^2 + 7(1)]$

$$= \frac{1}{2}[3 + 7] = \frac{1}{2} \times 10 = 5$$

Sum of 2 term $S_2 = \frac{1}{2}[3(2)^2 + 7 \times 2]$

$$= \frac{1}{2}[12 + 14] = \frac{1}{2} \times 26 = 13$$

Now $a_1 = S_1 = 5$

$$a_2 = S_2 - S_1 = 13 - 5 = 8$$
$$d = a_2 - a_1 = 8 - 5 = 3$$

Now, A.P. is $5, 8, 11, ....$

$n^{th}$ term, $a_n = a + (n-1)d$

$$= 5 + (n-1)3$$
$$= 5 + (20-1)(3)$$

$$= 5 + 57$$
$$= 62$$

Hence, $a_2 = 62$

**7.** In an A.P., if the $12^{th}$ term is $-13$ and the sum of its first four terms is 24, find the sum of its first ten terms.

**Ans :** [Foreign Set I, II, 2015]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

$$a_{12} = a + 11d = -13 \quad ...(1)$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $S_4 = 2[2a + 3d] = 24$

$$2a + 3d = 12 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$(2a + 22d) - (2a + 3d) = -26 - 12$$
$$19d = -38$$
$$d = -2$$

Substituting the value of $d$ in (1) we get

$$a + 11 \times -2 = -13$$
$$a = -13 + 22$$
$$a = 9$$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{10} = \frac{10}{2}(2 \times 9 + 9 \times -2)$$
$$= 5 \times (18 - 18) = 0$$

Hence, $S_{10} = 0$

**8.** The tenth term of an A.P., is $-37$ and the sum of its first six terms is $-27$. Find the sum of its first eight terms.

**Ans :** [Foreign Set III, 2015]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

$$a_n = a + (n-1)d$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$a + 9d = -37 \quad ...(1)$$
$$3(2a + 5d) = -27$$
$$2a + 5d = -9 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$(2a + 18d) - (2a + 5d) = -74 + 9$$
$$13d = -65$$
$$d = -5$$

Substituting the value of $d$ in (1) we get

$$a + 9 \times -5 = -37$$
$$a = -37 + 45$$
$$a = 8$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{8}{2}[2 \times 8 + (8-1)(-5)]$$
$$= 4[16 - 35]$$
$$= 4 \times -19 = -76$$

Hence, $S_n = -76$

**9.** Find the sum of first seventeen terms of A.P. whose $4^{th}$ and $9^{th}$ terms are $-15$ and $-30$ respectively.

**Ans :** [Board Term-2, 2014]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

Now $a_4 = a + 3d = -15 \quad ...(1)$

$$a_9 = a + 8d = -30 \quad ...(2)$$

Subtracting eqn (1) from eqn (2), we obtain

$$(a + 8d) - (a + 3d) = -30 - (-15)$$
$$5d = -15 \Rightarrow d = \frac{-15}{5} = -3$$

Substituting the value of $d$ in (1) we get

$$a + 3d = -15$$
$$a + 3(-3) = -15$$
$$a = -15 + 9 = -6$$

Now $S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$

$$= \frac{17}{2}[-\,-12 + 16 \times (-3)]$$
$$= \frac{17}{2}[-12 - 48]$$
$$= \frac{17}{2}[-60] = 17 \times (-30)$$
$$= -510$$

Thus $S_{17} = -510$

**10.** The common difference of an A.P. is $-2$. Find its sum, if first term is 100 and last term is $-10$.

**Ans :** [Board Term-2, 2014]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

We have $a = 100, d = -2, t_n = -10$

Now $a_n = a + (n-1)d$

$$-10 = 100 + (n-1)(-2)$$
$$-10 = 100 - 2n + 2$$
$$2n = 112$$
$$n = 56$$

Thus $56^{th}$ term is $-10$ and number of terms in A.P. are 56.

Now $S_n = \frac{n}{2}(a + 1)$

$$S_{56} = \frac{56}{2}(100 - 10)$$
$$= \frac{56}{2}(90) = 56 \times 45 = 2520$$

Thus $S_n = 2520$

**11.** The $16^{th}$ term of an A.P. is finve times its third term. If its $10^{th}$ term is 41, then find the sum of its first fifteen terms.

**Ans :** [Outside Delhi CBSE, 2015, Set II]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

We have, $a_{16} = 5a_3$

$$a + 15d = 5(a + 2d)$$

$$4a = 5d \quad ...(1)$$

and $$a_{10} = 41$$

$$a + 9d = 41 \quad ...(2)$$

Solving (1) and (2), we get

$$a = 5, d = 4$$

Now $$S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 4]$$

$$= \frac{15}{2}[10 + 56]$$

$$= \frac{15}{2} \times 66 = 15 \times 33 = 495$$

Thus $S_{15} = 495$

**12.** The $13^{th}$ term of an A.P. is four times its $3^{rd}$ term. If the fifth term is 16, then find the sum of its first ten terms.

**Ans :** [Outside Delhi, 2015 Set III]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

Here $a_{13} = 4a_3$

$$a + 12d = 4(a + 2d)$$

$$3a = 4d \quad ...(1)$$

and $$a_5 = 16$$

$$a + 4d = 16 \quad ...(2)$$

Substituting the value of $a = \frac{4}{3}d$ in (2)

$$\frac{4}{3}d + 4d = 16$$

$$16d = 48 \Rightarrow d = 3$$

Thus $a = 4$ and $d = 3$

Now $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2 \times 4 + (10-1)3]$$

$$= 5[8 + 27] = 5 \times 35 = 175$$

Thus $S_{10} = 175$

**13.** The $n^{th}$ term of an A.P. is given by $(-4n + 15)$. Find the sum of first 20 terms of this A.P.

**Ans :** [Board Term-2, 2013]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

We have $$a_n = -4n + 15$$

$$a_1 = -4 \times 1 + 15 = 11$$

$$a_2 = -4 \times 2 + 15 = 7$$

$$a_3 = -4 \times 3 + 15 = 3$$

$$d = a_2 - a_1 = 7 - 11 = -4$$

Now, we have $a = 11$, $d = -4$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2 \times 11 + (20-1) \times (-4)]$$

$$= 10[22 - 76]$$

$$= 10 \times (-54) = -540$$

Thus $S_{20} = -540$

**14.** The sum of first 7 terms of an A.P. is 63 and sum of its next 7 terms is 161. Find $28^{th}$ term of A.P.

**Ans :** [Foreign Set I, II, III, 2014]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now, $$S_7 = 63$$

$$\frac{7}{2}[2a + 6d] = 63$$

$$2a + 6d = 18 \quad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$

$$\frac{14}{2}[2a + 13d] = 224$$

$$2a + 13d = 32 \quad (2)$$

Subtracting (1) form (2)

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of $d$ in (1) we get

$$a = 3$$

Now $$a_n = a + (n-1)d$$

$$a_{28} = 3 + 2 \times (27)$$

$$= 57$$

Thus $28^{th}$ term is 57.

**15.** The sum of first $n$ terms of an A.P. is given by $S_n = 3n^2 - 4n$. Determine the A.P. and the $12^{th}$ term.

**Ans :** [Delhi CBSE Term-2, 2014] [Board Term-2, 2012 set (13)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

$$S_n = 3n^2 - 4n$$

$$S_1 = 3(1)^2 - 4(1) = -1$$

$$S_2 = 3(2)^2 - 4(2) = 4$$

$$a_1 = S_1 = -1$$

$$a_2 = S_2 - S_1 = 4 - (-1) = 5$$

$$d = a_2 - a_1 = 5 - (-1) = 6$$

Thus AP is $-1, 5, 11, ....$

Now $$a_{12} = a + 11d$$

$$= -1 + 11 \times 6 = 65$$

**16.** Find the sum of all two digit natural numbers which are divisible by 4.

**Ans :** [Delhi Compt. Set-III, 2017]

First two digit multiple of 4 is 12 and last is 96

So, $a=12, d=4$. Let $n^{th}$ term be last term $a_n=96$

$$\begin{aligned} \text{Now} \quad a+(n-1)d &= a_n \\ 12+(n-1)4 &= 96 \\ (n-1)4 &= 96-12=84 \\ n-1 &= 21 \\ n &= 21+1=22 \\ \text{Now,} \quad S_{22} &= \frac{22}{2}[12+96] \\ &= 11\times 108 \\ &= 1188 \end{aligned}$$

**17.** Find the sum of the following series.

$5+(-41)+9+(-39)+13+(-37)+17+....+(-5)+81+(-3)$

**Ans :** [Board foreign set-I, 2017]

The series can be written as

$(5+9+13+....+81)+(-41)+(-39)+(-37)+(-35)...(-5)+(-3)$

For the series $(5+9+13.....81)$

$$\begin{aligned} a &= 5 \\ d &= 4 \\ \text{and} \quad a_n &= 81 \\ \text{Now} \quad a_n &= 5+(n-1)4=81 \\ 81 &= 5+(n-1)4 \\ (n-1)4 &= 76 \\ n &= 20 \\ S_n &= \frac{20}{2}(5+81)=860 \end{aligned}$$

For series $(-41)+(-39)+(-37)+...+(-5)+(-3)$

$$\begin{aligned} a_n &= -3 \\ a &= -41 \\ d &= 2 \\ a_n &= -41+(n-1)(2) \\ -3 &= -41+2n-2 \Rightarrow n=20 \\ \text{Now} \quad S_n &= \frac{20}{2}[-41+-3]=-440 \end{aligned}$$

Sum of the series $=860-440=420$

**18.** Find the sum of $n$ terms of the series

$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+.......$

**Ans :** [CBSE Board Delhi Set-I, II, III, 2017]

Let sum of n term be $S_n$

$s_n=\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+.......$ up to $n$term

$=(4+4+4+.....$ up to $n$ terms$)+$

$+\left(-\frac{1}{n}-\frac{2}{n}-\frac{3}{n}-......\right.$ up to $n$ terms $\Big)$

$=(4+4+4+....$ up to $n$ terms$)+$

$-\frac{1}{n}(1+2+3+.....$ up to $n$ terms$)$

$$= 4n-\frac{1}{n}\times\frac{n(n+1)}{2}$$

$$= 4n-\frac{n+1}{2}=\frac{7n-1}{2}$$

Hence, sum of $n$ terms $=\frac{7n-1}{2}$

**19.** Find the number of multiple of 9 lying between 300 and 700.

**Ans :** [Outside Delhi Compt. Set-II, 2017]

The numbers, multiple of 9 between 300 and 700 are 306, 315, 324, .... 693.

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n=693$

$$\begin{aligned} a_n &= 306+(n-1)9 \\ 693 &= 306+(n-1)9 \\ (n-1)9 &= 693-306=387 \\ n-1 & \frac{387}{9}=43 \\ n &= 43+4=44 \end{aligned}$$

Hence there are 44 terms.

**20.** If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10 find it $20^{th}$ term.

**Ans :** [Board Outside Delhi Compt. Set-III, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

We have $a=10$, and $S_{14}=1050$

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ S_{14} &= \frac{14}{2}[2\times 10+(14-1)d] \\ 1050 &= 7[20+13d] \\ 20+13d &= \frac{1050}{7}=150 \\ 13d &= 130 \Rightarrow d=10 \\ a_{20} &= a+(n-1)d \\ &= 10+19\times 10=200 \end{aligned}$$

Hence $a_{20}=200$

**21.** If the tenth term of an A.P. is 52 and the $17^{th}$ term is 20 more than the $13^{th}$ term, find A.P.

**Ans :** [Board Outside Delhi Set-I, 2017]

Let the first term be $a$, common difference be $d$ and $n$th term be $a_n$.

$$\begin{aligned} \text{Now} \quad a_{10} &= 52 \\ a+9d &= 52 \quad ...(1) \\ \text{Also} \quad a_{17}-a_{13} &= 20 \\ a+16d-(a+12d) &= 20 \\ 4d &= 20 \\ d &= 5 \end{aligned}$$

Substituting this valued $d$ in (1), we get

$$a=7$$

Hence AP is 7, 12, 17, 22, ...

**22.** Find the sum of all odd number between 0 and 50.

**Ans :** [Delhi Compt. Set-III, 2017]

Let the first term be $a$, common difference be $d$, $n$th

term be $a_n$ and sum of $n$ term be $S_n$.
Given AP is $1+3+5+7+.....+49$
Let total number of terms be $n$.

$$a_n = 1+(n-1)\times 2$$
$$49 = 1+2n-2$$
$$50 = 2n \Rightarrow n = 25$$

Now
$$S_{25} = \frac{n}{2}(a+a_n)$$
$$= \frac{25}{2}(1+49)$$
$$= 25\times 25 = 625$$

Hence, Sum of odd number is 625

**23.** Find the sum of first 15 multiples of 8.

**Ans :** [Delhi Compt. Set-I, 2017]

Let the first term be $a=8$, common difference be $d=8$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.
First term of given A.P. Be 8 and common difference be 8. Than

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{15} = \frac{15}{2}[2\times 8+(15-1)8]$$
$$= \frac{15}{2}[16+112]$$
$$= \frac{15}{2}\times 128 = 996$$

Hence, the sum of 15 terms is 960.

**24.** If $m^{th}$ term of an AP is $\frac{1}{n}$ and $n^{th}$ term is $\frac{1}{m}$ find the sum of first $mn$ terms.

**Ans :** [CBSE Board Set-I, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.

Now
$$a_m = a+(m-1)d = \frac{1}{n} \qquad ...(1)$$
$$a_n = a+(n-1)d = \frac{1}{m} \qquad ...(2)$$

Subtracting (2) from (1) we get
$$(m-n)d = \frac{1}{n}-\frac{1}{m} = \frac{m-n}{mn}$$
$$d = \frac{1}{mn}$$

Substituting this valued $d$ in (1), we get
$$a = \frac{1}{mn}$$

Now,
$$S_{mn} = \frac{mn}{2}\left(\frac{2}{mn}+(mn-1)\frac{1}{mn}\right)$$
$$= 1+\frac{mn}{2}-\frac{1}{2} = \frac{1}{2}+\frac{mn}{2}$$
$$= \frac{1}{2}[mn+1]$$

Hence, the sum on $mn$ term is $\frac{1}{2}[mn+1]$.

**25.** How many terms of an A.P. $9, 17, 25, ....$ must be taken to give a sum of 636?

**Ans :**

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.
We have $a=9, d=8, S_n=636$

Now
$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$636 = \frac{n}{2}[18+(n-1)8]$$
$$636 = n[9+(n-1)4]$$
$$636 = n(9+4n-4)$$
$$636 = n(5+4n)$$
$$636 = 5n+4n^2$$
$$4n^2+5n-636 = 0$$
$$4n^2-48n+53n-636 = 0$$
$$4n(n-12)+53(n-12) = 0$$
$$(4n+53)(n-12) = 0$$

Thus
$$n = \frac{-53}{4} \text{ or } 12$$

As $n$ is a natural number $n=12$. Hence 12 terms are required to give sum 636.

## LONG ANSWER TYPE QUESTIONS

**1.** The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months. If the sum of ages of all the participants is 168 years, find the age of eldest participant in the paining competition.

**Ans :** [Board Sample Paper, 2016]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$.
We have $a=8, d=4$ month $=\frac{1}{3}$ years, $S_n=168$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$168 = \frac{n}{2}\left[2(8)+(n-1)\frac{1}{3}\right]$$
$$n^2+47n-1008 = 0$$
$$n^2+63n-16n-1008 = 0$$
$$(n-16)(n+63) = 0$$
$$n = 16 \text{ or } n = -63$$

As $n$ cannot be negative, we take $n=16$

Age of the eldest participant $= a+15d = 13$ years

**2.** A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after, the thief to catch him. He goes with a speed of 100/minute in the first minute and increased his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

**Ans :** [Delhi Set I, II, 2016]

Let total time to catch the thief be $n$ minutes
Total distance covered by thief $=(100n)$

Total distance covered by policeman

$$= 100 + 110 + 120 + .... + (n-1) \text{ terms}$$

$$100n = \frac{n-1}{2}[200 + (n-2)10]$$

$$n^2 - 3n - 18 = 0$$

$$(n-6)(n+3) = 0$$

$$n = 6$$

Policeman takes 5 minutes to catch the thief.

**3.** If $S_n$ denotes the sum of first $n$ terms of an A.P., Prove that, $S_{30} = 3(S_{20} - S_{10})$

**Ans :** [Delhi 2015 Set III, Foreign Set I, II, III, 2014]

Let the first term be $a$, and common difference be $d$.

Now $$S_{30} = \frac{30}{2}(2a + 29d) \quad ...(1)$$

$$= 15(2a + 29d)$$

$$3(S_{20} - S_{10}) = 3[10(2a + 19d) - 5(2a + 9d)]$$

$$= 3[20a + 190d - 10a - 45d]$$

$$= 3[10a + 145d]$$

$$= 15[2a + 29d] \quad ...(2)$$

Hence $$S_{30} = 3(S_{20} - S_{10})$$

**4.** The sum of first 20 terms of an A.P. is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

**Ans :** [Board Term-2, 2015]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We know $$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $$S_{20} = \frac{20}{2}(2a + 19d)$$

$$400 = \frac{20}{2}(2a + 19d)$$

$$400 = 10[2a + 19d]$$

$$2a + 19d = 40 \quad (1)$$

Also, $$S_{40} = \frac{40}{2}(2a + 39d)$$

or, $$1600 = 20[2a + 39d]$$

or, $$2a + 39d = 80 \quad (2)$$

Solving (1) and (2), we get $a = 1$ and $d = 2$.

Now $$S_{10} = \frac{10}{2}[2 \times 1 + (10-1)(2)]$$

$$= 5[2 + 9 \times 2]$$

$$= 5[2 + 18]$$

$$= 5 \times 20 = 100$$

**5.** Find $\left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(10 - \frac{3}{n}\right) + .....$ upto $n$ terms.

**Ans :** [Board Term-2, 2015]

Let sum of n term be $S_n$, then we have

$$s_n = \left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(40 - \frac{3}{n}\right) + ..... \text{ upto } n \text{ terms.}$$

$$= (4 + 7 + 10 + ..... + n\ terms) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} ..... + 1\right)$$

$$= (4 + 7 + 10 + .... + n\ terms) - \frac{1}{n}(1 + 2 + 3 + ...n)$$

$$= \frac{n}{2}[2 \times 4 + (n-1)(3)] - \frac{1}{n} \times \frac{n}{2}[2 \times 1 + (n-1)(1)]$$

$$= \frac{n}{2}[8 + 3n - 3] - \frac{1}{2}[2 + n - 1]$$

$$= \frac{n}{2}(3n + 5) - \frac{1}{2}(n + 1)$$

$$= \frac{3n^2 + 5n - n - 1}{2}$$

$$= \frac{3n^2 + 4n - 1}{2}$$

**6.** Find the $60^{th}$ term of the A.P. $8, 10, 12, ....,$ if it has a total of 60 terms and hence find the sum of its last 10 terms.

**Ans :** [Outside Delhi, CBSE Board, 2015 Set I, II]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have $a = 8, d = 10 - 8 = 2$

$$a_n = a + (n-1)d$$

Now $$a_{60} = 8 + (60-1)2 = 8 + 59 \times 2 = 126$$

and $$a_{51} = 8 + 50 \times 2 = 8 + 100 = 108$$

Sum of last 10 terms,

$$S_{51-60} = \frac{n}{2}(a_{51} + a_{60})$$

$$= \frac{10}{2}(108 + 126)$$

$$= 5 \times 234 = 1170$$

Hence sum of last 10 terms is 1170.

**7.** An arithmetic progression $5, 12, 19, .....$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

**Ans :** [Outside, Delhi CBSE Board, 2015, Set III]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have $a = 5,\ d = 12 - 5 = 7$ and $n = 50$

$$a_{50} = 5 + (50-1)7$$

$$= 5 + 49 \times 7 = 348$$

Also the first term of the A.P. of last 15 terms be $a_{36}$

$$a_{36} = 5 + 35 \times 7$$

$$= 5 + 245 = 250$$

Now, sum of last 15 terms

$$S_{36-50} = \frac{15}{2}[S_{36} + S_{50}]$$

$$= \frac{15}{2}[250 + 348]$$

$$= \frac{15}{2} \times 598 = 4485$$

Hence, sum of last 15 terms is 4485.

**8.** Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle terms separately.

**Ans :** [Foreignset I, 2015]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

The three digit numbers which leaves 3 as remainder when divided by 4 are: $103, 107, 111, ..... 999$

Now, the first number $a = 103$, last number $a_n = 999$ and common differenced $d = 4$

Let the number of terms in this sequence be $n$.

$$a_n = a + (n-1)d$$
$$999 = 103 + (n-1)4$$
$$896 = (n-1)4$$
$$(n-1) = \frac{896}{4} = 224$$
$$n = 224 + 1 = 225$$

$$\text{Middle term} = \frac{225+1}{2}$$
$$= 113^{th} \text{ term}$$
$$a_{113} = 103 + 112 \times 4 = 551$$

and $$a_{112} = 551 - 4 = 547$$

Sum of Ist 112 terms

$$S_{112} = \frac{112}{2}(a + a_{112})$$
$$= 56(103 + 547)$$
$$= 56 \times 650 = 36400$$

and $$a_{114} = 551 + 4 = 555$$

The sum of last 112 terms

$$= \frac{112}{2}(s_{114} + a_{225})$$
$$= 56(555 + 999)$$
$$= 56 \times 1554 = 87024$$

**9.** Find the middle term of the sequence formed all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately.

**Ans :** [Foreign Set II, 2015]

The sequence is 10, 13, .... 94

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

$$94 = 10 + (n-1)3$$
$$84 = (n-1)3$$
$$n = \frac{84}{3} + 1 = 29$$

Therefore $\frac{29+1}{2} = 15^{th}$ term is the middle term.

Middle term $$a_{15} = a + (15-1)d$$
$$= 10 + 14 \times 3 = 52$$
$$a_{16} = 52 + 3 = 55$$

Sum of first 14 terms,

$$s_{14}$$
$$= \frac{14}{2}[2 \times 10 + (14-1) \times 3]$$
$$= 7[20 + 13 \times 3] = 413$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Sum of the last 14 terms,

$$= \frac{14}{2}[2s_{16} + (14-1)d]$$
$$= \frac{14}{2}[2 \times 55 + (14-1) \times 3]$$
$$= 7[110 + 13 \times 3]$$
$$= 1043$$

**10.** Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also find the sum of all numbers on both sides of the middle term separately.

**Ans :** [Foreign Set III, 2014, 2015]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

The sequence is $103, 110, ...., 999$

Here $a = 103$, $d = 7$, $a_n = 999$

$$a_n = a + (n-1)d$$
$$999 = 103 + (n-1) \times 7$$
$$n = \frac{999-103}{7} + 1 = 129$$

Therefore $\frac{129+1}{2} = 65^{th}$ term is the middle term.

Middle term $$a_{65} = 103 + (64 \times 7) = 551$$
$$a_{66} = 551 + 7 = 558$$

Sum of first 64 terms,

$$S_{64} = \frac{64}{2}[2a + (64-1)d]$$
$$= 32[2 \times 103 + 63 \times 7]$$
$$= 32[206 + 441] = 20704$$

Sum of last 64 terms

$$S_{66-129} = \frac{64}{2}(558 + 999)$$
$$= 32 \times 1557$$
$$= 49824$$

**11.** If the sum of first $n$ term of an an A.P. is given by $S_n = 3n^2 + 4n$. Determine the A.P. and the $n^{th}$ term.

**Ans :** [Board Term-2, 2014]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have $$S_n = 3n^2 + 4n.$$
$$a_1 = S_1 3(1)^2 + 4(1) = 7$$
$$a_1 + a_2 = S_2 = 3(2)^2 + 4(2)$$
$$= 12 + 8 = 20$$
$$a_2 = S_2 - S_1 = 20 - 7 = 13$$
$$a + d = 13$$

or, $$7 + d = 13$$

Thus $$d = 13 - 7 = 6$$

Hence AP is $7, 13, 19, .......$

Now, $a_n = a+(n-1)d$

$$= 7+(n-1)(6)$$

$$= 7+6n-6$$

$$= 6n+1$$

$$a_n = 6n+1$$

**12.** The sum of the $3^{rd}$ and $7^{th}$ terms of an A.P. is 6 and their product is 8. Find the sum of first 20 terms of the A.P.

**Ans :** [Board Term-2, 2012 Set (21)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have $a_3 + a_7 = 6$

$$a+2d+a+6d = 6$$

$$a+4d = 3 \quad (1)$$

and $a_3 \times a_7 = 8$

$$(a+2d)(a+6d) = 8 \quad (2)$$

Substituting the value $a = (3-4d)$ in (2) we get

$$(3-4d+2d)(3-4d+6d) = 8$$

or, $(3+2d)(3-2d) = 8$

or, $9-4d^2 = 8$

$$4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE 1 : Substituting $d = \frac{1}{2}$ in equation (1), $a = 1$.

$$S_{20} = \frac{n}{2}[2a+(n-1)d]^2$$

$$= \frac{20}{2}\left[2+\frac{19}{2}\right] = 115$$

Thus $d = \frac{1}{2}, a = 1$ and $S_{20} = 115$

CASE 2 : Substituting $d = -\frac{1}{2}$ in equation (1) $a = 5$

$$S_{20} = \frac{20}{2}\left[2\times 5+19\times\left(-\frac{1}{2}\right)\right]$$

$$= 10\left[10-\frac{19}{2}\right] = 15$$

Thus $d = -\frac{1}{2}, a = 5$ and $S_{20} = 15$

**13.** A sum of Rs. 280 is to be used towards four prizes. If each prize after the first is Rs. 20 less than its preceding prize, find the value of each of the prizes.

**Ans :** [Board Term-2, 2012(44)]

Let $I^{st}$ prize be Rs. $x$, then series of prize is
$x, x-20, x-40, x-60, \ldots\ldots.$
Here series is AP and $a = x, d = -20, S_n = 280, n = 4$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$280 = \frac{4}{2}[2x+3(-20)]$$

$$280 = 2[2x-60]$$

$$140 = 2x-60$$

$$x = \frac{140+40}{2} = 100$$

Thus prizes are Rs. 100, Rs. 80, Rs. 60, Rs. 40.

**14.** In a garden bed, there are 23 rose plants in the first row, 21 are in the $2^{nd}$, 19 in $3^{rd}$ row and so on. There are 5 plants in the last row. How many rows are there of rose plants> also find the total number of roes plants in the garden.

**Ans :** [Board Term-2, 2012(1)]

The number of rose plants in the $1^{st}, 2^{nd}, \ldots..$ are $23, 21, 19, \ldots\ldots., 5$.
Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here $a = 23, d = -2, a_n = 5$

$$a_n = a+(n-1)d$$

$$5 = 23+(n-1)(-2)$$

$$n = 10$$

Total number of roes plants in the flower bed,

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{10} = 5(46-18) = 140$$

**15.** A sum of Rs. 1890 is to be used to given seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 50 less than its preceding prize, find the value of each of the prizes.

**Ans :** [Board Term-2, 2012(5)]

Let $I^{st}$ prize be Rs. $x$, then series of prize is
$x, x-50, x-100, x-150, \ldots\ldots$
Here series is AP and
$a = s, d = -50, S_n = 1890, n = 7$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$1890 = \frac{7}{2}[2x+(-50)\times 6]$$

$$270 = x+(-50)\times 3 = x-150$$

$$x = 270+150 = 420$$

The prizes are Rs. 420, Rs. 370, Rs. 320, Rs. 270, Rs. 220, Rs. 170, Rs. 120.

**16.** If the sum of first $m$ terms of an A.P. is same as the sum of its first $n$ terms $(m \neq n)$, show that the sum of its first $(m+n)$ terms is zero.

**Ans :** [Board Term-2, 2012 (12)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$, and sum of $n$ term be $S_n$

Now $S_m = S_n$

$$\frac{m}{2}[2a+(m-1)d] = \frac{n}{2}[2a+(n-1)d]$$

$$2a(m-n)+\{(m^2-n^2)-m-nd\} = 0$$

$$2a(m-n)+\{(m-n)(m+n)-(m-n)d\} = 0$$

$$(m-n)[2a+(m+n-1)d] = 0$$

$$2a+(m+n-1)d = 0 \quad [m-n \neq 0]$$

$$S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$$

$$= \frac{m+n}{2} \times 0 = 0$$

**17.** A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?

**Ans :** [Board Term-2, 2012 Set (34)]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here $a = 20, d = 15$

Now $S_n = 3250$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$3250 = \frac{n}{2}[2a + (n-1) \times 15]$$

$$3250 \times 2 = n[40 + 15n - 15]$$

$$6500 = n[25 + 15n]$$

$$1300 = n[5 + 3n]$$

$$3n^2 + 65n - 60n - 1300 = 0$$

$$n(3n+65) - 20(3n+65) = 0$$

$$(n-20)(3n+65) = 0$$

Since $n = -65/3$, is not possible, $n = 20$

Man will repay loan in 20 months.

**18.** If $1 + 4 + 7 + 10 ..... + x = 287$, Find the value of $x$.

**Ans :** [Board Foreign Set-I, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have $a = 1, d = 3$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{n}{2}[2 \times 1 + (n-1)3] = 287$$

$$\frac{n}{2}[2 + (3n-3)] = 287$$

$$3n^2 - n = 574$$

$$3n^2 - n - 574 = 0$$

$$3n^2 - 42n + 41n - 574 = 0$$

$$3n(n-14) + 41(n-14) = 0$$

$$(n-14)(3n+41) = 0$$

Since negative value is not possible, $n = 14$

$$a_{14} = a + (n-1)d$$

$$= 1 + 13 \times 3 = 40$$

**19.** Find the sum of first 24 terms of an A.P. whose $n^{th}$ term given by $a_n = 3 + 2n$.

**Ans :** [Board Outside Delhi Comptt. Set I, II, III, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have $a_n = 3 + 2n$

$$a_1 = 3 + 2 \times 1 = 5$$

$$a_2 = 2 + 2 \times 2 = 7$$

$$a_3 = 3 + 2 \times 3 = 9$$

Thus the series is 5, 7, 9, ..... in which

$$a = 5 \text{ and } d = 2$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{24} = \frac{24}{2}(2 \times 5 + 23 \times 2)$$

$$= 12 \times 56$$

Hence, $S_{24} = 672$

## HOTS QUESTIONS

**1.** Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

**Ans :** [CBSE O.D. 2014]

The sequence goes like 110, 120, 130, ......... 990

Since they have a common difference of 10, they form an A.P.

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here $a = 110, a_n = 990, d = 10$

$$a_n = a + (n-1)d$$

$$990 = 110 + (n-1) \times 10$$

$$990 - 110 = 10(n-1)$$

$$880 = 10(n-1)$$

$$88 = n - 1$$

$$n = 88 + 1 = 89$$

Hence, there are 89 terms between 101 and 999 divisible by both 2 and 5.

**2.** How many thee digit natural numbers are divisible by 7?

**Ans :** [Board Term-2, 2013]

Let A.P. is 105, 112, 119, ........., 994 which is divisible by 7.

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here, $a = 105, d = 112 - 105 = 7, t_n = 994$, then

$$a_n = a + (n-1)d$$

$$994 = 105 + (n-1) \times 7$$

$$889 = (n-1) \times 7$$

$$n - 1 = \frac{889}{7} = 127$$

$$n = 127 + 1 = 128$$

Hence, there 128 terms divisible by 7 in A.P.

**3.** How many two digit numbers are divisible by 7?

**Ans :** [Board Sample paper, 2016]

Two digit numbers which are divisible by 7 are 14, 21, 28, ..... 98. It forms an A.P.

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here $a = 14,\ d = 7, a_n = 98$

Now $a_n = a + (n-1)d$

$$98 = 14 + (n-1)7$$

$$98 - 14 = 7n - 7$$

$$84+7 = 7n$$
$$7n = 91 \Rightarrow n = 13$$

**4.** How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case?

**Ans :** [Board Term-2, 2012 Set (1)]

When a three digit number divided by 7 and leave 3 as remainder are 101, 108, 115, ..... 997
These are in A.P.
Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here $a = 101, d = 7, a_n = 997$

Now
$$a_n = a+(n-1)d$$
$$997 = 101+(n-1)7$$
$$997-101 = 896 = (n-1)7$$
$$\frac{896}{7} = n-1$$
$$n = 128+1 = 129$$

Hence, 129 numbers are divided by 7 which leaves remainder is 3.

**5.** How many multiples of 4 lie between 11 and 266?

**Ans :** [Board Term-2, 2012, Set (21)]

First multiple of 4 is 12 and last multiple of 4 is 264. It forms a AP. Let multiples of 4 be $n$.
Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here, $a = 12, a_n = 264, d = 4$

$$a_n = a+(n-1)d$$
$$264 = 12+(n-1)4$$
$$n = \frac{264-12}{4}+1$$

Hence, there are 64 multiples of 4 that lie between 11 and 266.



**6.** Prove that the $n^{th}$ term of an A.P. can not be $n^2+1$. Justify your answer.

**Ans :** [Board Term-2, 2015]

Let $n^{th}$ term of A.P.

$$a_n = n^2+1$$

Substituting the value of $n = 1, 2, 3, .....$ we get

$$a_1 = 1^2+1 = 2$$
$$a_2 = 2^2+1 = 5$$
$$a_3 = 3^2+1 = 10$$

The obtained sequence is 2, 5, 10, 17,......

Its common difference

$$a_2-a_1 = a_3-a_2 = a_4-a_3$$
$$5-2 \neq 10-5 \neq 17-10$$
$$3 \neq 5 \neq 7$$

Since the sequence has no. common difference, $n^2+1$ is not a form of $n^{th}$ term of an A.P.

**7.** Find the sum of all two digits odd positive numbers.

**Ans :** [KVS 2014]

The list of 2 digits odd positive numbers are 11, 13 ...... 99. It forms an AP.
Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here $a = 11, d = 2, l = 991$

Now
$$a_n = a+(n-1)d$$
$$99 = 11+(n-1)2$$
$$88 = (n-1)2$$
$$n = 44+1 = 45$$
$$S_n = \frac{n}{2}[a+a_n]$$
$$= \frac{45}{2}[11+99]$$
$$S_n = \frac{15\times 108}{2} = 2475$$

Hence the sum of given A.P. is $S_n = 2475$



**8.** Find the sum of the two digits numbers divisible by 6.

**Ans :** [Board Term-2, 2013]

Series of two digits numbers divisible by 6 is:
12, 18, 24, ...........96. It forms and AP.
Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here $a = 12, d = 18-12 = 6, a_n = 96$

$$a_n = a+(n-1)d$$
$$96 = 12+(n-1)\times 6$$
$$84 = 6(n-1)$$
$$n = 14+1 = 15$$
$$S_n = \frac{n}{2}[a+a_n]$$
$$= \frac{15}{2}[12+96]$$
$$= \frac{15\times 2}{2}[8]$$
$$= 15\times 54 = 810$$

Hence the sum of given AP is 810.

**9.** Find the sum of the integers between 100 and 200 that are divisible by 6.

**Ans :** [Board Term-2, 2012 Set (5)]

The series as per question is 102, 108, 114, ......... 198. which is an A.P.
Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$
Here $a = 102, d = 6$ and $l = 198$

Now
$$198 = 102+(n-1)6$$
$$96 = (n-1)6$$
$$\frac{96}{6} = n-1$$
$$n = 17$$

Now
$$S_{17} = \frac{n}{2}(a+l)$$
$$= \frac{17}{2}[102+198]$$

$$= \frac{17}{2} \times 300 = 17 \times 150 = 2550$$

Hence the sum of given AP is 2550.

**10.** Find the number of terms of the A.P. $-12, -9, -6, ........, 21$. If 1 is added to each term of this A.P., then find the sum of all the terms of the A.P. thus obtained.

**Ans :** [Board Term-2, 2013]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

We have
$$a = -12, d = 9-(-12) = 3$$
$$a_n = a+(n-1)d$$
$$21 = -12+(n-1)\times 3$$
$$21+12 = (n-1)\times 3$$
$$33 = (n-1)\times 3$$
$$n-1 = 11$$
$$n = 11+1 = 12$$

Now, if 1 is added to each term we have a New A.P. with

$-12+1, -a+1, -6+1......21+1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained A.P.

$$S_{12} = \frac{12}{2}[-11+22]$$
$$= 6 \times 11 = 66$$

Hence the sum of new AP is 66.

**11.** How many terms of the A.P. $-6, \frac{11}{2}, -5, .....$ are needed to given the sum $-25$? Explain the double answer.

**Ans :** [Board Term-2, 2012 Set (13)]

A.P. is $-6, -\frac{11}{2}, -5 .......$

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Here we have
$$a = -6$$
$$d = -\frac{11}{2}+\frac{6}{1} = \frac{1}{2}$$
$$S_n = -25$$
$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$-25 = \frac{n}{2}\left[-12+(n-1)\times\frac{1}{2}\right]$$
$$-50 = n\left[\frac{-24+(n-1)}{2}\right]$$
$$-100 = n[n-25]$$
$$n^2-25n+100 = 0$$
$$(n-20)(n-5) = 0$$
$$n = 20, 5$$

or, $S_{20} = S_3$

Here we have got two answers because two value of $n$some of AP is same. Since $a$ is negative and $d$ is positive; the sum of the terms from $6^{th}$ to $20^{th}$ is zero.

**12.** If $S_1, S_2, S_3$ be the sum of $n, 2n, 3n$ terms respectively of an A.P. Prove that $S_3 = 3(S_2 - S_1)$.

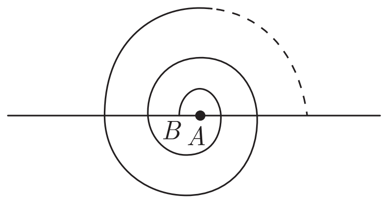
**Ans :** [Board Term-2, 2012 Set (59)]

Let the first term be $a$, and common difference be $d$.

Now
$$S_1 = \frac{n}{2}[2a(n-1)d]$$
$$S_2 = \frac{2n}{2}[2a+(2n-1)d]$$
$$S_3 = \frac{3n}{2}[2a+(3n-1)d]$$

$3(S_2 - S_1)$
$$= 3\left[\frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]\right]$$
$$= 3\left[\frac{n}{2}[4a+2(2n-1)d] - [2a+(n-1)d]\right]$$
$$= 3\left[\frac{n}{2}(4a+4nd-2d-2a-nd+d)\right]$$
$$= 3\left[\frac{n}{2}(2a+3nd-d)\right]$$
$$= \frac{3n}{2}[2a+(3n-1)d] = S_3$$

**13.** A spiral is made up of successive semi-circles with centres alternately a A and B starting with A, of radii 1 cm, 2 cm, 3 cm, ...... as shown in the figure. What is the total length of spiral made up of eleven consecutive semi-circles? (Use $\pi = 3.14$)




**Ans :** [Board Term-2, 2012 Set (50); [NCERT]]

Let $r_1, r_2$............ be the radii of semi-circles and $l_1, l_2$,........... be the lengths of circumferences of semi-circles, than

$$l_1 = \pi r_1 = \pi(1) = \pi\ cm$$
$$l_2 = \pi r_2 = \pi(2) = 2\pi\ cm$$
$$l_3 = 3\pi\ cm$$
..............
..............
$$l_{11} = 11\pi\ cm$$

Total length of spiral
$$L = l_1 + l_2 + ........ + l_{11}$$
$$= \pi + 2\pi + 3\pi + .......... + 11\pi$$
$$= \pi(1+2+3+.......+11)$$
$$= \pi \times \frac{11 \times 12}{2}$$
$$= 66 \times 3.14$$
$$= 207.24 \text{ cm}$$

**14.** The ratio of the sums of first $m$ and first $n$ terms of an A.P. is $m^2:n^2$. Show that the ratio of its $m^{th}$ and $n^{th}$ terms is $(2m-1):(2n-1)$.

**Ans :** [CBSE Board Delhi Set I, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

$$\frac{S_m}{n_n} = \frac{m^2}{n^2}$$

$$\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2}\times\frac{n}{m} = \frac{m}{n}$$

$$m(2a+(n-1)d) = n[2a+(m-1)d]$$

$$2ma+mnd-md = 2na+nmd-nd$$

$$2ma-2na = md-nd$$

$$d = 2a$$

Now,
$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$$

$$= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$$

$$= \frac{a+2ma-2a}{a+2na-2a}$$

$$= \frac{2ma-a}{2na-a} = \frac{a(2m-1)}{a(2n-1)}$$

$$= 2m-1:2n-1$$

**15.** If the $p^{th}$ term of an A.p. is $\frac{1}{q}$ and $q^{th}$ term is $\frac{1}{p}$. Prove that the sum of first $pq$ term of the A.P. is $\left[\frac{pq+1}{2}\right]$

**Ans :** [CBSE Board Delhi Set III, 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

$$a_p = a+(p-1)d = \frac{1}{q} \quad ...(1)$$

and
$$a_q = a+(q-1)d = \frac{1}{p} \quad ...(2)$$

Solving (1) and (2) we get

$$a = \frac{1}{pq} \text{ and } d = \frac{1}{p}$$

$$S_{pq} = \frac{pq}{2}\left[2\times\frac{1}{8q}+(pq-1)\frac{1}{pq}\right]$$

$$= \frac{pq+1}{2}$$

**16.** If the ratio of the $11^{th}$ term of an A.P. to its $18^{th}$ term is 2 : 3, find the ratio of the sum of the first five term of the sum of its first 10 terms.

**Ans :** [Delhi Compt. Set I, II, III 2017]

Let the first term be $a$, common difference be $d$, $n$th term be $a_n$ and sum of $n$ term be $S_n$

Now
$$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

$$2(a+17d) = 3(a+10d)$$

$$a = 4d \quad ...(1)$$

Now,
$$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]} = \frac{(a+2d)}{[2a+9d]}$$

Substituting the value $a = 4d$ we have

or,
$$\frac{S_5}{S_{10}} = \frac{4d+2d}{8d+9d} = \frac{6}{17}$$

Hence $S_5:S_{10} = 6:17$

**17.** An A.P. Consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the A.P.

**Ans :** [Sample Paper 2017]

Let the middle most terms of the A.P. be $(x-d), x, (x+d)$

We have
$$x-d+x+x+d = 225$$

$$3x = 225$$

or,
$$x = 75$$

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

Thus AP is

$(x-18d), ....(x-2d), (x-d), x, (x+d), (x+2d), .......(x-18d)$

Sum of last three terms,

$$(x+18d)+(x+17d)+(x+16d) = 429$$

$$3x+51d = 429$$

$$225+51d = 429 \text{ or, } d = 4$$

First term $a_1 = x-18d = 75-18\times 4 = 3$

$$a_2 = 3+4 = 7$$

Hence A.P. = 3, 7, 11, ......., 147.