CHAPTER 4

Quadratic Equation

TOPIC 1 : SOLUTION OF QUADRATIC EQUATIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the positive root of $\sqrt{3x^2+6}=9$.

Ans : [Board Term-2, 2015, Set UDICCY2]

We have $\sqrt{3x^2+6} = 9$

$3x^2+6 = 81$

$3x^2 = 81-6=75$

$x^2 = \frac{75}{3}=25$

Thus $x = \pm 5$

Hence 5 is positive root.

2. If $x=-\frac{1}{2}$, is a solution of the quadratic equation $3x^2+2kx-3=0$, find the value of k.

[Board Term-2, 2015, Delhi CBSE (Set, I, II, III)]

Ans : [CBSE Marking Scheme, 2015]

We have $3x^2+2kx-3 = 0$

Putting $x=-\frac{1}{2}$ we get

$$3\left(-\frac{1}{2}\right)^2+2k\left(-\frac{1}{2}\right)-3 = 0$$

$$\frac{3}{4}-k-3 = 0$$

$$k = \frac{3}{4}-3$$

$$= \frac{3-12}{4} = \frac{-9}{4}$$

Hence $k = \frac{-9}{4}$

3. Find the roots of the quadratic equation $\sqrt{3}\,x^2-2x-\sqrt{3}$.

Ans : [Board Term-2, 2012, (35)2011 (A1)]

We have $\sqrt{3}\,x^2-2x-\sqrt{3} = 0$

$\sqrt{3}\,x^2-3x+x-\sqrt{3} = 0$

$\sqrt{3}\,x(x-\sqrt{3})+1(x-\sqrt{3}) = 0$

$(x-\sqrt{3})(\sqrt{3}+1) = 0$

Thus $x = \sqrt{3}, \frac{-1}{\sqrt{3}}$

4. Find the value of k, for which one root of the quadratic equation $kx^2-14x+8=0$ is six times the other.

Ans : [Board Term-2, 201]

[Board Sample Paper 2016]

We have $kx^2-14x+8=0$

Let one root be α and other root be 6α.

Sum of roots $\alpha+6\alpha = \frac{14}{k}$

$7\alpha = \frac{14}{k}$ or $\alpha = \frac{2}{k}$ ...(1)

Product of roots $\alpha(6\alpha) = \frac{8}{k}$

or, $6\alpha^2 = \frac{8}{k}$ ...(2)

Solving (1) and (2), we obtain

$$6\left(\frac{2}{k}\right)^2 = \frac{8}{k}$$

$$6\times\frac{4}{k^2} = \frac{8}{k}$$

$$\frac{3}{k^2} = \frac{1}{k}$$

$$3k = k^2$$

$$3k-k^2 = 0$$

$$k[3-k] = 0$$

$$k = 0 \text{ or } k=3$$

Since $k = 0$ is not possible, therefore $k = 3$.

5. If one root of the quadratic equation $6x^2-x-k=0$ is $\frac{2}{3}$, then find the value of k.

Ans : [Board Term-II foreign-2, 2017]

We have $6x^2-x-k=0$

Substituting $x=\frac{2}{3}$, we get

$$6\left(\frac{2}{3}\right)^2-\frac{2}{3}-k = 0$$

$$6\times\frac{4}{3}-\frac{2}{3}-k = 0$$

$$k = 6\times\frac{4}{9}-\frac{2}{3}=\frac{24-6}{9}=2$$

Thus $k = 2$.

6. Find the value (s) of k if the quadratic equation $3x^2-k\sqrt{3}\,x+4=0$ has real roots.

Ans : [Sample Question Paper 2017]

If discriminant of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $3x^2-k\sqrt{3}\,x+4 = 0$

Compare with $ax^2+bx+c = 0$

$D = b^2-4ac$

For real roots $b^2-4ac \geq 0$

$(-k\sqrt{3})^2-4\times3\times4 \geq 0$

$$3k^2 - 48 \geq 0$$
$$k^2 - 16 \geq 0$$
$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

SHORT ANSWER TYPE QUESTIONS - I

1. Find the roots the quadratic equation $6x^2 - x - 2 = 0$.

Ans : [Board Term-2, 2012, Set (13)]

We have
$$6x^2 - x - 2 = 0$$
$$6x^2 + 3x - 4x - 2 = 0$$
$$3x(2x+1) - 2(2x+1) = 0$$
$$(2x+1)(3x-2) = 0$$
$$3x - 2 = 0 \text{ or } 2x + 1 = 0$$
$$x = \frac{2}{3} \text{ or } x = -\frac{1}{2}$$

Hence roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

2. Find the roots of the following quadratic equation : $15x^2 - 10\sqrt{6}\,x + 10 = 0$

Ans : [Board Term-2, 2012 Set (1)]

We have
$$15x^2 - 10\sqrt{6}\,x + 10 = 0$$
$$3x^2 - 2\sqrt{6}\,x + 2 = 0$$
$$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$$
$$\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$$
$$(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$$

Thus $x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

3. Solve the following quadratic equation for x : $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

Ans : [Board term-2, 2013, 2012, Set (22)]

We have
$$4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$$
$$4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$4x(\sqrt{3}\,x + 2) - \sqrt{3}(\sqrt{3}\,x + 2) = 0$$
$$(\sqrt{3}\,x + 2)(4x - \sqrt{3}) = 0$$

Thus $x = -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

4. Solve for x : $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

Ans : [Foreign Set, II, III, 2015

We have
$$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$$
$$x^2 - \sqrt{3}\,x - 1x + \sqrt{3} = 0$$
$$x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$
$$(x - \sqrt{3})(x - 1) = 0$$

Thus $x = \sqrt{3}, x = 1$

5. Find the roots of the following quadratic equation : $(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

Ans : [Board Term-2, 2012, Set (52), 2011 Set (A1)]

We have
$$(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$$
$$x^2 + 2x - 3 = 3x - 1$$
$$x^2 - x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x-2) + 1(x-2) = 0$$
$$(x-2)(x+1) = 0$$

Thus $x = 2, -1$

6. Find the roots of the following quadratic equation : $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

Ans : [Board Term-2, 2012 Set (40)]

We have
$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$$
$$\frac{2x^2 - 5x - 3}{5} = 0$$
$$2x^2 - 5x - 3 = 0$$
$$2x^2 - 6x + x - 3 = 0$$
$$2x(x-3) + 1(x-3) = 0$$
$$(2x+1)(x-3) = 0$$

Thus $x = -\frac{1}{2}, 3$

7. Solve the following quadratic equation for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$

Ans : [Delhi CBSE Term-2, 2015 (Set I, II)]

We have $4x^2 - 4a^2x + (a^4 - b^4) = 0$

Compare with $Ax^2 + Bx + C = 0$ we have

$$A = 4, B = -4a^2, C = (a^4 - b^4)$$
$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$= \frac{4a^2 \pm \sqrt{(-4a^2)^2 - 4 \times 4(a^4 - b^4)}}{2 \times 4}$$

$$= \frac{4a^2 \pm \sqrt{16a^2 - 16a^4 + 16b^4}}{8}$$

$$= \frac{4a^2 \pm \sqrt{16b^4}}{8}$$

or, $x = \frac{4a^2 \pm 4b^2}{8} = \frac{a^2 \pm b^2}{2}$

Thus either $x = \frac{a^2 + b^2}{2}$ or $x = \frac{a^2 - b^2}{2}$

8. Solve the following quadratic equation for x :

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

Ans : [Delhi CBSE Term-2, 2015 (Set III)]

We have $9x^2 - 6b^2x - (a^4 - b^4) = 0$

Compare with $ax^2 + bx + c = 0$ we have

$$a = 9, b = -6b^2, c = -(a^4 - b^4)$$

$$x = \frac{6b^2 \pm \sqrt{(-6b^2)^2 - 4 \times 9 \times \{(a^4 - b^4)\}}}{2 \times 9}$$

$$= \frac{6b^2 \pm \sqrt{36b^4 + 36a^4 - 36b^4}}{18}$$

$$= \frac{6b^2 \pm \sqrt{36a^4}}{18}$$

$$= \frac{6b^2 \pm 6a^2}{18}$$

Thus $x = \frac{a^2 + b^2}{3}, \frac{b^2 - a^2}{3}$

9. Solve the following equation for x :

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

Ans : [Outside Delhi CBSE, 2015 Set I,II,III]

We have $4x^2 + 4bx + b^2 - a^2 = 0$

$$(2x + b)^2 - a^2 = 0$$

$$(2x + b + a)(2x + b - a) = 0$$

$$x = \frac{-(a + b)}{2}, x = \frac{a - b}{2}$$

10. Solve the following quadratic equation for x :

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

Ans : [Outside Delhi CBSE, 2015 Set III]

We have $x^2 - 2ax - (4b^2 - a^2) = 0$

$$x^2 - 2ax + a^2 - 4b^2 = 0$$

$$(x - a)^2 - (2b)^2 = 0$$

$$(x - a + 2b)(x - a - 2b)$$

$= 0$

Thus $x = a - 2b, x = a + 2b$

11. Solve the quadratic equation, $2x^2 + ax - a^2 = 0$ for x. [Delhi CBSE, Term-2, 2014]

We have $2x^2 + ax - a^2 = 0$

Compare with $Ax^2 + Bx + C = 0$ we have

$$A = 2, B = a, C = -a^2$$

Now $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$= \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a^2)}}{2 \times 2}$$

$$= \frac{-a \pm \sqrt{a^2 + 8a^2}}{4}$$

$$= \frac{-a \pm \sqrt{9a^2}}{4}$$

$$= \frac{-a \pm 3a}{4}$$

$$x = \frac{-4 + 3a}{4}, \frac{-a - 3a}{4}$$

Thus $x = \frac{a}{2}, -a$

12. Find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2) = 0$

Ans : [Board Term-2, 2014]

We have $4x^2 - 4px + (p^2 - q^2) = 0$

Compare with $ax^2 + bx + c = 0$ we get

$$a = 4, b = -4p, c = (p^2 - q^2)$$

The roots are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4p \pm \sqrt{16p^2 - 4 \times 4 \times (p^2 - q^2)}}{2 \times 4}$$

$$= \frac{4p \pm \sqrt{16p^2 - 16p^2 + 16q^2}}{8}$$

$$= \frac{4p \pm 4q}{8}$$

Thus roots are $\frac{p + q}{2}, \frac{p - q}{2}$.

13. Sum of the areas of two squares is 468 m^2.If the difference of their perimeter is 24 m, find the sides of the squares.

Ans : [Board Term-2, 2012 Set (28), 2011 Set (A1)]

Let the side of the smaller square be y and be the side of the longer square be x, then we have

$$4x - 4y = 24$$

$$x - y = 6$$

$$x = y + 6$$

According to the question

$$x^2 + y^2 = 468$$

$$(y + 6)^2 + y^2 = 468$$

$$2y^2 + 12y + 36 = 468$$

$$2y^2 + 12y - 432 = 0$$

$$y^2 + 6y - 216 = 0$$

$$(y + 18)(y - 12) = 0$$

$$y = -18, 12$$

As side can not be negative, $y = 12$ and $x = 12 + 6 = 18$ Hence, the side of larger square 18 m and that of smaller square 12 m.

14. Solve for x (in ferms of a and b) :

$$\frac{a}{x-b}+\frac{b}{x-a}=2, x\neq a, b$$

Ans : [Board Term-2 Foreign Set II, 2016]

We have $\dfrac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$

$$\begin{aligned} a(x-a)+b(x-b) &= 2\left[x^2-(a+b)x+ab\right] \\ ax-a^2+bx-b^2 &= 2x^2-2(a+b)x+2ab \\ 2x^2-3(a+b)x+(a+b)^2 &= 0 \\ 2x^2-2(a+b)x-(a-b)x+(a+b)^2 &= 0 \\ \left[2x-(a+b)\right]\left[x-(a+b)\right] &= 0 \end{aligned}$$

Thus $x = a+b, \dfrac{a+b}{2}$

15. Solve for x : $\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3}=0$

Ans : [Board Term-2 Foreign Set II, 2016]

We have $\sqrt{3}\,x^2-3\sqrt{2}\,x+\sqrt{2}\,x-2\sqrt{3}=0$

$$\begin{aligned} \sqrt{3}\,x\left[x-\sqrt{6}\right]+\sqrt{2}\left[x-\sqrt{6}\right] &= 0 \\ (x-\sqrt{6})(\sqrt{3}\,x+\sqrt{2}) &= 0 \end{aligned}$$

Thus $x\ \sqrt{6}, -\sqrt{\dfrac{2}{3}}$

16. If $x=\dfrac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of a and b.

Ans : [Board Term-2 Delhi Set I, II, III, 2016]

We have $ax^2+7x+b=0$

Substituting $x=\dfrac{2}{3}$ in above equation we obtain

$$\begin{aligned} \frac{4}{9}a+\frac{14}{3}+b &= 0 \\ 4a+42+9b &= 0 \\ 4a+9b &= -42 \qquad (1) \end{aligned}$$

and substituting $x=-3$ we obtain

$$\begin{aligned} 9a-21+b &= 0 \\ 9a+b &= 21 \qquad (2) \end{aligned}$$

Solving (1) and (2), we get $a=3$ and $b=-6$

17. Solve for x : $\sqrt{6x+7}-(2x-7)=0$

Ans : [O. D. Set III, 2016]

We have $\sqrt{6x+7}-(2x-7)=0$

or, $\sqrt{6x+7}=(2x-7)$

Squaring both sides we get

$$\begin{aligned} 6x+7 &= (2x-7)^2 \\ 6x+7 &= 4x^2-28x+49 \\ 4x^2-34x+42 &= 0 \\ 2x^2-17x+21 &= 0 \\ 2x^2-14x-3x+21 &= 0 \\ 2x(x-7)-3(x-7) &= 0 \\ (x-7)(2x-3) &= 0 \end{aligned}$$

Thus $x=7$ and $x=\dfrac{2}{3}$

18. Find the roots of $x^2-4x-8=0$ by the method of completing square.

Ans : [Board Term-2, 2015]

We have $x^2-4x-8=0$

Squaring both side we have

$$\begin{aligned} (x-2)^2-8-4 &= 0 \\ (x-2)^2-12 &= 0 \\ (x-2)^2 &= 12 \\ (x-2)^2 &= \left(2\sqrt{3}\right)^2 \\ x-2 &= \pm 2\sqrt{3} \\ x &= 2\pm 2\sqrt{3} \end{aligned}$$

Thus $x=2+2\sqrt{3},\ 2-2\sqrt{3}$

19. A two digit number is four times the sum of the digits. It also equal to 3 times the product of digits. Find the number.

Ans : [Board Term-2, Foreign Set I, 2016]

Let units digit and tens digit of the two digit number be x and y respectively.

Thus number is $10y+x$

According to question, we have

$$\begin{aligned} 10y+x &= 4(y+x) \\ 10y+x &= 4y+4x \\ 10y-4y &= 4x-x \\ 6y &= 3x \\ 2y &= x \end{aligned}$$

Also, $10y+x = 3xy$

$$\begin{aligned} 10y+2y &= 3(2y)y \\ 12y &= 6y^2 \\ 6y^2-12y &= 0 \\ 6y(y-2) &= 0 \\ y &= 0 \text{ or } y=2 \end{aligned}$$

As the number can not be zero $x=4$ and $x=2y=4$.

Thus required number is 24.

20. In a cricket match, Harbhajan took three wickets less than twice the number of wickets taken by Zahir. The Product of the number of wickets taken by these two is 20. Represent the above situation in the form of quadratic equation.

[Board Term-2, 2015]

Ans : [CBSE Marking Scheme, 2015]

Let the number of wickets is taken by Zahir be x, then number of wickets taken by Harbhajan will be $2x-3$.

According to question,

$$x(2x-3) = 20$$
$$2x^2-3x = 20$$

Thus required quadratic equation,

$$2x^2-3x-20 = 0$$

21. Solve for x : $\sqrt{2x+9}+x=13$

Ans : [Board Term-2 Outside Delhi Set II 2016]

We have $\sqrt{2x+9}+x = 13$

$$\sqrt{2x+9} = 13-x$$

Squaring both side we have

$$2x+9 = (13-x)^2$$
$$2x+9 = 169+x^2-26x$$
$$0 \quad x^2+169-26x-9-2x$$
$$x^2-28x+160 = 0$$
$$x^2-20x-8x+160 = 0$$
$$x(x-20)-8(x-20) = 0$$
$$(x-8)(x-20) = 0$$

Thus $x = 8$ and $x = 20$.

22. Find the roots of the quadratic equation $\sqrt{2}\,x^2+7x+5\sqrt{2}=0$

Ans : [Board Term-II Outside Delhi, 2017]

We have $\sqrt{2}\,x^2+7x+5\sqrt{2} = 0$

$$\sqrt{2}\,x^2+2x+5x+5\sqrt{2} = 0$$
$$\sqrt{2}\,x(x+\sqrt{2})+5(x+\sqrt{2}) = 0$$
$$(x+\sqrt{2})(\sqrt{2}\,x+5) = 0$$

Thus $x=-\sqrt{2}$ and $=-\frac{5}{\sqrt{2}} = -\frac{5}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = -\frac{5}{2}\sqrt{2}$

23. Find the value of k for which the roots of the quadratic equation $2x^2+kx+8=0$ will have the equal roots ?

[Board Term-II Outside Delhi Compt., 2017]

Ans :

We have $2x^2+kx+8=0$

Compare with $ax^2+bx+c = 0$ we get

$$a = 2, b = k, \text{ and } c = 8$$

For equal roots, $D = 0$

$$b^2-4ac = 0$$
$$k^2-4\times 2\times 8 = 0$$
$$k^2 = 64$$
$$k = \pm\sqrt{64}$$

Thus $k = \pm 8$

24. Solve for x : $\sqrt{3}\,x^2+10x+7\sqrt{3}=0$

Ans : [Board Term-II Foreign 2017 Set-2]

We have $\sqrt{3}\,x^2+10x+7\sqrt{3} = 0$

$$\sqrt{3}\,x^2+3x+7x+7\sqrt{3} = 0$$
$$\sqrt{3x}(x+\sqrt{3})+7(x+\sqrt{3}) = 0$$
$$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$$

Thus $x = -\sqrt{3}$ and $x = -\frac{7}{\sqrt{3}}$

SHORT ANSWER TYPE QUESTIONS - II

1. Solve for x :

$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}; x \neq 1,-2,2$

Ans : [Board Term-2 Delhi Set II, 2016]

We have $\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$
$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$
$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$
$$5x^2+19x-30 = 0$$
$$(5x-6)(x+5) = 0$$
$$x = -5, \frac{6}{5}$$

2. Solve for x :

$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -\frac{3}{2}$

Ans : [Board Term-2, Delhi Set I, 2016]

We have $2x(2x+3)+(x-3)+(3x+9) = 0$

$$2x^2+5x+3 = 0$$
$$(x+1)(2x+3) = 0$$

Thus $x = -1, x = -\frac{3}{2}$

3. Solve for x : $\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}, x \neq 0, \frac{2}{3}, 2.$

Ans : [Board Term-2, Foreign Set II, 2016]

We have $\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}$

$$\frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$$
$$\frac{4x-3}{x(2x-3)} = \frac{1}{x-2}$$
$$(x-2)(4x-3) = 2x^2-3x$$
$$4x^2-11x+6 = 2x^2-3x$$
$$2x^2-8x+6 = 0$$
$$x^2-4x+3 = 0$$
$$(x-1)(x-3) = 0$$

Thus $x = 1, 3$

4. Solve the following quadratic equation for x :

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1 = 0$$

Ans : [Board Term-2, Delhi Set III, 2016]

We have $x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$

$$x^2+\frac{a}{a+b}x+\frac{a+b}{a}x+1=0$$

$$x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right)=0$$

$$\left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right)=0$$

Thus $x=\frac{-a}{a+b},\frac{-(a+b)}{a}$

5. Solve for x :

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3};x\neq 1,2,3$$

Ans : [Board Term-2, O.D. Set I, 2016]

We have $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$

$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\frac{2x=4}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\frac{2(x-2)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\frac{2}{(x-1)(x-3)}=\frac{2}{3}$$

$$3=(x-1)(x-3)$$

$$x^2-4x+3=3$$

$$x^2-4x=0$$

$$x(x-4)=0$$

Thus $x=0$ or $x=4$

6. Solve for x : $\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3}=0$

Ans : [Board Term-2, Outside Delhi CBSE, 2015 Set I, III, Foreign Set I, II, II, 2014]

We have $\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3}=0$

$$\sqrt{3}\,x^2-[3\sqrt{2}-\sqrt{2}]x-2\sqrt{3}=0$$

$$\sqrt{3}\,x^2-3\sqrt{2}\,x+\sqrt{2}\,x-2\sqrt{3}=0$$

$$\sqrt{3}\,x^2-\sqrt{3}\sqrt{3}\sqrt{2}\,x+\sqrt{2}\,x-\sqrt{2}\sqrt{2}\sqrt{3}=0$$

$$\sqrt{3}\,x[x-\sqrt{3}.\sqrt{2}]+\sqrt{2}[x-\sqrt{2}\sqrt{3}]=0$$

$$\sqrt{3}\,x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}]=0$$

$$(x-\sqrt{6})(\sqrt{3}\,x+\sqrt{2})=0$$

Thus $x=\sqrt{6}=-\sqrt{\frac{2}{3}}$

7. Solve for x : $2x^2+6\sqrt{3}\,x-60=0$

Ans : [Board Term-2, O.D. CBSE, 2015, Set II]

We have $2x^2+6\sqrt{3}\,x-60=0$

$$x^2+x\sqrt{3}\,x-30=0$$

$$x^2+5\sqrt{3}\,x-2\sqrt{3}\,x-30=0$$

$$x(x+5\sqrt{3})-2\sqrt{3}(x+5\sqrt{3})=0$$

$$(x+5\sqrt{3})(x-2\sqrt{3})=0$$

Thus $x=-5\sqrt{3},2\sqrt{3}$

8. Solve for x : $x^2+5x-(a^2+a-6)=0$

Ans : [Board Term-2 Foreign Set I, 2015]

$x^3+5x-(a^2+a-6)=0$

$$x=\frac{-5\pm\sqrt{25+4(a^2+a-6)}}{2}$$

$$\left[\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$$

$$=\frac{-5\pm(2a+1)}{2}$$

$$=\frac{2a-4}{2},\frac{-2a-6}{2}$$

$$x=a-2,x=-(a+3)$$

9. Solve for x : $x^2-(2b-1)x+(b^2-b-20)=0$

Ans : [Board Term-2 Foreign Set II, 2015]

We have $x^2-(2b-1)x+(b^2-b-20)=0$

Compare with $Ax^2+Bx+C=0$ we have

$$A=1,B=-(2b-1),C=(b^2-b-20)$$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{(2b-1)\pm\sqrt{(2b-1)^2-4(b^2-b-20)}}{2}$$

$$=\frac{(2b-1)\pm 9}{2}$$

$$=\frac{2b+8}{2},\frac{2b-10}{2}$$

$$=b+4,\ b-5$$

Thus $x=b+4$ and $x=b-5$

10. Solve for x : $\frac{16}{x}-1=\frac{15}{x+1};x\neq 0,-1$

Ans : [Board Term-2, OD 2014]

We have $\frac{16}{x}-1=\frac{15}{x+1}$

$$\frac{16}{x}-\frac{15}{x+1}=1$$

$$16(x+1)-15x=x^2+x$$

$$16x+16-15x=x^2+x$$

$$x+16=x^2+x$$

$$x^2-16=0$$

$$x^2=16$$

$$x=\pm 4$$

Thus $x=-4$ and $x=+4$

11. Solve the quadratic equation $(x-1)^2-5(x-1)-6=0$

Ans : [Board Term-2, 2015]

We have $(x-1)^2-5(x-1)-6 = 0$

$x^2-2x+1-5x+5-6 = 0$

$x^2-7x+6-6 = 0$

$x^2-7x = 0$

$x(x-7) = 0$

Thus $x = 0,7$

12. Solve the equation for x : $\frac{4}{3}-3 = \frac{5}{2x+3}; x \neq 0, \frac{-3}{2}$

Ans : [Board Term-2 Delhi CBSE, 2014]

We have $\frac{4}{x}-3 = \frac{5}{2x+3}$

$\frac{4}{x}-\frac{5}{2x+3} = 3$

$\frac{4(2x+3)-5x}{x(2x+3)} = 3$

$8x+12-5x = 3x(2x+3)$

$3x+12 = 6x^2+9x$

$6x^2+6x-12 = 0$

$x^2+x-2 = 0$

$x^2+2x-x-2 = 0$

$x(x+2)-(x+2) = 0$

$(x+2)(x-1) = 0$

Thus $x = -1,-2$

13. Find the roots of the equation $2x^2+x-4=0$, by the method of completing the squares.

Ans : [KVS, 2014]

We have $2x^2+x-4 = 0$

$x^2+\frac{x}{2}-2 = 0$

$x^2+2x\left(\frac{1}{4}\right)-2 = 0$

Adding and subtracting $\left(\frac{1}{4}\right)^2$, we get

$x^2+2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2 = 0$

$\left(x+\frac{1}{4}\right)^2-\left(\frac{1}{16}+2\right) = 0$

$\left(x+\frac{1}{4}\right)^2-\left(\frac{1+32}{16}\right) = 0$

$\left(x+\frac{1}{4}\right)^2-\frac{33}{16} = 0$

$\left(x+\frac{1}{4}\right)^2 = \frac{33}{16}$

$\left(x+\frac{1}{4}\right) = \pm\frac{\sqrt{33}}{4}$

Thud toots are $x = \frac{-1\pm\sqrt{33}}{4}, -\left(\frac{1+\sqrt{33}}{4}\right)$

14. Solve for x : $9x^2-6ax+(a^2-b^2) = 0$

Ans : [Delhi CBSE Term-2, Board Term-2, 2012, Set (40)]

We have $9x^2-6ax+a^2-b^2 = 0$

Compare with $Ax^2+Bx+C = 0$ we have

$A = 9, B = -6a, C = (a^2-b^2)$

$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

$x = \frac{6a\pm\sqrt{(-6a)^2-4\times 9kt(a^2-b^2)}}{2\times 9}$

$= \frac{6a\pm\sqrt{36a^2-36a^2+36b^2}}{18}$

$= \frac{6a+6b}{18}, x = \frac{6a-6b}{18}$

$= \frac{6(a+b)}{18}, x = \frac{6(a-b)}{18}$

$\frac{a+b}{3}, x = \frac{a-b}{3}$

15. Solve the equation $\frac{1}{x+4}-\frac{1}{x-7} = \frac{11}{30}, x \neq -4,7$ for x.

Ans : [Board Term-2, 2012 Set (1)]

We have, $\frac{1}{x+4}-\frac{1}{x-7} = \frac{11}{30}$

$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$

$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$

$\frac{-1}{(x+4)(x-7)} = \frac{1}{30}$

$(x+4)(x-7) = -30$

$x^2-3x-28 = -30$

$x^2-3x+2 = 0$

$x^2-2x-x+2 = 0$

$(x-1)(x-2) = 0$

Thus $x = 1,2$.

16. Find the roots of the quadratic equation :

$a^2b^2x^2+b^2x-a^2x-1 = 0$

Ans : [Board Term-2, 2012 (31)]

We have $a^2b^2x^2+b^2x-a^2x-1 = 0$

$b^2x(a^2x+1)-1(a^2x+1) = 0$

$(b^2x-1)(a^2x+1) = 0$

$x = \frac{1}{b^2}$, or $x = -\frac{1}{a^2}$

Hence, roots are $\frac{1}{b^2}$ and $-\frac{1}{a^2}$.

17. Solve the following quadratic equation for x :

$p^2x^2+(p^2-q^2)x-q^2 = 0$

Ans : [Board Term-2, 2012 Set (A1)]

We have $p^2x^2+(p^2-q^2)x-q^2 = 0$

Compare with $ax^2+bx+c = 0$ we get

$a = p^2, b = p^2-q^2, c = -q^2$

The roots are given by the quadratic formula

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$= \frac{-(p^2-q^2)-\sqrt{(p^2-q^2)^2-4(p^2)(-q^2)}}{2p^2}$$

$$= \frac{-(p^2-q^2)-\sqrt{p^4+q^4-2p^2q^2+4p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2)-\sqrt{p^4+q^4+2p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2)-\sqrt{(p^2+q^2)^2}}{2p^2}$$

$$= \frac{-(p^2-q^2)\pm(p^2+q^2)}{2p^2}$$

Thus $x = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$

and $x = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$

Hence, roots are $\frac{q^2}{p^2}, -1$

18. Solve the following quadratic equation for x :

$$9x^2-9(a+b)x+2a^2+5ab+2b^2 = 0$$

Ans : [Board Term-2, Foreign Set I, 2016]

We have $9x^2-9(a+b)x+2a^2+5ab+2b^2 = 0$

Now $2a^2+5ab+2b^2 = 2a^2+4ab+ab+2b^2$

$= 2a[a+2b]+b[a+2b]$

$= (a+2b)(2a+b)$

Hence the equation becomes

$$9x^2-9(a+b)x+(a+2b)(2a+b) = 0$$
$$9x^2-3[3a+3b]x+(a+2b)(2a+b) = 0$$
$$9x^2-3[(a+2b)+(2a+b)]x+(a+2b)(2a+b) = 0$$
$$9x^2-3(a+2b)x-3(2a+b)x+(a+2b)(2a+b) = 0$$
$$3x[3x-(a+2b)]-(2a+b)[3x-(a+2b)] = 0$$
$$[3x-(a+2b)][3x-(2a+b)] = 0$$
$$3x-(2a+b) = 0$$
$$x = \frac{a+2b}{3}$$

$$3x-(2a+b) = 0$$
$$x = \frac{2a+b}{3}$$

Hence, roots are $\frac{a+2b}{3}$ and $\frac{2a+b}{3}$.

19. Solve for x : $x^2+6x-(a^2+2a-8)$

Ans : [Board Term-2, Foreign Set III, 2015]

We have $x^2+6x-(a^2+2a-8)=0$

Compare with $Ax^2+Bx+C = 0$ we get

$$A = 1, B = 6, C = (a^2+2a-8)$$

The roots are given by the quadratic formula

$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$= \frac{-6\pm\sqrt{36+4(a^2+2a-8)}}{2}$$

$$= \frac{-6\pm(2a+2)}{2}$$

Thus $x = \frac{-6+(2a+2)}{2} = a-2$

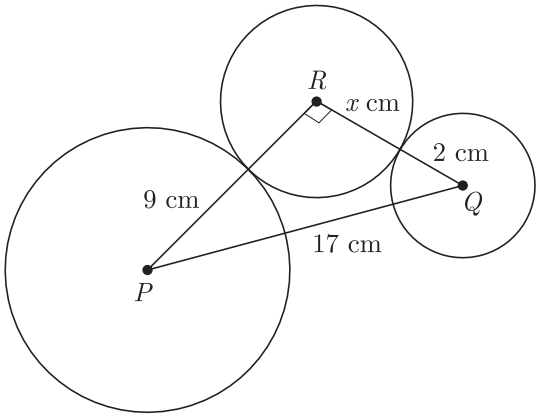
and $x = \frac{-6-(2a+2)}{2} = -a-4$

Thus $x = a-2, -a-4$

20. p and Q are centres of circles of radii 9 cm and 2 cm respectively. $PQ = 17$ cm. R is the centre of the circle of radius x cm which touch given circles externally. Given that angle PRQ is 90°. Write an equation in x and solve it.

Ans : [Board Term-2, SQP, 2016]

As per question statement figure is given below.



In right ΔPQR, by Pythagoras theorem

$$PQ^2 = PR^2+RQ^2$$

or, $17^2 = (x+9)^2+(x+2)^2$

$$289 = x^2+18x+81+x^2+4x+4$$
$$289 = 2x^2+22x+85$$
$$0 = 2x^2+22x+85-289$$
$$0 = 2x^2+22x-204$$
$$x^2+11x-102 = 0$$
$$x^2+17x-6x-102 = 0$$
$$x(x+17)-6(x+17) = 0$$
$$(x-6)(x+17) = 0$$
$$x = 6 \text{ or } x = -17$$

As x can't be negative, $x=6$.

21. Three consecutive natural number are such that the squre of the middle number exceeds the difference of the squares of the other two by 60. Find the number.

Ans : [Board Term-2, O.D. Set III, 2016]

Let the three consecutive natural numbers be $x, x+1$ and $x+2$.

$$\begin{aligned}\text{Now}\quad (x+1)^2 &= (x+2)^2-(x)^2+60\\ x^2+2x+1 &= x^2+4x+4-x^2+60\\ x^2-2x-63 &= 0\\ x^2-9x+7x-63 &= 0\\ x(x-9)+7(x-9) &= 0\\ (x-9)(x+7) &= 0\\ x &= 9 \text{ or } x=-7\end{aligned}$$

As x can't be negative, $x=9$.

Hence three numbers are 9, 10, 11.

22. If $(x^2+y^2)(a^2+b^2)=(ax+by)^2$. Prove that $\frac{x}{a}=\frac{y}{b}$

Ans : [Board Term-2, 2014]

$$\begin{aligned}\text{We have}\quad (x^2+y^2)(a^2+b^2) &= (ax+by)^2\\ x^2a^2+x^2b^2+y^2a^2+y^2b^2 &= a^2x^2+b^2y^2+2abxy\\ x^2b^2+y^2a^2-2abxy &= 0\\ (xb-ya)^2 &= 0\\ xb &= ya\end{aligned}$$

Thus $\frac{x}{a}=\frac{y}{b}$ Hence Proved.

23. The sum of ages (in years) of a son and his father is 35 years and product of theirs ages is 150 years, find their ages.

Ans : [Delhi Term-2, 2014, Term-2, 2012 Set (40)]

Let the age of father be x years and age of son be y years

$$\text{Now}\quad x+y = 35 \qquad (1)$$

$$\text{and}\quad xy = 150 \qquad (2)$$

Putting the value of y, from (1) we have

$$\begin{aligned}x(35-x) &= 150\\ x^2-35x+150 &= 0\\ (x-30)(x-5) &= 0\\ x &= 30, x=5 \text{ (Rejected)}\end{aligned}$$

Age of father cant be 5 years, so we reject $x=5$ and take $x=30$.

$$\text{Now}\quad y = 5$$

Hence the age of father is 30 years and the age of son is 5 years.

24. One fourth of a herd of camels was seen in forest. Twice of square root of the herd had gone to mountain and remaining 15 camels were seen on the bank of a river, find the total number of camels.

Ans : [Board Term-2, 2012 Set (1)]

Let the total number of camels be x.
According to the question,

$$\begin{aligned}\frac{x}{4}+2\sqrt{x}+15 &= x\\ 3x-8\sqrt{x}-60 &= 0\end{aligned}$$

Let $\sqrt{x}=y$, then we have

$$\begin{aligned}3y^2-8y-60 &= 0\\ 3y^2-18y+10y-60 &= 0\\ 3y(y-6)+10(y-6) &= 0\\ (3y+10)(y-6) &= 0\\ y &= 6 \text{ or } y=-\frac{10}{3}\end{aligned}$$

Here $y=-\frac{10}{3}$ is not possible.

Thus $y=6$ or $y^2=36$, $\quad x=y^2=36$

Hence the number of camels is 36.

25. The sum of the squares of two consecutive naturals is 421. Find the numbers.

[Board Term-2, 2012 Set (12)]

Ans : [CBSE Marking Scheme, 2012]

Let the first natural number be x. The second consecutive natural will be $.x+1$

According to the question,

$$\begin{aligned}x^2+(x+1)^2 &= 421\\ x^2+x^2+2x+1 &= 421\\ x^2+x-210 &= 0\\ x^2+15x-14x-210 &= 0\\ x(x+15)-14(x+15) &= 0\\ (x+15)(x-14) &= 0\\ x+15 &= 0 \text{ or } x-14=0\\ x &= -15 \text{ or } x=14\end{aligned}$$

Rejecting negative value $x=14$.

Therefore first number is 14 and consecutive number is 15.

26. In a class test, the sum of the marks obtained by a student in mathematics and science is 28. Had he got 3 marks more in mathematics and 4 marks less in science, the product of the marks would have been 180. Find his marks in two subjects.

Ans : [Board Term-2 2012, Set (21)]

Let marks obtained in maths be x, the marks obtained in science will be $28-x$

$$\begin{aligned}\text{Now}\quad (x+3)(28-x-4) &= 180\\ (x+3)(24-x) &= 180\\ 24x-x^2+72-3x &= 180\\ x^2-21x+108 &= 0\\ (x-9)(x-12) &= 0\\ x &= 9 \text{ or } x=12\end{aligned}$$

Case I : $x=9$

Marks obtained in maths $=9$

Marks obtained in science $=28-9=19$

Case II : $x=12$

Marks obtained in maths $=12$

Marks obtained in science $= 28 - 12 = 16$

27. If the roots of the equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, prove that $\frac{a}{b}, \frac{c}{d}$.

Ans : [Board 2016]

We have $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$A = (a^2+b^2), B = -2(ac+bd), C = (c^2+d^2)$

If roots are equal, $D = B^2 - 4AC = 0$

or $B^2 = 4AC$

Now $[-2(ac+bd)]^2 = 4(a^2+b^2)(c^2+d^2)$

$(a^2c^2 + 2abcd + b^2d^2) = 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$2abcd = a^2d^2 + b^2c^2$

$0 = a^2d^2 - 2abcd + b^2c^2$

$0 = (ad - bc)^2$

$0 = ad - bc$

$ad = bc$

$\frac{a}{b} = \frac{c}{d}$ Hence Proved

LONG ANSWER TYPE QUESTIONS

1. Solve for x : $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

Ans : [CBSE S.A.-2, 2016, Set-HODM4OL]

We have $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y$ then we have

$y^2 + 5y - 24 = 0$

$(y+8)(y-3) = 0$

$y = 3, -8$

Taking $y = 3$ we have

$\frac{2x}{x-5} = 3$

$2x = 3x - 15$

$x = 15$

Taking $y = -8$ we have

$\frac{2x}{x-5} = -8$

$2x = -8x + 40$

$10x = 40$

$x = 4$

Hence, $x = 15, 4$

2. Solve for x : $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$ $x \neq -1, -2, -4$

Ans : [Bord Term-2 O.D. Set I, 2016]

We have $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$

$\frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$

$(3x+4)(x+4) = 4(x^2+3x+2)$

$3x^2 + 16x + 16 = 4x^2 + 12x + 8$

$x^2 - 4x - 8 = 0$

Now $x = \frac{-b\sqrt{b^2-4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2 \times 1}$

$= \frac{4 \pm \sqrt{16+32}}{2}$

$= \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$

$= 2 \pm 2\sqrt{3}$

Hence, $x = 2 + 2\sqrt{3}$ and $2 - 2\sqrt{3}$

3. Find x in terms of a, b and c :

$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$

Ans : [Board Term-2, Delhi Set 1, 2016]

We have $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

$a(x-b)(x-c) + b(x-a)(x-c) = 2c(x-a)(x-b)$

$ax^2 - abx - acx + abc + bx^2 - bax - bcx + abc = 2cx^2 - 2cxb - 2cxa + 2abc$

$ax^2 + bx^2 - 2cx^2 - abx - acx - bax - bcx + 2cbx + 2acx = 0$

$x^2(a+b-2c) - 2abx + acx + bcx = 0$

$x^2(a+b-2c) + x(-2ab + ac + bc) = 0$

Thus $x = -\left(\frac{ac+bc-2ab}{a+b-2c}\right)$

4. Solve for x : $\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}; x =, -1, \frac{1}{4}$

Ans : [Delhi CBSE Board, 2015 (Set 3)]

We have $\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$

$\frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1}$

$$\frac{7x+1}{x^2-1} = \frac{29}{4x-1}$$

$$(7x+1)(4x-1) = 29x^2-29$$
$$28x^2-7x+4x-1 = 29x^2-29$$
$$-3x = x^2-28$$
$$x^2+3x-28 = 0$$
$$x^2+7x-4x-28 = 0$$
$$x(x+7)-4(x+7) = 0$$
$$(x+7)(x-4) = 0$$

Hence, $x = 4, -7$

5. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately.

Ans : [O. D. Set III, 2016]

Let time taken by pipe A be x minutes and time taken by pipe B be $x+5$ minutes.

In one minute pipe A will fill $\frac{1}{x}$ tank.

In one minute pipe B will fill $\frac{1}{x+5}$ tank.

Thus pipes $A+B$ will fill $\frac{1}{x}+\frac{1}{x+5}$ tank in one minute.

As per question, two pipes running together can fill a tank in $11\frac{1}{9} = \frac{100}{9}$ minutes, in one minute $\frac{9}{100}$ tank will be filled.

Now according to the question we have

$$\frac{1}{x}+\frac{1}{x+5} = \frac{9}{100}$$

$$\frac{x+5+x}{x(x+5)} = \frac{9}{100}$$

$$100(2x+5) = 9x(x+5)$$
$$200x+500 = 9x^2+45x$$
$$9x^2-155x-500 = 0$$
$$9x^2-180x+25x-500 = 0$$
$$9x(x-20)+25(x-20) = 0$$
$$(x-20)(9x+25) = 0$$
$$x = 20, \frac{-25}{9}$$

As time can't be negative we take $x = 20$ minutes

and $x+5 = 25$ minutes

Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

6. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction.

Ans : [Delhi Set III, 2016]

Let the speed while going be x km/h

Speed while returning $= (x+10)$ km/h

According to question we have

$$\frac{150}{x}-\frac{150}{x+10} = \frac{5}{2}$$

$$x^2+10x-600 = 0$$
$$(x+30)(x-20) = 0$$
$$x = 20$$

Speed while going is 20 km/h and speed while returning will be $= 20+10 = 30$ km/h

7. The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Ans : [Foreign Set III, 2016]

Let numerator be x then fraction will be $\frac{x}{2x+1}$

As per the question we have

$$\frac{x}{2x+1}+\frac{2x+1}{x} = 2\frac{16}{21} = \frac{58}{21}$$

$$21\left[x^2+(2x+1)^2\right] = 58(2x^2+x)$$

or, $11x^2-26x-21 = 0$

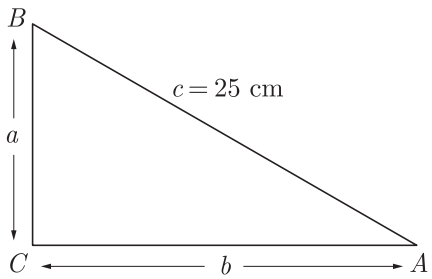
$$11x^2+33x+7x-20 = 0$$
$$x = 3, -\frac{7}{11} \text{ (rejected)}$$

We reject $x = -\frac{7}{3}$, thus $x = 3$ and fraction will be $\frac{3}{6+1} = \frac{3}{7}$

8. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

Ans : [Delhi Set II, 2016]

As per question statement figure is given below.



Here $a+b+c = 60, c = 25$

$$a+b = 60-c = 60-25 = 35$$

Using Pythagoras theorem

$$a^2+b^2 = 25^2 = 625$$

Substituting the values in $(a+b)^2 = a^2+b^2+2ab$,

$$35^2 = 625+2ab$$
$$1225-625 = 2ab$$

or, $ab = 300$

Hence, Area of ΔABC

$$\frac{1}{2}ab = 150 \text{ cm}^2.$$

9. Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank. Find the time in which each tap can separately fill the tank.

Ans : [Foreign Set III, 2016]

Let the tap with smaller diameter fills the tank in x hours, then the other tap fills the tank in $(x-8)$

hours

In one hour small tap will fill $\frac{1}{x}$ tank.

In one hour large tap will fill $\frac{1}{x-8}$ tank.

Thus both tap will fill $\frac{1}{x}+\frac{1}{x-8}$ tank in one hour.

9 hours 36 minutes$= 9+\frac{36}{60}=9+\frac{3}{5}=\frac{48}{3}$

Since two water taps together can fill a tank in $\frac{48}{3}$ hour, tank fill by both pipe in one hour is $\frac{1}{\frac{48}{5}}=\frac{5}{48}$.

Thus $\frac{1}{x}+\frac{1}{x-8}=\frac{5}{48}$

$$\frac{x-8+x}{x(x-8)}=\frac{5}{48}$$

$$5x(x-8)=(2x-8)48$$

$$5x^2-136+384=0$$

$$x=\frac{136\pm\sqrt{(136)^2-4\times5\times384}}{2.5}$$

$$=\frac{136\pm\sqrt{18496-7680}}{10}$$

$$x=\frac{136\pm104}{10}=24,\frac{16}{5}$$

There is no possibility of $x=\frac{16}{5}$ because it is less than 8 Hours.

Thus smaller tap can fill the tank in 24 hours and larger tap can fill in 24 hrs.

10. The denominator of a fraction is two more than its numerator. If the sum of the fraction and its reciprocal is $\frac{34}{15}$, find the fraction.

Ans : [Board Term-2, 2012 Set (1)]

Let numerator be x, then denominator will be $x+2$.

and fraction $=\frac{x}{x+2}$

Now $\frac{x}{x+2}+\frac{x+2}{x}=\frac{34}{15}$

$$15(x^2+x^2+4x+4)=34(x^2+2x)$$

$$30x^2+60x+60=34x^2+68x$$

$$4x^2+8x-60=0$$

$$x^2+2x-15=0$$

$$x^2+5x-3x-15=0$$

$$x(x+5)-3(x+5)=0$$

$$(x+5)(x-3)=0$$

We reject the $x=-5$. Thus $x=3$ and fracrtion $=\frac{3}{5}$

11. Solve for x : $\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{33}{5x};x\neq0,-1,2$

Ans : [Board Term-2, Delhi 2015, Set I, II]

We have $\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}$

$$\frac{2x}{x+1}+\frac{3x}{2(x-2)}=\frac{23}{5}$$

$$\frac{2\times2x(x-2)+3x(x+1)}{2(x+1)(x-2)}=\frac{23}{5}$$

$$\frac{4x^2-8x+3x^2+3x}{2(x^2-x-2)}=\frac{23}{5}$$

$$\frac{7x^2-5x}{2(x^2-x-2)}=\frac{23}{5}$$

$$35x^2-25x=46x^2-46x-92$$

$$11x^2-21x-92=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{21\pm\sqrt{(-21)^2-4(11)(-92)}}{2\times11}$$

$$=\frac{21\pm\sqrt{441+4048}}{22}$$

$$=\frac{21\pm67}{22}$$

$$x=\frac{21+67}{22}\text{ or }x=\frac{21-67}{22}$$

Thus $x=4,\frac{-23}{11}$

12. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

Ans : [Board Term-2, Delhi, 2015 Set I, III]

Let the denominator be x, then numerator will $x-3$

So the fraction will be $\frac{x-3}{x}$

By the given condition, new fraction will

$$\frac{x-3+2}{x+2}=\frac{x-1}{x+2}$$

Now $\frac{x-3}{x}+\frac{x-1}{x+2}=\frac{29}{20}$

$$20[(x-3)(x+2)+x(x-1)]=29(x^2+2x)$$

$$20(x^2-x-6+x^2-x)=29x^2+58x$$

$$20(2x^2-2x-6)=29x^2+58x$$

$$40x^2-40x-240=29x^2+58x$$

$$11x^2-98x-120=0$$

$$11x^2-110x+12x-120=0$$

$$(11x+20)(x-10)=0,$$

We take $x=10$ and fraction will be is $\frac{10-3}{10}=\frac{7}{10}$.

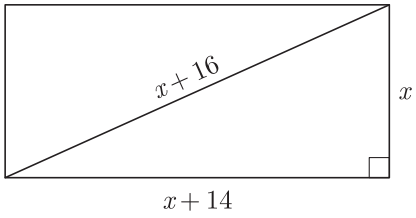
13. The diagonal of a rectangular field is 16 metre more than the shorter side. If the longer side is 14 metre more than shorter side, then find the length of the of the field.

Ans : [Board Term, O.D., 2015 Set I, II, III]

Let the length of shorter side be x m.

Length of diagonal $=(x+16)$ m

and, Length of longer side $=(x+14)$ m



Now as per question we have

$$\begin{aligned} x^2+(x+14)^2 &= (x+16)^2 \\ x^2+x^2+28x+196 &= x^2+32x+256 \\ x^2-4x-60 &= 0 \\ x^2+6x-10x-60 &= 0 \\ x(x+6)-10(x+6) &= 0 \\ x &= -6, x=10 \end{aligned}$$

As lenght can't be negative $x = 10$ m. Therefore length of sides are 10 m and 24 m.

14. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ?

Ans : [Board Term-2, O.D., 2015 Set I, II]

Let the speed of the train be x km/hr. for first 54 km. and for next 63 km, speed $= (x+6)$ km/hr.
According to the question

$$\frac{54}{x}+\frac{63}{x+6} = 3$$

$$\frac{54(x+6)+63x}{x(x+6)} = 3$$

$$\begin{aligned} 54x+324+63x &= 3x(x+6) \\ 117x+324 &= 3x^2+18x \\ 3x^2-99x-324 &= 0 \\ x^2-33x-108 &= 0 \\ x^2-36x+3x-108 &= 0 \\ x(x-36)+3(x-36) &= 0 \\ (x-36)(x+3) &= 0 \\ x &= -3, 36 \end{aligned}$$

Negative value is rejected, thus first speed of train is 36 km/h.

15. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

Ans : [Board Term-2, O.D., 2015, Set II]

Let the average speed of the truck be x km/hr. for first 150 km and for next 200 km, speed $(x+20)$ km/hr.

Now $$\frac{150}{x}+\frac{200}{x+20} = 5$$

$$\begin{aligned} 150x+3000+200x &= 5x(x+20) \\ x^2-50x-600 &= 0 \\ x^2-60x+10x-600 &= 0 \\ x(x-60)+10(x-60) &= 0 \\ (x-60)(x+10) &= 0 \end{aligned}$$

Negative value is rejected, thus first speed of truck is 60 km/h.

16. The total cost of a certain length of cloth is Rs 200. If the piece was 5 m longer and each metre of cloth coast Rs 2 less, the cost of the piece would have remained unchaged. How longer is the piece and what is its original rate per metre ?

Ans : [Foreign Set I, II, 2015]

Let the length of the cloth be x m.

$$\text{cost per metre} = \frac{200}{x}$$

$$\text{New length of the cloth} = (x+5) \text{ m}$$

$$\text{New cost per metre} = \left(\frac{200}{x}-2\right)$$

Since cost of the piece have remained unchanged,

$$(x+5)\left(\frac{200}{x}-2\right) = 200$$

$$\frac{(x+5)(200-2x)}{x} = 200$$

$$\begin{aligned} 200x-2x^2+1000-10x &= 200x \\ x^2+5x-500 &= 0 \\ (x+25)(x-20) &= 0 \\ x &= 20, 25 \end{aligned}$$

Negative value is rejected, thus length of the piece is 20 m.

Original cost per metre is $\frac{200}{20} = 10$ Rs.

17. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans : [CBSE O.D. 2014]

Let the speed of stream be x km/h
Then the speed of boat upstream $= (18-x)$ km/h
Speed of boat downstream $= (18+x)$ km/h
According to the question,

$$\frac{24}{18-x}-\frac{24}{18+x} = 1$$

$$\frac{24[(18+x)-(18-x)]}{18^2-x^2} = 1$$

$$\begin{aligned} 48x &= 324-x^2 \\ x^2+48x-324 &= 0 \\ x^2+54x-6x-324 &= 0 \\ x(x+54)-6(x+54) &= 0 \\ (x+54)(x-6) &= 0 \\ x+54 &= 0, x-6=0 \\ x &= -54, x=6 \end{aligned}$$

Since speed cannot be negative, we reject $x = -54$.

The speed of steam is 6 km/h.

18. Solve for x : $\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3}; x \neq 4,6$

Ans :

We have $\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3}$

$$\frac{(x-3)(x-6)+(x-4)(x-5)}{(x-4)(x-6)}=\frac{10}{3}$$

$$\frac{x^2-9x+18+x^2-9x+20}{x^2-10x+24}=\frac{10}{3}$$

$$\begin{aligned}3(2x^2-18x+38) &= 10x^2-100x+240\\ 6x^2-54x+114 &= 10x^2-100x+240\\ 4x^2-46x+126 &= 0\\ 2x^2-23x+63 &= 0\\ 2x^2-14x-9x+63 &= 0\\ 2x(x-7)-9(x-7) &= 0\\ (2x-9)(x-7) &= 0\\ 2x-9=0, x-7 &= 0\\ x &= \tfrac{9}{2}, x=7\end{aligned}$$

19. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans : [Board Term-2, O.D., Set II 2016]

Ans :

Let the speed of stream be x km/h
Then the speed of boat upstream $=(24-x)$ km/h
Speed of boat downstream $=(24+x)$ km/h
According to the question,

$$\frac{32}{24-x}-\frac{32}{24+x}=1$$

$$32\left[\frac{1}{24-x}-\frac{1}{24+x}\right]=1$$

$$32\left[\frac{24+x-24+x}{576-x^2}\right]=1$$

$$\begin{aligned}32(24+x-24+x) &= 576-x^2\\ 64x &= 576-x^2\\ x^2+64x-576 &= 0\\ x^2+72x-8x-576 &= 0\\ x(x+72)-8(x+72) &= 0\\ (x-8)(x+72) &= 0\\ x &= 8,-72\end{aligned}$$

Since speed cannot be negative, we reject $x=-72$.

The speed of steam is 8 km/h.

20. A student scored a total of 32 marks in class tests in mathematics ans science. Had he scored 2 marks less in science and 4 more in mathematics, the product of his marks would have been 253. Find his marks in two subjects.

Ans : [Board Term-2, 2012 Set (50)]

Let marks in Mathematics be x, then marks in science will be $32-x$

According to question,

$$\begin{aligned}(32-x-2)(x+4) &= 253\\ (30-x)(x+4) &= 253\\ 26x-x^2+120 &= 253\\ x^2-26x+133 &= 0]\text{or},\\ x^2-19x-7x+133 &= 0\\ x(x-19)-7(x-19) &= 0\end{aligned}$$

or, $x=7$ or $x=19$

If $x=7$ then marks in mathematics $=7$, and marks in science $=25$
If $x=19$, then marks in mathematics $=19$ and marks in science $=13$.

21. The sum of squares of two consecutive multiples of 7 is 637. Find the multiples.

Ans : [Foreign Set II, 2014]

Let $7x$ and $7x+7$ be two consecutive multiples of 7.

According to question,

$$\begin{aligned}(7x)^2+(7x+7)^2 &= 637\\ 49x^2+49x^2+49+98x &= 637\\ 98x^2+98x-588 &= 0\\ x^2+x-6 &= 0\\ (x+3)(x-2) &= 0\\ x &= -3,2\end{aligned}$$

Neglecting negative value, $x=2$

Therefore multiples are 14 and 21.

22. Solve for x : $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$ where $x \neq -\frac{1}{2},1$

Ans : [Out Side Delhi Set-III 2017]

We have $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=0$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1}=\frac{1}{y}$
Substituting this value we obtain

$$y+\frac{1}{y}=2$$

$$\begin{aligned}y^2+1 &= 2y\\ y^2-2y+1 &= 0\\ (y-1)^2 &= 0\\ y &= 1\end{aligned}$$

Putting $y=\frac{x-1}{2x+1}$ we have

$$\frac{x-1}{2x+1}=1 \text{ or } x-1=2x+1$$

or $x=-2$

23. Find for x : $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}; x \neq 0,1,2$

Ans : [Board Outside Delhi Compt. Set-I, II 2017]

We have $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}$

$$\frac{x-1+2x-4}{(x-2)(x-1)}=\frac{6}{x}$$

$$3x^2 - 5x = 6x^2 - 18x + 12$$
$$3x^2 - 13x + 12 = 0$$
$$3x^2 - 4x - 9x + 12 = 0$$
$$x(3x-4) - 3(3x-4) = 0$$
$$(3x-4)(x-3) = 0$$
$$x = \frac{4}{3} \text{ and } 3$$

Hence, $x = 3, \frac{4}{3}$

24. Solve, for x : $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

Ans : [Board Foreign II, III 2017]

We have
$$\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$$
$$\sqrt{3}\,x^2 + 3x + 7x + 7\sqrt{3} = 0$$
$$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$$
$$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$$
$$x = -\sqrt{3} \text{ and } x = \frac{-7}{\sqrt{3}}$$

Hence roots $x = -\sqrt{3}$ or $x = \frac{-7}{\sqrt{3}}$

25. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers

Ans : [Board Term-2, 2014, Delhi]

Let the first number be x, then second number will be $x+5$

Now according to the question
$$\frac{1}{x} - \frac{1}{x+5} = \frac{1}{10}$$
$$\frac{x+5-x}{x(x+5)} = \frac{1}{10}$$
$$50 = x^2 + 5x$$
$$x^2 + 5x - 50 = 0$$
$$x^2 + 10x - 5x - 50 = 0$$
$$x(x+10) - 5(x+10) = 0$$
$$(x+10)(x-5) = 0$$
$$x = 5, -10$$

Rejecting the negative value, numbers are 5 and 10.

26. The sum of squares of two consecutive even numbers is 340. Find the numbers.

Ans : [Foreign Set I, 2014]

Ans :

Let the number be x and $x+2$

Now
$$(x)^2 + (x+2)^2 = 340$$
$$x^2 + x^2 + 4 + 4x = 340$$
$$2x^2 + 4x - 336 = 0$$
$$x^2 + 2x - 168 = 0$$
$$(x+14)(x-12) = 0$$
$$x = 12$$

Thus numbers are 12 and 14.

27. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Ans : [Foreign Set I, 2014] [Board Term-2, 2012 Set(12)]

Let the odd number be $2x+1$, then consecutive odd number will be $2x+1+2 = 2x+3$

Now, according to question
$$(2x+1)^2 + (2x+3)^2 = 394$$
$$4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 394$$
$$8x^2 + 16x - 384 = 0$$
$$x^2 + 2x - 48 = 0$$
$$x^2 + 8x - 6x - 48 = 0$$
$$x(x+8) - 6(x+8) = 0$$
$$x = -8, 6$$

Rejecting the negative value,
$$\text{Ist number} = 2 \times 6 + 1 = 13$$
$$\text{and second odd number} = 15$$

28. Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares.

Ans : [Board Term-2, 2013]

Let the sides of two squares be a and b,

then
$$a^2 + b^2 = 400 \quad (1)$$
and
$$4(a-b) = 16$$
$$a - b = 4$$
$$a = 4 + b$$

From equations (1) and (2), we obtain
$$(4+b)^2 + b^2 = 400$$
$$16 + b^2 + 8b = b^2 = 400$$
$$2b^2 + 8b - 384 = 0$$
$$b^2 = 4b - 192 = 0$$
$$b^2 + 16b - 12b - 192 = 0$$
$$b(b+16) - 12(b+16) = 0$$
$$(b+16)(b-12) = 0$$
$$b = -16, 12$$

Rejecting the negative value, $b = 12$ cm

then
$$a = 4 + 12 = 16 \text{ cm.}$$

29. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Ans : [Board Term-2, 2012, Set(22)]

Let the usual speed of train be x km/hr.

According to question we have

$$\frac{300}{x}-\frac{300}{x+5}=2$$

$$x^2+5x-750=0$$
$$x^2=30x-25x-750=0$$
$$(x=30)(x-25)=0$$
$$x=-30 \text{ or } x=25$$

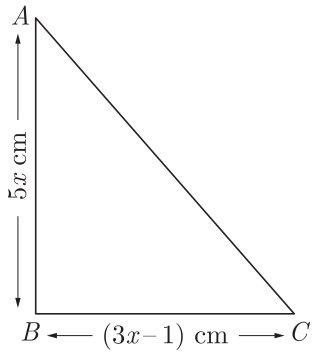
Since, speed cannot be negative $x \neq -30$.

Thus speed of train is 25 km/hr.

30. The length of the sides forming right angle of a right triangle are $5x$ cm and $(3x-1)$ cm. If the area of the triangle is 60 cm^2. Find its hypotenuse.

Ans : [Board Term-2, 2012 Set (44)]

According to the question we have drawn figure below.



Now Area of triangle $=\frac{1}{2}\times$ base $\times$ height

$$60=\frac{1}{2}\times 5x\times(3x-1)$$
$$15x^2-5x=120$$
$$3x^2-x-24=0$$
$$3x^2-9x+8x-24=0$$
$$3x(x-3)+8(x-3)=0$$
$$(x-3)(3x+8)=0$$
$$x=3, x=-\frac{8}{3}$$

Length can't be negative, so $x=3$

Now $AB=5\times 3=15$ cm,

$BC=3x-1=9-1=8$ cm

Now $AC=\sqrt{15^2+8^2}$

$=\sqrt{225+64}$

$=\sqrt{289}=17$ cm.

Hence hypotenuse $=17$ cm.

31. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

Ans : [Board Term-2, 2012 Set (5)]

Suppose B alone finish the work in x days and A alone takes $(x-6)$ days.

B's one day work $=\frac{1}{x}$

A' one day work $=\frac{1}{x-6}$

and (A+B)'s one day work $=\frac{1}{4}$

According to the question,

$$\frac{1}{x}+\frac{1}{x-6}=\frac{1}{4}$$

$$x^2-14x+24=0$$
$$x^2-12x-2x+24=0$$
$$x(x-12)-2(x-12)=0$$
$$(x-12)(x-2)=0$$
$$x=12 \text{ or } x=2$$

,

But x cannot be less than 6. So $x=12$

Hence B can finish the work in 12 days.

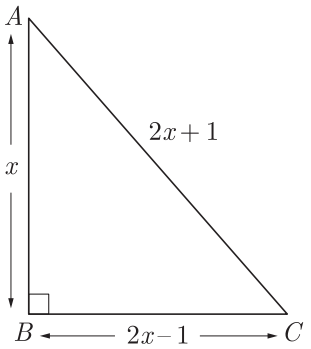
32. The length of the hypotenuse of a right triangle exceeds the length of its base by 2 cm and exceeds twice the length of altitude by 1 cm. Find the length of each side of the triangle.

Ans : [Board Term-2, 2012 Set (12)]

et altitude of triangle be x.

Hypotenuse of triangle $=2x+1$

and base of triangle $=2x-1$



Using Pythagoras theorem,

$$(2x+1)^2=x^2+(2x-1)^2$$
$$4x^2+1+4x=x^2+4x^2+1-4x$$
$$x^2-8x=0$$
$$x(x-8)=0$$

Rejecting $x=0$, we get $x=8$

Thus altitude of triangle is 8 cm

Hypotenuse of triangle is $2\times 8+1=17$ cm

and base of triangle is $2\times 8-1=15$ cm

33. The perimeter of a rectangular field is 82 m and its area is 400 square metre. Find the length and breadth of the rectangle.

Ans : [Board Term-2, 2012 Set (21)]

We have Perimeter $=2(l+b)=82$ m

or, $l+b=41$ m

Let length be x m, then breadth $=(41-x)$ m.

$$\text{Area} = l \times b = 400 \text{ m}^2$$
$$x(41-x) = 400$$
$$41x - x^2 = 400$$
$$x^2 - 41x + 400 = 0$$
$$(x-16)(x-25) = 0$$
$$x = 16 \text{ or } x = 25$$

If length is 16 m, then breadth will be 25 m.
If length is 25 m, then breadth will be 16 m.

34. The product of Tanay's age (in years) five years ago and his age ten years later is 16. Determine Tanay's present age.

Ans : [Board Term-2, 2012 Set (31)]

Let the present age of Tanay's be x years.

According to question we have

$$(x-5)(x+10) = 16$$
$$x^2 + 5x - 50 = 16$$
$$x^2 + 5x - 66 = 0$$
$$x^2 + 11x - 6x - 66 = -66$$
$$x(x+11) - 16(x-11) = 0$$
$$(x+11)(x-6) = 0$$
$$x = -11, 6.$$

As age cannot be negative, we reject $x = -11$. Thus present age of Tanay is 6 years.

35. Solve for x : $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}; x \neq 0.2$

Ans : [Delhi Compt. Set-I 2017]

We have $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

$$\frac{x(x+3)-(1-x)(x-2)}{x(x-2)} = \frac{17}{4}$$
$$\frac{(x^2+3x)-(-x^2+3x-2)}{x^2-2x} = \frac{17}{4}$$
$$\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$$
$$8x^2 + 8 = 17x^2 - 34x$$
$$9x^2 - 34x - 8 = 0$$
$$9x^2 - 36x + 2x - 8 = 0$$
$$9x(x-4) + 2(x-4) = 0$$
$$(x-4)(9x+2) = 0$$
$$x = 4 \text{ or } x = -\frac{2}{9}$$

Hence, $x = 4, -\frac{2}{9}$

36. Solve for x : $4x^2 + 4bx - (a^2 - b^2) = 0$

Ans : [Board Foreign Set- III 2017]

We have $4x^2 + 4bx - (a^2 - b^2) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = 4, B = 4b \text{ and } C = b^2 - a^2$$
$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$= \frac{-4b \pm \sqrt{(4b)^2 - 4.4(b^2 - a^2)}}{2.4}$$
$$= \frac{-4b \pm \sqrt{16b^2 - 16b^2 + 16a^2}}{8}$$
$$= \frac{-4b \pm 4a}{8}$$
$$= -\frac{(a+b)}{2}, \frac{(a-b)}{2}$$

Hence the roots are $-\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$

SHORT ANSWER TYPE QUESTIONS - I

1. Find k so that the quadratic equation $(k+1)x^2 - 2(k+1)x + 1 = 0$ has equal roots.

Ans : [Board Term-2, 2016 Set HODM40L]

We have $(k+1)x^2 - 2(k+1)x + 1 = 0$

Compare with $Ax^2 + Bx + C = 0$ we get
$A = (k+1), B = -2(k+1), C = 1$
If roots are equal, $D = 0$, i.e.

$$B^2 = 4AC$$
$$4(k+1)^2 = 4(k+1)$$
$$k^2 + 2k + 1 = k + 1$$
$$k^2 + k = 0$$
$$k(k+1) = 0$$
$$k = 0, -1$$

$k = -1$ does not satisfy the equation, thus $k = 0$

2. If 2 is a root or the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

Ans : [Board Sample Paper 2016]

We have $x^2 + kx + 12 = 0$

If 2 is the root of above equation, it must satisfy it.

$$(2)^2 + 2k + 12 = 0$$
$$2k + 16 = 0$$
$$k = -8$$

Substituting $k = -8$ in $x^2 + kx + q = 0$ we have

$$x^2 - 8x + q = 0$$

For equal roots,

$$(-8)^2 - 4(1)q = 0$$
$$64 - 4q = 0$$
$$4q = 64$$
$$q = 16$$

3. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has euqal roots.

Ans : [CBSE Delhi, O.D. 2014]

We have $9x^2 - 3kx + k = 0$

Compare with $ax^2 + bx + c = 0$ we get

$$a = 9, b = -3k, c = k$$

Since roots of the equation are equal, $b^2 - 4ac = 0$

$$(-3k)^2-(4\times 9\times k)=0$$
$$9k^2-36k=0$$
$$k^2-4k=0$$
$$k(k-4)=0$$
$$k=0 \text{ or } k=4$$

Hence, $k=4$.

4. If the equation $kx^2-2kx+6=0$ has equal roots, then find the value of k.

Ans : [Board Term-2, 2012 Set (22)]

We have $kx^2-2kx+6=0$

Compare with $ax^2+bx+c=0$ we get

$a=k, b=-2k, c=6$

Since roots of the equation are equal, $b^2-4ac=0$

$$(-2k)^2-4(k)(6)=0$$
$$4k^2-24k=0$$
$$4k(k-6)=0$$
$$k=0,6$$

But $k\neq 0$, as coefficient of x^2 can not be zero

$k=6$

5. Find the values of p for which the quadratic equation $4x^2+px+3=0$ has equal roots.

Ans : [Board Term-2, 2014]

We have $4x^2+px+3=0$

Compare with $ax^2+bx+c=0$ we get

$a=4, b=p, c=3$

Since roots of the equation are equal, $b^2-4ac=0$

$$p^2-4\times 4\times 3=0$$
$$p^2-48=0$$
$$p^2=48$$
$$p=\pm 4\sqrt{3}$$

6. Find the nature of the roots of the quadratic equation :

$13\sqrt{3}\,x^2+10x+\sqrt{3}=0$

Ans : [Board Term-2, 2012, (12)]

We have $13\sqrt{3}\,x^2+10x+\sqrt{3}=0$

Compare with $ax^2+bx+c=0$ we get

$$a=13\sqrt{3}, b=10, c=\sqrt{3}$$
$$b^2-4ac=(10)^2-4(13\sqrt{3})(\sqrt{3})$$
$$=100-156$$
$$=-56$$

As $D<0$, the equation has not real roots.

SHORT ANSWER TYPE QUESTIONS - II

1. If 2 is a root of the quadratic equation $3x^2+px-8=0$ and the quadratic equation $4x^2-2px+k=0$ has equal roots, find k.

Ans : [Foreign Set II, 2014]

We have $3x^2+px-8=0$

Since 2 is a root of above equation, it must satisfy it. Substituting $x=2$ in $3x^2+px-8=0$ we have

$$12+2p-8=0$$
$$p=-2$$

Since $4x^2-2px+k=0$ has equal roots,

or $4x^2+4x+k=0$ has equal roots,

$$D=b^2-4ac=0$$
$$4^2-4(4)(k)=0$$
$$16-16k=0$$
$$16k=16$$

Thus $k=1$

2. For what value of k, the roots of the quadratic equation $kx(x-2\sqrt{5})+10=0$ are equal ?

Ans : [Delhi CBSE, Term-2, 2014] [Delhi 2013]

We have $kx(x-2\sqrt{5})+10=0$

or, $kx^2-2\sqrt{5}\,kx+10=0$

Compare with $ax^2+bx+c=0$ we get

$a=k, b=-2\sqrt{5}, c=10$

Since, roots are equal, $D=b^2-4ac=0$

$$(-2\sqrt{5}\,k)^2-4\times k\times 10=0$$
$$20k^2-40k=0$$
$$20k(k-2)=0$$
$$k(k-2)=0$$

Since $k\neq 0$, we get $k=2$

3. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them : $3x^2-4\sqrt{3}\,x+4=0$

Ans : [Board Term-2, 2012 Set (5)]

We have $3x^2-4\sqrt{3}\,x+4=0$

Compare with $ax^2+bx+c=0$ we get

$a=3k, b=-4\sqrt{3}, c=4$

$$b^2-4ac=(-4\sqrt{3})^2-4(3)(4)$$
$$=48-48=0$$

Thus roots are real and equal.

Roots are $\left(-\frac{b}{2a}\right),\left(-\frac{b}{2a}\right)$ or $\frac{2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}$

4. Determine the positive value of 'k' for which the equation $x^2+kx+64=0$ and $x^2-8x+k=0$ will both have real and equal roots.

Ans : [Board Term-2, 2012 Set (44)] [Delhi CBSE Term-2, 2014]

We have $x^2+kx+64=0$

Compare with $ax^2+bx+c=0$ we get

$a=1, b=k, c=64$

For real and equal roots, $b^2-4ac=0$

Thus $k^2-4\times 1\times 64=0$

$$k^2-256=0$$
$$k=\pm 16$$

(1)

Now for equation $x^2-8x+k=0$ we have

$$\begin{aligned} b^2-4ac &= 0 \\ (-8)^2-4\times 1\times k &= 0 \\ 64 &= 4k \\ k &= \frac{64}{4}=16 \end{aligned}$$

(2)

From (1) and (2), we get $k=16$. Thus for $k=16$, given equations have equal roots.

5. Find that non-zero value of k, for which the quadratic equation $kx^2+1-2(k-1)x+x^2=0$ has equal roots. Hence find the roots of the equation.

Ans : [Delhi CBSE Board Term-2, 2015, Set I, III]

We have $kx^2+1-2(k-1)x+x^2=0$

$$(k+1)^2-2(k-1)x+1=0$$

Compare with $ax^2+bx+c=0$ we get

$$a=k+1, b=-2(k-1), c=1$$

For real and equal roots, $b^2-4ac=0$

$$\begin{aligned} 4(k-1)^2-4(k+1)\times 1 &= 0 \\ 4k^2-8k+4-4k-4 &= 0 \\ 4k^2-12k &= 0 \\ 4k(k-3) &= 0 \end{aligned}$$

As k can't be zero, thus $k=3$.

6. Find the value of k for which the quadratic equation $(k-2)x^2+2(2k-3)x+(5k-6)=0$ has equal roots.

Ans : Board Term-2, 2015]

We have $(k-2)x^2+2(2k-3)x+(5k-6)=0$

Compare with $ax^2+bx+c=0$ we get

$$a=k-2, b=2(2k-3), c=(5k-6)$$

For real and equal roots, $b^2-4ac=0$

$$\begin{aligned} \{2(2k-3)\}^2-4(k-2)(5k-6) &= 0 \\ 4(4k^2-12k+9)-4(k-2)(5k-6) &= 0 \\ 4k^2-12k+9-5k^2+6k+10k-12 &= 0 \\ k^2-4k+3 &= 0 \\ k^2-3k-k+3 &= 0 \\ k(k-3)-1(k-3) &= 0 \\ (k-3)(k-1) &= 0 \end{aligned}$$

Thus $k=1,3$

7. If the roots of the quadratic equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal, prove that $2a=b+c$.

Ans : [Outside Delhi, Set-II, 2016]

We have $(a-b)x^2+(b-c)x+(c-a)=0$

Compare with $ax^2+bx+c=0$ we get

$$a=(a-b,), b=(b-c), c=c-a$$

For real and equal roots, $b^2-4ac=0$

$$\begin{aligned} (b-c)^2-4(a-b)(c-a) &= 0 \\ b^2+c^2-2bc-4(ac-a^2-bc+ab) &= 0 \\ b^2+c^2-2bc-4ac+4a^2+4bc-4ab &= 0 \\ 4a^2+b^2+c^2+2bc-4ab-4ac &= 0 \end{aligned}$$

Using $a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2$ we have

$$\begin{aligned} (-2a+b+c)^2 &= 0 \\ -2a+b+c &= 0 \end{aligned}$$

Hence, $b+c=2a$

8. If the quadratic equation, $(1+a^2)b^2x^2+2abcx+(c^2-m^2)=0$ in x has equal roots, prove that $c^2=m^2(1+a^2)$

Ans : [Board Term-2, 2014]

We have $(1+a^2)b^2x^2+2abcx+(c^2-m^2)=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=(1+a^2), B=2abc, C=(c^2-m^2)$$

If roots are equal, $B^2-4AC=0$

$$\begin{aligned} (2abc)^2-4(1+a^2)b^2(c^2-m^2) &= 0 \\ 4a^2b^2c^2-(4b^2+4a^2b^2)(c^2-m^2) &= 0 \\ 4a^2b^2c^2-[4b^2c^2-4b^2m^2+4a^2b^2c^2-4a^2b^2m^2] &= 0 \\ 4a^2b^2c^2-4b^2c^2+4b^2m^2-4a^2b^2c^2+4a^2b^2m^2 &= 0 \\ 4b^2[a^2m^2+m^2-c^2] &= 0 \\ c^2 &= a^2m^2+m^2 \\ c^2 &= m^2(1+a^2) \end{aligned}$$

9. If -3 is a root of quadratic equation $2x^2+px-15=0$, while the quadratic equation $x^2-4px+k=0$ has equal roots. Find the value of k.

Ans : [Outside Delhi Compt. Set II, III 2017]

Given -3 is a root of quadratic equation

We have $2x^2+px-15=0$

Since 3 is a root of above equation, it must satisfy it. Substituting $x=3$ in above equation we have

$$\begin{aligned} 2(-3)^2+p(-3)-15 &= 0 \\ 2\times 9-3p-15 &= 0 \\ p &= 1 \end{aligned}$$

Since $x^2-4px+k=0$ has equal roots,

or $x^2-4x+k=0$ has equal roots,

$$\begin{aligned} b^2-4ac &= 0 \\ 4^2-4k &= 0 \\ 16-4k &= 0 \\ 4k &= 16 \\ k &= 4 \end{aligned}$$

10. If $ad\neq bc$, then prove that the equation $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$ hs no real roots.

Ans : [Board outside Delhi Set-I 2017]

We have $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=(a^2+b^2), B=2(ac+bd) \text{ and } C=(c^2+d^2)$$

For no real roots, $D=B^2-4AC<0$

$$\begin{aligned}D &= B^2 - 4AC\\ &= \left[2(ac+bd)\right]^2 - 4(a^2+b^2)(c^2+d^2)\\ &= 4\left[a^2c^2 + 2abcd + b^2d^2\right] - 4\left[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2\right]\\ &= 4\left[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2\right]\\ &= -4\left[a^2d^2 + b^2c^2 - 2abcd\right]\\ &= -4(ad-bc)^2\end{aligned}$$

Since $ad \neq bc$, therefore $D \neq 0$ and always negative. Hence the equation has no real roots.

11. Find the value of c for which the quadratic equation $4x^2 - 2(c+1)x + (x+1) = 0$ has equal roots.

Ans : [Delhi Comp. Set-III 2017]

We have $4x^2 - 2(c+1)x + (x+1) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = 4, B = 2(c+1), C = (c+1)$$

If roots are equal, $B^2 - 4AC = 0$

$$\begin{aligned}[2(c+1)]^2 - 4 \times 4(c+1) &= 0\\ 4(c^2 + 2c + 1) - 4(4c+4) &= 0\\ 4(c^2 + 2c + 1 - 4c - 4) &= 0\\ c^2 - 2c - 3 &= 0\\ c^2 - 3c + c - 3 &= 0\\ c(c-3) + 1(c-3) &= 0\\ (c-3)(c+1) &= 0\\ c &= 3, -1\end{aligned}$$

Hence for equal roots $c = 3, -1$.

12. Show that if the roots of the following equation are equal that $ad = bc$ or $\frac{a}{b} = \frac{c}{d}$.

$$x^2(a^2+b^2) + 2(ac+bd)x + c^2 + d^2 = 0$$

Ans : [Delhi Compt. Set I, II 2017 Outside Delhi Set II]

We have $x^2(a^2+b^2) + 2(ac+bd)x + c^2 + d^2 = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = a^2 + b^2, B = 2(ac+bd), C = c^2 + d^2$$

If roots are equal, $B^2 - 4AC = 0$

$$\begin{aligned}\left[2(ac+bd)\right]^2 - 4(a^2+b^2)(c^2+d^2) &= 0\\ 4(a^2c^2 + 2abcd + b^2d^2) - 4(a^2c^2 + a^2b^2 + b^2c^2 + b^2d^2) &= 0\\ 4(a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2) &= 0\\ -4(a^2d^2 + b^2c^2 - 2abcd) &= 0\\ (ad-bc)^2 &= 0\\ ad &= bc\\ \frac{a}{b} &= \frac{c}{d}\end{aligned}$$ Hence Proved.

LONG ANSWER TYPE QUESTIONS

1. If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal.

Ans : [Foreign Set II, 2016]

We have $x^2 + 2px + mn = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = 1, B = 2p, C = mn$$

If roots are equal, $B^2 - 4AC = 0$

$$4p^2 - 4mn = 0$$

or, $$p^2 = mn \quad (1)$$

Now we have

$$x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = 1, B = -2(m+n), C = (m^2 + n^2 + 2p^2)$$

If roots are equal, $B^2 - 4AC = 0$

$$\begin{aligned}4(m+n)^2 - 4(m^2 + n^2 + 2p^2) &= 0\\ m^2 + n^2 + 2mn - m^2 - n^2 - 2p^2 &= 0\\ 2mn - 2p^2 &= 0\\ p^2 &= mn\end{aligned}$$

Thus if roots of $x^2 + 2px + mn = 0$ are equal then those of $x^2 - 2a(m+n)x + (m^2n^2 + 2p^2) = 0$ are also equal.

2. Find the positive values of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have the real roots.

Ans : [Foreign Set I-2016]

(i) For $x^2 + kx + 64 = 0$ to have real roots

$$\begin{aligned}k^2 - 256 &\geq 0\\ k^2 &\geq 256\\ k &\geq 16 \text{ or } k < -16\end{aligned}$$

(ii) For $x^2 - 8x + k = 0$ to have real roots

$$\begin{aligned}64 - 4k &\geq 0\\ 16 - k &\geq 0\\ 16 &\geq k\end{aligned}$$

For (i) and (ii) to hold simultaneously

$$k = 16$$

3. Find the positive value of k for which $x^2 - 8x + k = 0$, will have real roots.

Ans : [Board Term-2, 2014]

We have $x^2 - 8x + k = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = 1, B = -8, C = k$$

If roots are equal, $B^2 - 4AC = 0$

Since the given equation has real roots, $B^2 - 4AC > 0$

$$\begin{aligned}(-8)^2 - 4(1)(k) &\geq 0\\ 64 - 4k &\geq 0\\ 16 - k &\geq 0\\ 16 &\geq k\end{aligned}$$

Thus $k \leq 16$

4. Find the values of k for which the equation $(3k+1)^2 + 2(k+1)x + 1$ has equal roots. Also find the roots.

Ans : [Board Term-2, 2014]

We have $(3k+1)^2 + 2(k+1)x + 1$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = (3k+1), B = 2(k+1), C = 1$$

If roots are equal, $B^2-4AC=0$

$$\begin{aligned}\left[2(k+1)\right]^2-4(3k+1)(1) &= 0\\ 4(k^2+2k+1)-(12k+4) &= 0\\ 4k^2+8k+4-12k-4 &= 0\\ 4k^2-4k &= 0\\ 4k(k-1) &= 0\\ k &= 0,1.\end{aligned}$$

Substituting $k=0$, in the given equation,

$$\begin{aligned}x^2+2x+1 &= 0\\ (x+1)^2 &= 0\\ x &= -1\end{aligned}$$

Again substituting $k=1$, in the given equation,

$$\begin{aligned}4x^2=4x+1 &= 0\\ (2x+1)^2 &= 0\end{aligned}$$

or, $$x = -\frac{1}{2}$$

Hence, roots $=-1,-\frac{1}{2}$

5. Find the values of k for which the quadratic equations $(k+4)x^2+(k+1)x+1=0$ has equal roots. Also, find the roots.

Ans : [Delhi CBSE, Term-2, 2014]

We have $(k+4)x^2+(k+1)x+1=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=(k+4),B=(k+1),C=1$$

If roots are equal, $B^2-4AC=0$

$$\begin{aligned}(k+1)^2-4(k+4)(1) &= 0\\ k^2+1+2k-4k-16 &= 0\\ k^2-2k-15 &= 0\\ (k-5)(k+3) &= 0\\ k &= 5,-3\end{aligned}$$

For $k=5$, equation becomes

$$\begin{aligned}9x^2+6x+1 &= 0\\ (3x+1)^2 &= 0\end{aligned}$$

or $$x = -\frac{1}{3}$$

For $k=-3$, equation becomes

$$\begin{aligned}x^2-2x=1 &= 0\\ (x-1)^2 &= 0\\ x &= 1\end{aligned}$$

Hence roots are 1 and $-\frac{1}{3}$.

6. If $x=-2$ is a root of the equation $3x^2+7x+p=0$, find the value of k so that the roots of the equation $x^2+k(4x+k-1)+p=0$ are equal.

Ans : [Foreign Set I, II, 2015]

We have $3x^2+7x+p=0$

Since $x=-2$ is the root of above equation. It must satisfy it.

Thus $$\begin{aligned}3(-2)+7(-2)+p &= 0\\ p &= 2\end{aligned}$$

Since roots of the equation $x^2+4kx+k^2-k+2=0$ are equal.

$$\begin{aligned}16k^2-4(k^2-k+2) &= 0\\ 16k^2-4k^2+4k-8 &= 0\\ 12k^2+4k-8 &= 0\\ 3k^2+k-2 &= 0\\ (3k-2)(k+1) &= 0\\ k &= \frac{2}{3},-1\end{aligned}$$

Hence, roots $=\frac{2}{3},-1$

7. If $x=-4$ is a root of the equation $x^2+2x+4p=0$, find the values of k for which the equation $x^2+px(1+3k)+7(3+2k)=0$ has equal roots.

Ans : [Foreign Set III, 2015]

We have $x^2+2x+4p=0$

Since $x=-4$ is the root of above equation. It must satisfy it.

$$\begin{aligned}(-4)^2+(2\times-4)+4p &= 0\\ p &= -2\end{aligned}$$

Since equation $x^2-2(1+3k)x+7(3+2k)=0$ has equal roots.

$$\begin{aligned}4(1+3k)^2-28(3+2k) &= 0\\ 9k^2-8k-20 &= 0\\ (9k+10)(k-2) &= 0\\ k &= \frac{-10}{9},2\end{aligned}$$

Hence, the value of k are $-\frac{10}{9}$ and 2.

8. Find the value of p for which the quadratic equation $(p+1)x^2-6(p+1)x+3(p+9)=0$, $p\neq-1$ has equal roots. Hence find the roots of the equation.

Ans : [Board Term-2, 2015 Set II]

We have $(p+1)x^2-6(p+1)x+3(p+9)=0$

Compare with $ax^2+bx+c=0$ we get

$$a=p+1,b=-6(p+1),c=3(p+9)$$

For real and equal roots, $b^2-4ac=0$

$$\begin{aligned}36(p+1)^2-4(p+1)\times3(p+9) &= 0\\ 3(p^2+2p+1)-(p+1)(p+9) &= 0\\ 3p^2+6p+3-(p^2+9p+p+9) &= 0\\ 2p^2-4p-6 &= 0\\ p^2-2p-3 &= 0\\ p^2-3p+p-3 &= 0\\ p(p-3)+1(p-3) &= 0\\ (p-3)(p+1) &= 0\\ p &= -1,3\end{aligned}$$

Neglecting $p\neq-1$ we get $p=3$

Now the equation becomes

$$\begin{aligned}4x^2-24x+36 &= 0\\ x^2-6x+9 &= 0\\ (x-3)(x-3) &= 0\\ x &= 3,3\end{aligned}$$

or,

Thus roots are 3 and 3.

9. If the equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$ has equal roots, prove that $c^2=a^2(1+m^2)$

Ans : [Delhi CBSE Board, 2015]

We have $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

Compare with $Ax^2+Bx+C=0$ we get

$A=1+m^2, B=2mc, C=(c^2-a^2)$

If roots are equal, $B^2-4AC=0$

$$(2mc)^2-4\big(1+m^2(c^2-a^2)\big)=0$$
$$4m^2c^2-4(1+m^2)(c^2-a^2)=0$$
$$m^2c^2-(c^2-a^2+m^2c^2-m^2a^2)=0$$
$$m^2c^2-c^2+a^2-m^2c^2+m^2a^2=0$$
$$-c^2+a^2+m^2a^2=0$$
$$c^2=a^2(1+m^2)$$

Hence Proved.

10. If (-5) is a root of the quadratic equation $2x^2+px+15=0$ and the quadratic equation $p(x^2+x)+k=0$ has equal roots, then find the values of p and k.

Ans : [Delhi CBSE Board, 2015 (Set II)]

We have $2x^2+px-15=0$

Since $x=-5$ is the root of above equation. It must satisfy it.

$$2(-5)^2+p(-5)-15=0$$
$$50-5p-15=0$$
$$5p=35 \Rightarrow p=7$$

Now $p(x^2+x)+k=0$ has equal roots

or $7x^2+7x+k=0$

Taking $b^2-4ac=0$ we have

$$7^2-4\times7\times k=0$$
$$7-4k=0$$
$$k=\frac{7}{4}$$

Hence $p=7$ and $k=\frac{7}{4}$.

11. If the roots of the quadratic equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal. Then show that $a=b=c$.

Ans : [Delhi CBSE Board, 2015 (Set II)]

We have

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$
$$x^2-ax-bx+ab+$$
$$+x^2-bx-cx+bc+$$
$$+x^2-cx-ax+ac=0$$
$$3x^2-2ac-2bx-2cx+ab+bc+ca=0$$

For equal roots $B^2-4AC=0$

$$\{-2(a+b+c)\}^2-4\times3(ab+bc+ca)=0$$
$$4(a+b+c)^2-12(ab+bc+ca)=0$$
$$a^2+b^2+c^2-3(ab+bc+ca)=0$$
$$a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac=0$$
$$a^2+b^2+c^2-ab-ac-bc=0$$
$$\tfrac{1}{2}[2a^2+2b^2+2c^2-2ab-2ac-2bc]=0$$
$$\tfrac{1}{2}[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)]=0$$
$$\tfrac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]=0$$

or, $(a-b)^2+(b-c)^2+(c-a)^2=0$

If $a\neq b\neq c$

$(a-b)^2>0,\ (b-c)^2>0,\ (c-a)^2>0$

If $(a-b)^2=0\Rightarrow a=b$

$(a-c)^2=0\Rightarrow b=c$

$(c-a)^2=0\Rightarrow c=a$

Thus $a=b=c$

Hence Proved

12. If the roots of the quadratic equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ in x are equal then show that either $a=0$ or $a^3+b^3+c^3=3abc$

Ans : [Board Outside Delhi Set II, III 2017]

Ans :

We have $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$

Compare with $Ax^2+Bx+C=0$ we get

$A=(c^2-ab), B=(a^2-bc), C=(b^2-ac)$

If roots are equal, $B^2-4AC=0$

$$\big[2(a^2-bc)\big]^2-4(c^2-ab)(b^2-ac)=0$$
$$4[a^4+b^2c^2-2a^2bc]-4(b^2c^2-c^3a-ab^3-a^2bc)=0$$
$$4[a^4+b^2c^2-2a^2bc-b^2c^2+c^3a+ab^3-a^2bc]=0$$
$$4[a^4+ac^3+ab^3-3a^2bc]=0$$
$$a(a^3+c^3+b^3-3abc)=0$$
$$a=0 \text{ or } a^3+b^3+c^3=3abc$$

13. Solve for x : $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$ where $a+b+x\neq0$ and $a,b,x\neq0$

Ans : [Board Foreign Set II, III 2017]

We have $\frac{1}{a+b+c}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

$$\frac{-(a+b)}{x^2+(a+b)x}=\frac{b+a}{ab}$$
$$x^2+(a+b)x+ab=0$$
$$(x+a)(x+b)=0$$
$$x=-a, x=-b$$

Hence $x=-a,-b$

14. The difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between their areas of the two circles in 1078 sq. cm. Find the radius of the smaller circle.

Ans : [Board Comp. I, II, III 2017]

We have $r_2 - r_1 = 7$ cm, $r_2 > r_1$ (1)

and $\pi(r_2^2 - r_1^2) = 1078$ cm^2

$\pi(r_2 - r_1)(r_2 + r_1) = 1078$

$\frac{22}{7} \times 7(r_2 + r_1) = 1078$

$r_2 + r_1 = \frac{1078}{22} = 49$ (2)

Adding (1) and (2) we get

$2r_2 = 56$

$r_2 = 28$ cm

and $r_1 = 49 - 28 = 21$

Hence radii of two circles are 28 cm and 21 cm.

15. A train travelling at a uniform speed for 360 km have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train

Ans : [Sample Paper 2017]

Ans :

Let the original speed of the train be x km/hr.

$$\text{Time taken} = \frac{Distance}{Speed} = \frac{360}{x} \text{ hours}$$

$$\text{Time taken at increase speed} = \frac{360}{x+5} \text{ hours.}$$

According to the question

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$$360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5}$$

$$\frac{1800}{x^2+5x} = \frac{4}{5}$$

$$x^2 + 5x - 2250 = 0$$

$$x^2 + (50 - 45)x - 2250 = 0$$

$$x^2 - 50x - 45x - 2250 = 0$$

$$(x+50)(x-45) = 0$$

$$x = -50 \text{ or } x = 45$$

As speed can not be negative, original speed of train is 45 km/hr.

16. Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.

Ans : [Sample Question Paper 2017]

We have $5x^2 - 6x - 2 = 0$

Compare with $ax^2 + bx + c = 0$ we get

$a = 5, b = (-6)$ and $c = (-2)$

$b^2 - 4ac = (-6)^2 - 4 \times 5 \times -2$

$= 36 + 40 = 36 > 0$

So the equation has real and two distinct roots.

$5x^2 - 6x = 2$

Dividing both the sides by 5 we get

$$x^2 - \frac{6}{5}x = \frac{2}{5}$$

$$x^2 - 2x\left(\frac{3}{5}\right) = \frac{2}{5}$$

Adding square of the half of coefficient of x

$$x^2 - 2x\left(\frac{3}{5}\right) + \frac{9}{25} = \frac{2}{5} + \frac{9}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$

$$x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$

$$x = \frac{3+\sqrt{19}}{5} \text{ or } \frac{3-\sqrt{19}}{5}$$

Verification :

$$5\left[\frac{3+\sqrt{19}}{5}\right]^2 - 6\left[\frac{3+\sqrt{19}}{5}\right] - 2$$

$$= \frac{9 + 6\sqrt{19} + 19}{5} - \left(\frac{18 + 6\sqrt{19}}{5}\right) - 2$$

$$= \frac{28 + 6\sqrt{19}}{5} - \frac{18 + 6\sqrt{19}}{5} - 2$$

$$= \frac{28 + 6\sqrt{19} - 18 - 6\sqrt{19} - 10}{5}$$

$$= 0$$

Similarly

$$5\left[\frac{3-\sqrt{19}}{5}\right]^2 - 6\left[\frac{3-\sqrt{19}}{5}\right] - 2 = 0$$

Hence verified.

HOTS QUESTIONS

1. Solve $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, a + b \neq 0.$

Ans : [Sample Paper, 2016]

We have $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{x - (a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{x - a - b - x}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$x(a+b+x) = -ab$$

$$x^2 + (a+b)x + ab = 0$$

$$(x+a)(x+b) = 0$$

$$x = -a \text{ or } x = -b$$

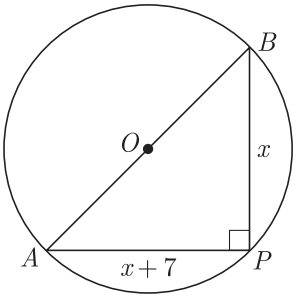
2. A pole has to erected at a point on the boundary of a circular park of diameter 17 m in such a way that

the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 meters. Find the distances from the two gates where the pole is to be erected.

[Foreign Set I, II, 2016]

Ans : [CBSE Marking Scheme, 2011, 2012]

As per question the figure is shown below.



Let p be the location of the pole such that its distance from gate B, x metres.

Thus $AP = x+7$

Here AP is diameter or, $\angle APB = 90^\circ$ and $AB = 17$ m

$$x^2+(x+7)^2 = (17)^2$$
$$x^2+x^2+14x-240 = 0$$
$$x^2+7x-120 = 0$$
$$x = \frac{-7\pm\sqrt{49+480}}{2}$$
$$= \frac{-2\pm 23}{2} = 8, -15$$

Thus $x = 8$ m and $x+7 = 15$ m

Hence distance between two gates are 8 m and 15 m.

3. Find the value of k for which the distance between $(9,2)$ and $(3,k)$ is 10 units.

Ans : [Board Term-2, 2012 set (43); Set 2011 set (B1)]

Ans : [CBSE Marking Scheme, 2011, 2012]

$$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Thus $\sqrt{(3-9)^2+(k-2)^2} = 10$

$$(-6)^2+k^2-4k+4 = 100$$
$$k^2-4k+40 = 100$$
$$k^2-4k-60 = 0$$
$$k^2-10k+6k-60 = 0$$
$$k(k-10)+6(k-10) = 0$$
$$(k-10)(k+6) = 0$$
$$k = 10, -6$$

4. A shopkeeper buys a number of books for Rs. 1200. If he had bought 10 more books for the same amount, each book would have cost him Rs. 20 less. How many books did he buy?

Ans : [Board Term-2, 2012 Set (22)]

Let the number of books bought be x.

As per question we have

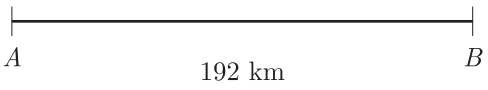
$$\frac{1200}{x}-\frac{1200}{x+10} = 20$$
$$x^2+10x-600 = 0$$
$$(x+30)(x-20) = 0$$
, $$x = -30 \text{ or } x = 20$$

Since number of books cannot be negative, $x = 20$

Thus number of books bought is 20.

5. A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more, find the speed of the faster and the passanger train.

Ans : [Board Term-2, 2012 Set (28)]



Thus speed of super-fast train $= (x+16)$ km/h

Now $T_{passenger} = \frac{192}{x}$

and $T_{superfast} = \frac{192}{(x+16)}$

As per question we have

$$\frac{192}{x}-\frac{192}{x+16} = 2$$
$$192(x+16)-192x = 2(x^2+16x)$$
$$192x+192\times 16-192x = 2(x^2+16x)$$
$$x^2+16x-1536 = 0$$
$$x^2+48x-32x-1536 = 0$$
$$x(x+48)-32(x+48) = 0$$
$$(x-32)(x+48) = 0$$
$$x = 32 \text{ or } -48$$

Since speed can't be negative, therefore -48 is not possible. Speed of passenger train is 32 km/h

6. If the price of a book is reduced by Rs. 5, a person can buy 5 more book for Rs.300. Find the original list price of the book.

Ans : [Board Term-2, 2012 sEt (17)]

Let the original list price be Rs. x

No. of books bought for Rs. 300 $= \frac{300}{x}$

Reduced list price of the book $= (x-5)$

No. of books brough for Rs. 300 $= \frac{300}{x-5}$

According to questions, we have

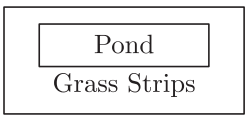
$$\frac{300}{x-5}-\frac{300}{x} = 5$$
$$x^2-5x-300 = 0$$
$$(x-20)(x+15) = 0$$

Since price cannot be negative, $x = 20$

Thus original list price is 20 rs.

7. In a rectangular part of dimension $50m\times 40m$ a rectangular pond is constructed so that the area of grass strip of uniform breadth surrounding the pond

would be $1184m^2$. Find the length and breadth of the pond.



Ans : [Board Foregin Set-I, III, 2017]

Let width of grass strip be x mts.

Length of pond $=(50-2x)$mt

and breadth of pond $=(40-2x)$mt

and area of park – area of pond = area of grass strip

$$\begin{aligned}(50\times 40)-(50-2x)(40-2x) &= 1184\\ 2000-2000+180x-4x^2 &= 1184\\ x^2-45x+296 &= 0\\ x^2-37x-8x+296 &= 0\\ x(x-37)-8(x-37) &= 0\\ x &= 8,37\end{aligned}$$

Here 37 is not possible, thus $x=8$

$$\begin{aligned}\text{Length of pond} &= 50-2\times 8\\ &= 34 \text{ m}\\ \text{and breadth of pond} &= 40-2\times 8\\ &= 24 \text{ m.}\end{aligned}$$

8. A car covers a distance of 2592 km with a uniform speed. The number of hours taken for journey is one half the number representing the speed in km/hour. Find the time taken to ever the distances.

Ans : [Delhi Compt Set-I, III 2017]

Let the speed of the car be x km/hr.

Therefore time taken $=\frac{x}{2}$ hour

Now $\text{Speed} = \frac{Distance}{Time}$

$$\begin{aligned}x &= \frac{2592}{\frac{x}{2}}\\ x^2 &= 2592\times 2=5184\\ x &= \sqrt{5184}\ =72\end{aligned}$$

Hence the time taken $\frac{72}{2}=36$ hours.

9. Speed of a boat in still water is 15 km/hour. It goes 30 km up stream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

Ans : [Board Delhi Set-I, III, 2017]

Let the speed of the Stream be x km/hr.
Speed of boat up stream $=15-x$
and speed of boat down stream $=15+x$
According to the question

$$\frac{30}{10-x}+\frac{30}{15+x}=4\frac{1}{2}$$

$$\frac{30(15+x)+30(15-x)}{15^2-x^2}=\frac{9}{2}$$

$$\begin{aligned}900\times 2 &= 9(15^2-x^2)\\ 9x^2 &= 2025-1800=225\\ x^2 &= 25\\ x &= \pm 5\end{aligned}$$

Hence, the speed of the stream $=5$ km/hr

10. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it ?

Ans : [Board 2016]

Let B complete a work in x days.
Then A takes $x-6$ days to complete it.
Together they complete it in 4 days.
According to work done per day,

$$\begin{aligned}\frac{1}{x-6}+\frac{1}{x} &= \frac{1}{4}\\ \frac{x+x-6}{x(x-6)} &= \frac{1}{4}\\ 4(2x-6) &= x(x-6)\\ 8x-24 &= x^2-6x\\ x^2-14x+24 &= 0\\ x^2-12x-2x+24 &= 0\\ x(x-12)-2(x-12) &= 0\\ (x-2)(x-12) &= 0\\ x &= 2 \text{ or } 12\end{aligned}$$

Here $x=2$ is not possible because $x-6$ is (-4) which is not possible.

Thus $x=12$ and B takes 12 days to finish the work.

11. In a class test Raveena got a total of 30 mark in English and Mathematics. Had she got 2 more marks in Mathematics and 3 marks less in English then the product of her marks obtained would have be 210. Find the individual marks obtained in two subjects.

Ans : [Outside Delhi Compt. I, II, III 2017]

Let Raveena got marks in English be x.
Marks in Mathematics $=(30-x)$
Marks in English $=(x-3)$
According to problem

$$\begin{aligned}(x-3)(30-x+2) &= 210\\ 35x-96-x^2 &= 210\\ x^2-35x+306 &= 0\\ x^2-18x-17x+306 &= 0\\ x(x-18)-17(x-18) &= 0\\ (x-18)(x-17) &= 0\\ x &= 18,17\end{aligned}$$

Hence, if she got 18 marks in English, then she got 12 in mathematics.
If she got 17 marks in English, then she got 13 marks in mathematics.

12. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

Ans : [Outside Delhi I 2017]

Tow tap running together fill the tank in $3\frac{1}{13}$ hr.

$$= \frac{40}{13} \text{ hours}$$

Thus It will fill in 1 hour $= \frac{13}{40}$ tank

If first tap alone fills the tank in x hrs, then second tap alone fills it in $(x+3)$ hr.

Now, $$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$$

$$\frac{x+3+x}{x(x+3)} = \frac{13}{40}$$

$$80x + 120 = 13x^2 + 39x$$

or, $$13x^2 - 41x - 120 = 0$$

$$13x^2 - (65-24)x + 120 = 0$$

$$(x-5)(13x+24) = 0$$

$$x = 5,\ x = -\frac{24}{13}$$

Here $x = -\frac{24}{13}$ is not possible. Hence, 1st tap takes 5 hours and 2nd tap takes $= 5+3 = 8$ hours

13. Two taps running together can fill a cistern in $2\frac{8}{11}$ minutes. If one tap takes 1 minute more than the other to fill the cistern, find the time in which each tap separately can fill the cistern.

Ans : [Outside Delhi Set-III 2017]

Two taps together fill the cistern in $2\frac{8}{11}$ minutes

$$= \frac{30}{11} \text{ minutes}$$

Thus It will fill in 1 minute $= \frac{11}{30}$ cistern

Let first tap fills the same cistern in x minutes

and 2nd tap will take $= (x+1)$ minutes

Thus $$\frac{1}{x} + \frac{1}{(x+1)} = \frac{11}{30}$$

$$\frac{x+1+x}{x(x+1)} = \frac{11}{30}$$

$$60x + 30 = 11x^2 + 11x$$

$$11x^2 - 49x - 30 = 0$$

$$11x^2 - (55-6)x - 30 = 0$$

$$11x^2 - 55x + 6x - 30 = 0$$

$$11x(x-5) + 6(x-5) = 0$$

$$(x-5)(11x+6) = 0$$

$$x = 5,\ x = -\frac{6}{11}$$

Here $x = -\frac{6}{11}$ is not possible. Hence, 1st tap takes 5 minute and 2nd tap takes 6 minute.

14. A and B working together can do a work in 6 days. If A takes 5 days less than B to finish the work, in how many days B alone can do it alone?

Ans : [Outside Delhi Compt. Set-I]

Since $A+B$ finish the work in 6 days.

They will finish in one day $= \frac{1}{6}$ work

Let B alone does the same work in x days, then A alone will finish in $(x-5)$ days.

Now, $$\frac{1}{x-5} + \frac{1}{x} = \frac{1}{6}$$

$$\frac{x+x-5}{x(x-5)} = \frac{1}{6}$$

$$12x - 30 = x^2 - 5x$$

$$x^2 - 17x + 30 = 0$$

$$x^2 - (15+2)x + 30 = 0$$

$$x^2 - 15x - 2x + 30 = 0$$

$$x(x-15) - (x-15) = 0$$

$$(x-15)(x-2) = 0$$

$$x = 15, x = 2$$

Here $x = 2$ is not possible. Hence, B finishes the work in 15 days.

15. Ram takes 6 days less than Bhagat to finish a place of work. If both of them together can finish the work in 4 days, in how many days Bhagat alone can finish the work ?

Ans : [Delhi Compt. Set-III 2017, Outside Delhi Set-II2017]

Ram and Bhagat together do the work in 4 days

Ram and Bhagat will do in one days $= \frac{1}{4}$ work

Let Bhagat alone does the same work in x days.

Ram will take $= (x-6)$ days

Now $$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$

$$\frac{x-6+x}{x(x-6)} = \frac{1}{4}$$

$$8x - 24 = x^2 - 6x$$

$$x^2 - 14x + 24 = 0$$

$$x^2 - (12+2)x + 24 = 0$$

$$x^2 - 12x - 2x + 24 = 0$$

$$x(x-12) - 2(x-12) = 0$$

$$(x-12)(x-12) = 0$$

$$x = 12, x = 2$$

If Bhagat complete the work in 2 days, Ram will take $= 2-6 = -4$ days that is impossible. Hence, Bhagat can finish in 12 days.