CHAPTER 3

Pair of Linear Equation in Two Variables

VERY SHORT ANSWER TYPE QUESTIONS

1. Find whether the pair of linear equations $y = 0$ and $y = -5$ has no solution, unique solution or infinitely many solutions.

Ans :

The given variable y has different values. Therefore the pair of equations $y = 0$ and $y = -5$ has no solution.

2. If $am = bl$, then find whether the pair of linear equations $ax + by = c$ and $lx + my = n$ has no solution, unique solution or infinitely many solutions.

Ans :

Since, $am = bl$, we have

$$\frac{a}{1}\ \frac{b}{m} \neq \frac{c}{n}$$

Thus, $ax + by = c$ and $lx + my = n$ has no solution.

3. If $ad \neq bc$, then find whether the pair of linear equations $ax + by = p$ and $cx + dy = q$ has no solution, unique solution or infinitely many solutions.

Ans :

Since $\quad ad \neq bc$ or $\frac{a}{c} \neq \frac{b}{d}$

Hence, the pair of given linear equations has unique solution.

4. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

Ans :

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $$\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

SHORT ANSWER TYPE QUESTIONS - I

1. Find whether the lines represented by $2x + y = 3$ and $4x + 2y = 6$ are parallel, coincident or intersecting.

Ans : [Board Term-1, 2016, MV98HN3]

Ans :

Here $a_1 = 2, b_1 = 1, c_1 = -3$ and $a_1 = 4, b_2 = 2, c_2 = -6$

If $$\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

then the lines are parallel.

Clearly $$\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$$

Hence lines are coincident.

2. Find whether the following pair of linear equation is consistent or inconsistent:
$3x + 2y = 8, \quad 6x - 4y = 9$

Ans : [Board Term-1, 2016 ORDAWEZ]

We have $$\frac{3}{6} \neq \frac{2}{-4}$$

i.e., $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, the pair of linear equation is consistent.

3. Is the system of linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ consistent? Justify your answer.

Ans : [Board Term-1, 2012 set-66]

For the equation $2x + 3y - 9 = 0$ we have

$$a_2 = 2, b_1 = 3 \text{ and } c_1 = -9$$

and for the equation, $4x + 6y - 18 = 0$ we have

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$

Here $$\frac{a_1}{a_1} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2}\ \frac{3}{6} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$$

Thus $$\frac{c_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, system is consistent and dependent.

4. Given the linear equation $3x + 4y = 9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:
(1) intersecting lines
(2) coincident lines.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

(1) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, one of the possible equation $3x - 5y = 10$

(2) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, one of the possible equation $6x + 8y = 18$

5. For what value of p does the pair of linear equations given below has unique solution ?
$4x+py+8=0$ and $2x+2y+2=0$.

Ans : [Board Term-1, 2012, Set-44]

We have
$$4x+py+8 = 0$$
$$2x+2y+2 = 0$$

The condition of unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, $\frac{4}{2} \neq \frac{p}{2}$ or $\frac{2}{1} \neq \frac{p}{2}$

Thus $p \neq 4$. The value of p is other than 4 it may be $1,2,3,-4$.....etc.

6. For what value of k, the pair of linear equations $kx-4y=3$, $6x-12y=9$ has an infinite number of solutions ?

Ans : [Board Term-1, 2012, Set-25]

We have
$$kx-4y-3 = 0$$
and
$$6x-12y-9 = 0$$
where, $a_1 = k, b_1 = 4, c_1 = -3$

$a_2 = 6, b_2 = -12, c_2 - 9$

Condition for infinite solutions:
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$$

Hence, $k = 2$

7. For what value of k, $2x+3y=4$ and $(k+2)x+6y=3k+2$ will have infinitely many solutions ?

Ans : [Board Term-1, 2012, Set-68]

We have
$$2x+3y-4 = 0$$
and
$$(k+2)x+6y-(3k+2) = 0$$
Here $a_1 = 2, b_1 = 3, c_1 = -4$

and $a_2 = k+2, b_2 = 6, c_3 = -(3k+2)$

For infinitely many solutions
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
or,
$$\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

From $\frac{2}{k+2} = \frac{3}{6}$ we have

$3(k+2) = 2\times 6 \Rightarrow (k+2) = 4 \Rightarrow k = 2$

From $\frac{3}{6} = \frac{4}{3k+2}$ we have

$3(3k+2) = 4\times 6 \Rightarrow (3k+2) = 8 \Rightarrow k = 2$

Thus $k = 2$

8. For what value of 'k', the system of equations $kx+3y=1$, $12x+ky=2$ has no solution.

Ans : [Board Term-1, 2011, Set-A2 NCERT]

The given equations can be written as
$kx+3y-1=0$ and $12x+ky-2=0$

Here, $a_1 = k, b_1 = 3, c_1 = -1$

and $a_2 = 12, b_2 = k, c_2 = -2$

The equation for no solution if
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
or,
$$\frac{k}{12} = \frac{3}{k} \neq \frac{-1}{-2}$$

From $\frac{k}{12} = \frac{3}{k}$ we have $k^2 = 36 \Rightarrow k \pm 6$

From $\frac{3}{k} \neq \frac{-1}{-2}$ we have $k \neq 6$

Thus $k = -6$

SHORT ANSWER TYPE QUESTIONS - II

1. Which of the following pairs of linear equations are consistent/inconsistent ? If consistent, obtain the solution graphically.

Ans : [NCERT]

(a)
$$x+y = 5$$
$$2x+2y = 10$$
(b)
$$x-y = 8$$
$$3x-3y = 16$$
(c)
$$2x+y-y = 0$$
$$4x-2y-4 = 0$$
(d)
$$2x-2y-2 = 0$$
$$4x-4y-5 = 0$$

Ans :

(a) The pair of linear equations is:
$$x+y = 5$$
or,
$$x+y-5 = 0$$
and
$$2x+2y = 10 \quad ...(1)$$
or,
$$2x+|\,2y-10 = 0 \quad ...(2)$$
We have, $a = 1, b_1 = 1, c_1 = -5$

$a_2 = 2, b_2 = 2, c_2 = -10$

Now $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}$, and $\frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the pair of linear equations is coincident having many, solutions. Thus, the equation is consistent to solve it graphically.

We have $x+y = 5$

or, $y = 5-x$

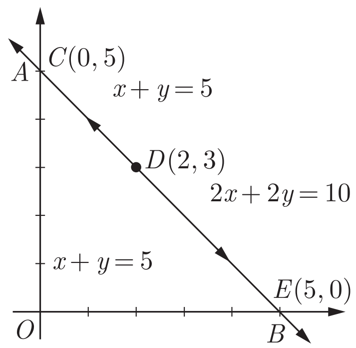
x	0	5
y	5	0

and $2x+2y = 10$

or, $y = \frac{10-2x}{2}$

x	0	2	5
y	5	3	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly lines are coincident having infinite many solutions.

(b) The pair of linear equation is:

$$x + y = 8$$

or $\quad x + y - 8 = 0 \quad$ ...(1)

and $\quad 3x - 3y = 16$

or $\quad 3x - 3y - 16 = 0 \quad$ ...(2)

We have $\quad a_1 = 1, b_1 = -1, c_1 = -8$

$$a_2 = 3, b_2 = -3, c_2 = -16$$

Now $\quad \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{1}{2}$

As $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

The lines are parallel having no solution. So, the pair of linear equations is inconsistent.

(c) The pair of linear equations are :

$2x + y - 6 = 0 \quad$ ...(1)

and $\quad 4x - 2y - 4 = 0 \quad$ ...(2)

where, $\quad a_1 = 2, b_1 = 1, c_1 = -6$

$$a_2 = 4, b_2 = -2, c_2 = -4$$

Now $\quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{1}{-2}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of linear equations is consistent.

We have, $2x + y - 6 = 0$

or, $\quad y = 6 - 2x$

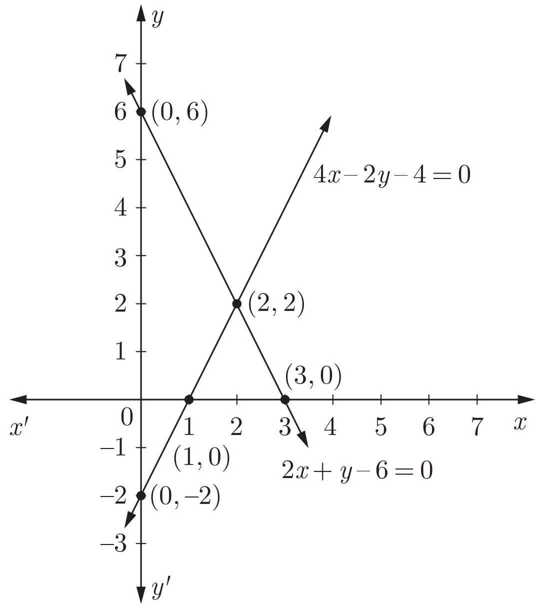
x	0	3	2
y	6	0	2

and $\quad 4x - 2y - 4 = 0$

or, $\quad y = 2x - 2$

x	0	1	2
y	-2	0	2

Plotting the above points and drawing lines joining them, we get the following graph. Two obtained lines intersect each other at (2, 2).



(d) $\quad 2x - 2y - 2 = 0$

$$4x - 4y - 5 = 0$$

The pair of linear equations is

$2x - 2y - 2 = 0 \quad$ ...(1)

and $\quad 4x - 4y - 5 = 0 \quad$ ...(2)

where, $\quad a_1 = 2, b_1 = -2, c_1 = -2$

$$a_2 = 4, b_2 = -4, c_2 = -5$$

Now $\quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$, and $\frac{c_1}{c_2} = \frac{-2}{-5}$

Since $\quad \left[\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right]$

Therefore, the pair of linear equations is inconsistent.

2. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

Ans : [NCERT]

Let the length of the garden be x m and its width be y m.

Perimeter of rectangular garden

$$p = 2(x + y)$$

Since half perimeter is given as 36 m,

$(x + y) = 36 \quad$ ...(1)

Also, $\quad x = y + 4$

or $\quad x - y = 4 \quad$ ...(2)

For $\quad x + y = 36$

$$y = 36 - x$$

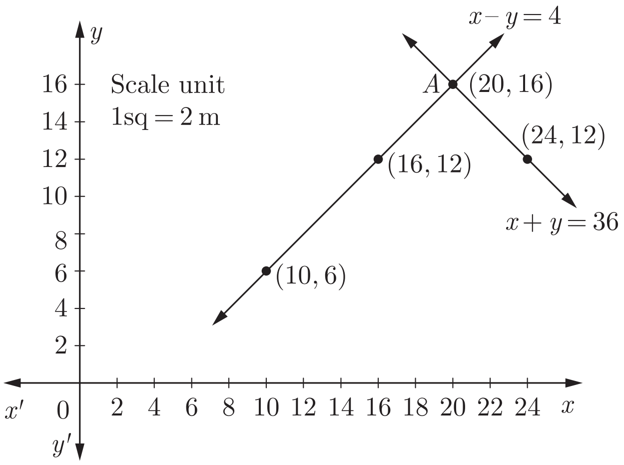
x	20	24
y	16	12

For $\quad x - y = 4$

or, $\quad y = x - 4$

x	10	16	20
y	6	12	16

Plotting the above points and drawing lines joining them, we get the following graph. we get two lines intersecting each other at (20, 16)



Hence, length is 20 m and width is 16 m.

3. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :
(a) intersecting lines
(b) parallel lines
(c) coincident lines.

Ans : [NCERT]

Given, linear equation is $2x + 3y - 8 = 0$ ...(1)

(a) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

To get its parallel line one of the possible equation may be taken as

$$5x + 2y - 9 = 0 \qquad (2)$$

(b) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

One of the possible line parallel to equation (1) may be taken as

$$6x + 9y + 7 = 0$$

(c) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

To get its coincident line, one of the possible equation may be taken as

$$4x + 6y - 16 = 0$$

4. Solve the pair of equations graphically :
$4x - y = 4$ and $3x + 2y = 14$

[Board Term-1, 2014, Set-A]

Ans :

We have $4x - y = 4$

or, $y = 4x - 4$

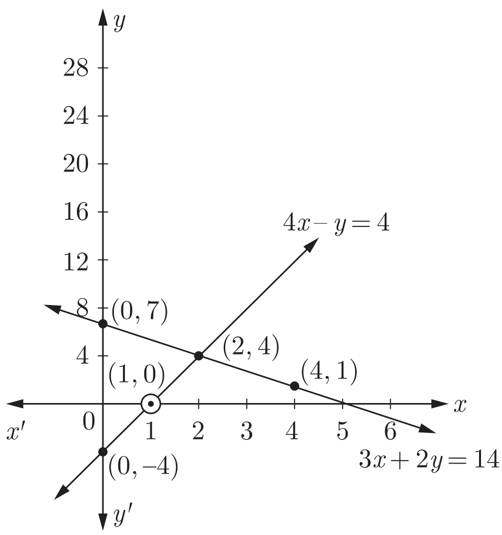
x	0	1	2
y	-4	0	4

and $3x + 2y = 14$

or, $y \frac{14 - 3x}{2}$

x	0	2	4
y	7	4	1

Plotting the above points and drawing lines joining them, we get the following graph. We get two obtained lines intersect each other at (2, 4).



Hence, $x = 2$ and $y = 4$.

5. Determine the values of m and n so that the following system of linear equation have infinite number of solutions :

$$(2m - 1)x + 3y - 5 = 0$$
$$3x + (n - 1)y - 2 = 0$$

Ans : [Board Term-1, 2013, VKH6FFC; 2011, Set-66

We have $(2m - 1)x + 3y - 5 = 0$ ...(1)

Here $a_1 = 2m - 1, b_1 = 3, c_1 = -5$

$$3x + (n - 1)y - 2 = 0 \qquad ...(2)$$

Here $a_2 = 3, b_2 = (n - 1), c_2 = -2$

For a pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2m - 1}{3} = \frac{3}{n - 1} = \frac{5}{2}$$

or $2(2m - 1) = 15$ and $5(n - 1) = 6$

Hence, $m = \frac{17}{4}, n = \frac{11}{5}$

6. Find the values of α and β for which the following pair of linear equations has infinite number of solutions : $2x + 3y = 7; 2\alpha x + (\alpha + \beta)\ y = 28$.

Ans : [Board Term-1, 2011, Set-25]

We have $2x + 3y = 7$ and $2\alpha x + (\alpha + \beta)y = 28$.

For a pair of linear equations to be consistent and having infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{2\alpha} = \frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$\frac{2}{2\alpha} = \frac{7}{28}$$

$$2\alpha \times 7 = 28 \times 2 \Rightarrow \alpha = 4$$

$$\frac{3}{\alpha+\beta} = \frac{7}{28}$$

$$7(\alpha+\beta) = 28\times 3$$

$$\alpha+\beta = 12$$

$$\beta = 12-\alpha = 12-4 = 8$$

Hence $\alpha = 4$, and $\beta = 8$

7. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$$x-5y = 6, 2x-10y = 12.$$

Ans : [Board Term-1, 2011, Set-5]

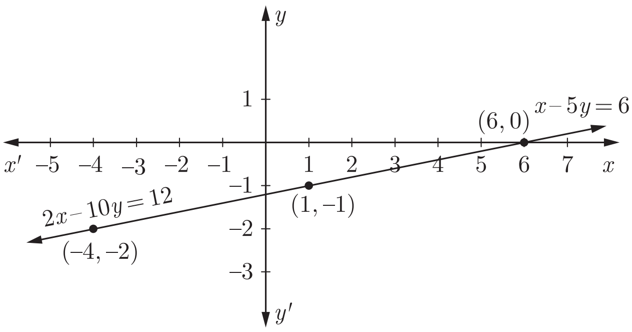
We have $x-5y = 6$ or $x = 5y+6$

x	6	1	-4
y	0	-1	-2

and $2x-10y = 12$ or $x = 5y+6$

x	6	1	-4
y	0	-1	-2

Plotting the above points and drawing lines joining them, we get the following graph.



Since the lines are coincident, so the system of linear equations is consistent with infinite many solutions.

8. For what value of p will the following system of equations have no solution ?

$$(2p-1)x+(p-1)y = 2p+1;\ y+3x-1 = 0$$

Ans : [Board Term-1, 2011, Set-28]

We have $(2p-1)x+(p-1)y-(2p+1) = 0$

Here $a_1 = 2p-1, b_1 = p-1$ and $c_1 = -(2p+1)$

Also $3x+y-1 = 0$

Here $a_2 = 3, b_2 = 1$ and $c_2 = -1$

The condition for no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{2p-1}{3} = \frac{p-1}{1} \neq \frac{2p+1}{1}$$

From $\frac{2p-1}{3} = \frac{p-1}{1}$ we have

$$3p-3 = 2p-1$$

$$3p-2p = 3-1$$

$$p = 2$$

From $\frac{p-1}{1} \neq \frac{2p+1}{1}$ we have

$$p-1 \neq 2p+1 \text{ or } 2p-p \neq -1-1$$

$$p \neq -2$$

From $\frac{2p-1}{3} \neq \frac{2p+1}{1}$ we have

$$2p-1 \neq 6p+3$$

$$4p \neq -4$$

$$p \neq -1$$

Hence, system has no solution when $p = 2$

9. Find the value of k for which the following pair of equations has no solution :
$x+2y = 3, (k-1)x+(k+1)y = (k+2)$.

Ans : [Board Term-1, 2011, Set-52]

For $x+2y = 3$ or $x+2y-3 = 0$
$a_1 = 1, b_1 = 2, c_1 = -3$
For $(k-1)x+(k+1)y = (k+2)$
or $(k-1)x+(k+1)y-(k-2) = 0$
$a_2 = (k-1), b_2 = (k+1), c_2 = -(k+2)$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{1}{k-1} = \frac{2}{k+1} \neq \frac{3}{k+2}$$

From $\frac{1}{k-1} = \frac{2}{k+1}$ we have

$$k+1 = 2k-2$$

$$3 = k$$

Thus $k = 3$.

LONG ANSWER TYPE QUESTIONS

1. For Uttarakhand flood victims two sections A and B of class contributed Rs. 1,500. If the contribution of X-A was Rs. 100 less than that of X-B, find graphically the amounts contributed by both the sections.

Ans : [Board Term-1, 2016, Set-MV98HN3]

Let amount contributed by two sections X-A and X-B be Rs. x and Rs. y.

$$x+y = 1,500 \qquad ...(1)$$

$$y-x = 100 \qquad ...(2)$$

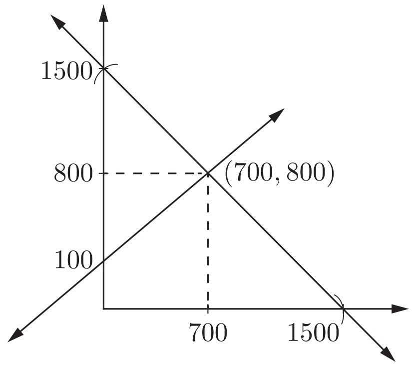
From (1) $y = 15000-x$

x	0	700	1,500
y	1,500	800	0

From (2) $y = 100+x$

x	0	700
y	100	800

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point (700, 800) Hence X-A contributes 700 Rs and X-B contributes 800 Rs.

2. Determine graphically whether the following pair of linear equations :

$$3x - y = 7$$
$$2x + 5y + 1 = 0 \text{ has :}$$

(1) a unique solution

(b) infinitely many solutions or

(c) no solution.

Ans : [Board Term-1, 2015, Set-DDE-E]

We have $\quad 3x - y = 7$

or $\quad 3x - y - 7 = 0 \qquad (1)$

Here $a_1 = 3, b_1 = 1, c_1 = -7$

$$2x + 5y + 1 = 0 \qquad (2)$$

Here $a_2 = 2, b_2 = 5, c_2 = 1$

Now $\quad \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-1}{5}$

Since $\frac{3}{2} \neq \frac{-1}{5}$, thus $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given pair of linear equations has a unique solution.

Now line (1) $\quad y = 3x - 7$

x	0	2	3
y	-7	-1	2

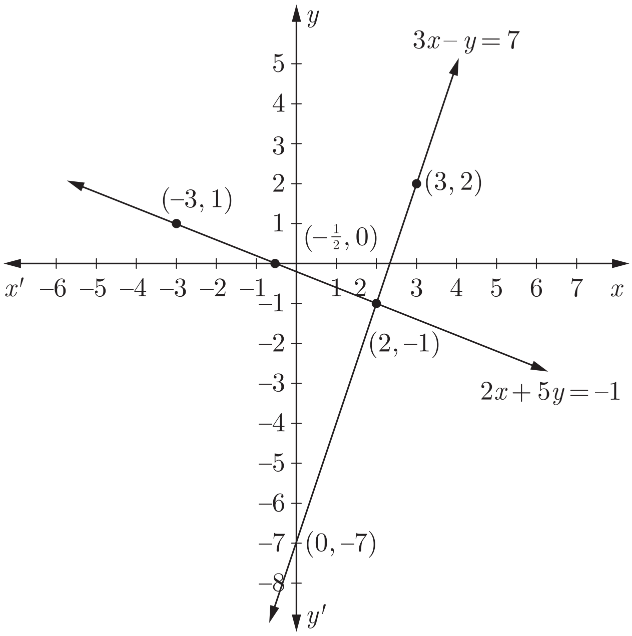
and line (2)

$$2x + 5y + 1 = 0$$

or, $\quad y = \frac{-1 - 2x}{5}$

x	2	-3
y	-1	1

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(2, -1)$. Hence $x = 2$ and $y = -1$

3. Draw the graphs of the pair of linear equations : $x + 2y = 5$ and $2x - 3y = -4$

Also find the points where the lines meet the $x-$axis.

Ans : [Board Term-1, 2015, Set-FHNQMGD]

We have $\quad x + 2y = 5$

or, $\quad y = \frac{5 - x}{2}$

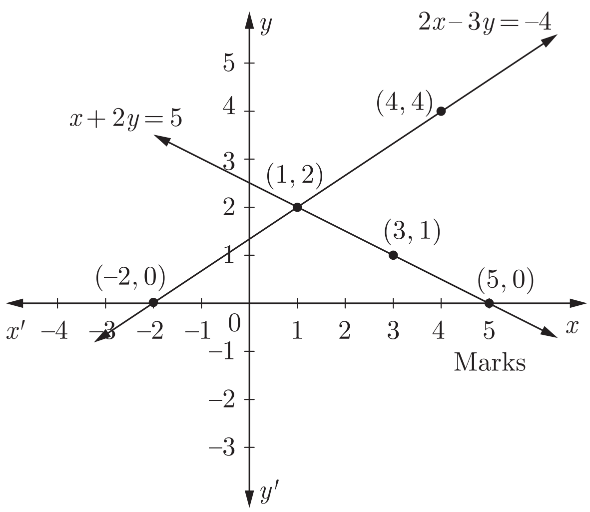
x	1	3	5
y	2	1	0

and $\quad 2x - 3y = -4$

or, $\quad y = \frac{2x + 4}{3}$

x	1	4	-2
y	2	4	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two lines meet x-axis at (5,0) and (-20)

respectively.

4. Solve graphically the pair of linear equations : $3x-4y+3=0$ and $3x+4y-21=0$
Find the co-ordinates of the vertices of the triangular region formed by these lines and x-axis. Also, calculate the area of this triangle.

Ans : [Board Term-1, 2015, Set-DDE-E]

We have $3x-4y+3=0$

or, $y=\frac{3x+3}{4}$

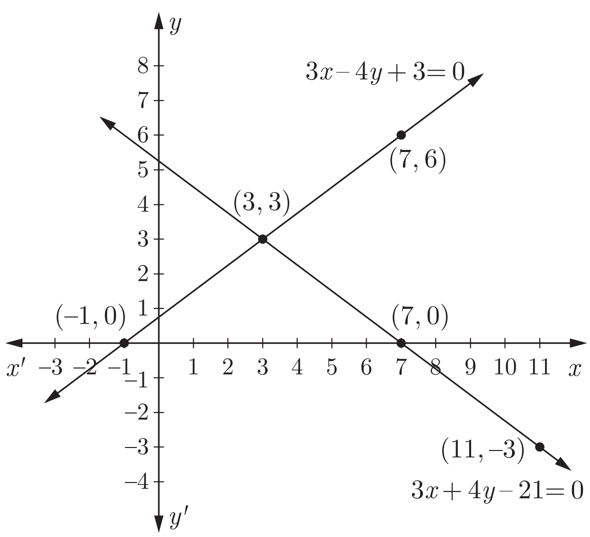
x	3	7	-1
y	3	6	0

and $3x+4y-21=0$

or, $y=\frac{21-3x}{4}$

x	3	7	11
y	3	0	-3

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(3,3)$.

(a) These lines intersect each other at point $(3,3)$. Hence $x=3$ and $y=3$

(b) The vertices of triangular region are $(3,3)$, $(-1,0)$ and $(7,0)$.

(c) Area of $\Delta=\frac{1}{2}\times8\times3=12$

Hence, Area of obtained Δ is 12 sq unit.

5. Aftab tells his daughter, '7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.' Represent this situation algebraically and graphically.

Ans : [NCERT]

Let the present age of Aftab be x years and the age of daughter be y years.

7 years ago father's(Aftab) age $=(x-7)$years

7 years ago daughter's age $=(y-7)$ years

According to the question,

$$(x-7)=7(y-7)$$

or, $$(x-7y)=-42 \quad (1)$$

After 3 years father's(Aftab) age $=(x+3)$ years

After 3 years daughter's age $=(y+3)$ years

According to the condition,

$$x+3=3(y+3)$$

or, $$x-3y=6 \quad (2)$$

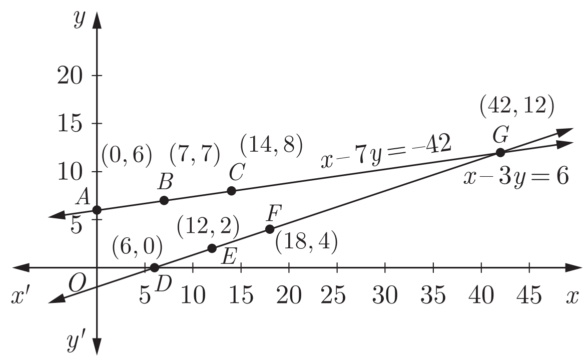
From equation(1) $x-7y=-42$

x	0	7	14
$y=\frac{x+42}{7}$	3	0	-3

From equation (2) $x-3y=6$

x	6	12	18
$y=\frac{x-6}{3}$	0	2	4

Plotting the above points and drawing lines joining them, we get the following graph.



Two lines obtained intersect each other at $(42,12)$

Hence, father's age $=42$ years

and daughter's age $=12$ years

6. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs. 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs. 300. Represent the situations algebraically and geometrically.

Ans : [NCERT]

Let the cost of 1 kg of apples be Rs.x and cost of 1 kg of grapes be Rs. y.

The given conditions can be represented given by the following equations :

$$2x+y=160 \quad ...(1)$$

$$4x+2y=300 \quad ...(2)$$

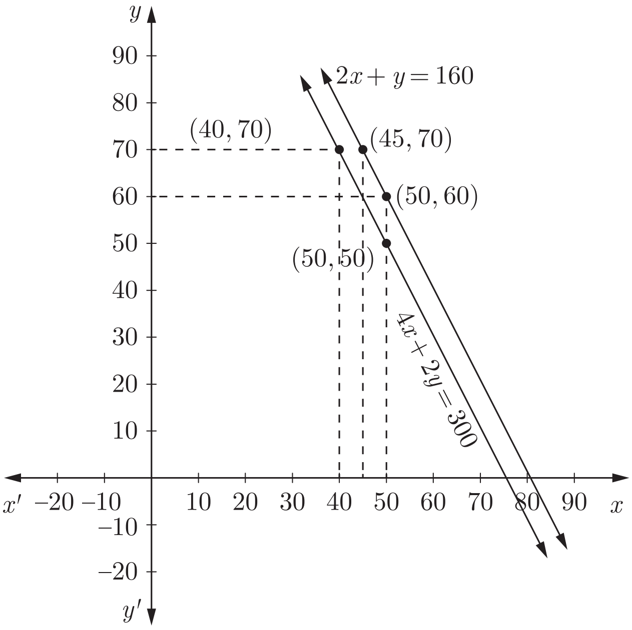
From equation (1) $y=160-2x$

x	50	45
y	60	70

From equation (2) $y=150-2x$

x	50	40
y	50	70

Plotting these points on graph, we get two parallel line as shown below.



NO NEED TO PURCHASE ANY BOOKS

7. Form the pair of linear equations in the following problems find their solutions graphically :

(a) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(b) 5 pencils and 7 pens together cost Rs. 50 whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Ans : [NCERT]

(a) Let the number of boys be x and the number of girls be y.

The given conditions can be represented given by the following equations :

$$x+y=10 \qquad ...(1)$$

and

$$y-x=4 \qquad ...(2)$$

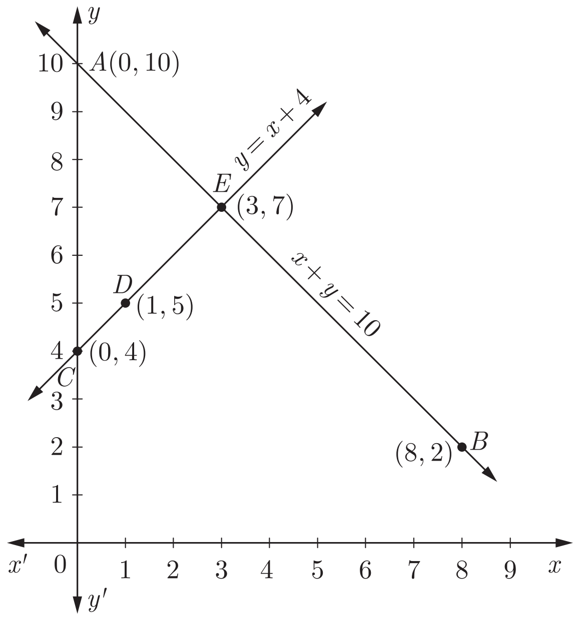
From equation (2) $y=10-x$

x	0	8	3
$y=10-x$	10	2	7

From equation (2) $y=x+4$

x	0	1	3
$y=x+4$	4	5	7

Plotting co-ordinates on graph paper, two lines intersect each other at point $E(3,7)$. So, $x=3$ and $y=7$.



Hence, number of boys is 3 and number of girls is 7.

(b) Let the cost of one pencil be Rs. x and one pen be Rs. y.

Then, the equations formed are

$$5x+7y=50 \qquad ...(1)$$

and

$$7x+5y=46 \qquad ...(2)$$

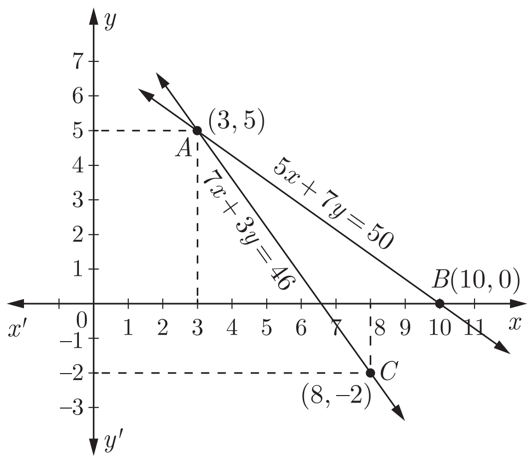
From equation (1) $y=\dfrac{50-5x}{7}$

x	10	3
y	0	5

From equation (2) $y=\dfrac{46-7x}{5}$

x	8	3
y	-2	5

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $A(3,5)$., Hence $x=3$ and $y=5$ is the required solution. Cost of one pencil is Rs. 3 and cost of one pen is Rs.5.

8. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Ans : [NCERT]

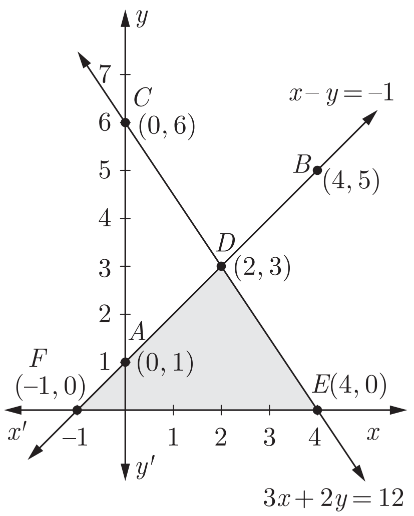
We have $\quad x-y+1=0 \qquad ...(1)$

x	0	4	2
$y=x+1$	1	5	3

and $\quad 3x+2y-12=0 \qquad ...(2)$

x	0	2	4
$y=\dfrac{12-3x}{2}$	6	3	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $D(2,3)$. Hence, $x=2$ and $y=3$ is the solution of the given pair of equations. The line CD intersects the x-axis at the point $E(4,0)$ and the line AB intersects the x-axis at the points $F(-1,0)$. Hence, the co-ordinates of the vertices of the triangle are $D(2,3)$, $E(4,0)$ and $F(-1,0)$.

9. Solve the following pair of linear equations graphically: $2x+3y=12$ and $x-y=1$
Find the area of the region bounded by the two lines representing the above equations and y-axis.

Ans : [Board Term-1, 2012, Set-58]

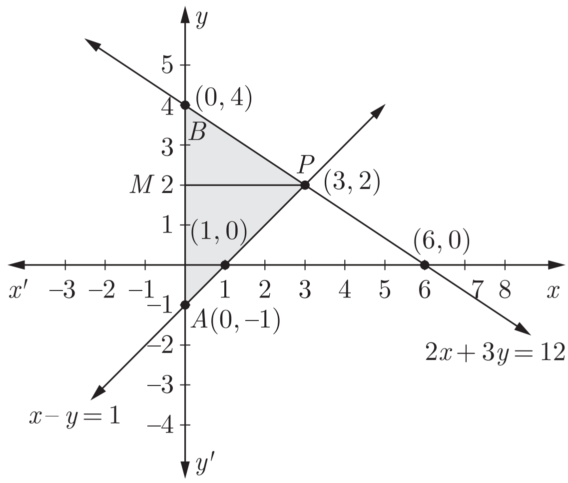
We have $\quad 2x+3y=12 \Rightarrow y=\dfrac{12-2x}{3}$

x	0	6	3
y	4	0	2

We have $\quad x-y=1 \Rightarrow y=x-1$

x	0	1	3
y	1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $p(3,2)$. Hence, $x=3$ and $y=2$
Area of shaded triangle region,

$$\text{Area of } \Delta PAB = \frac{1}{2}\times base \times height$$

$$= \frac{1}{2}\times AB \times PM$$

$$= \frac{1}{2}\times 5\times 3$$

$$= 7.5 \text{ square unit.}$$

10. Solve the following pair of linear equations graphically: $x+3y=12$, $2x-3y=12$
Also shade the region bounded by the line $2x-3y=2$ and both the co-ordinate axes.

Ans : [Board Term-1, 2013 FFC, 2012, Set-35, 48]

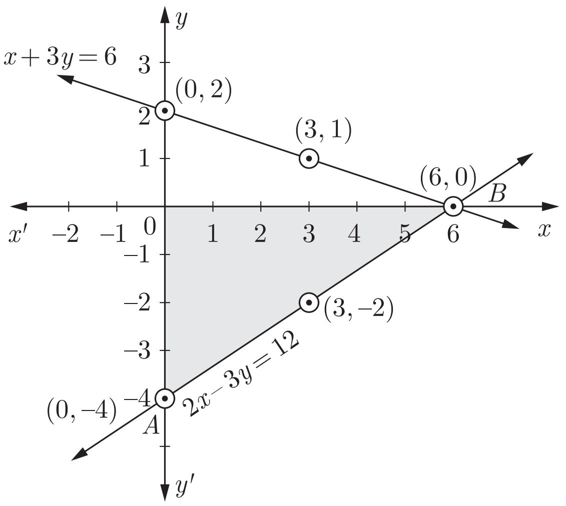
We have $\quad x+3y=6 \Rightarrow y=\dfrac{6-x}{3} \qquad ...(1)$

x	3	6	0
y	1	0	2

and $\quad 2x-3y=12 \Rightarrow y=\dfrac{23-12}{3}$

x	0	6	3
y	-4	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $B(6,0)$.
Hence, $x=6$ and $y=0$
Again ΔOAB is the region bounded by the line $2x-3y=12$ and both the co-ordinate ases. 1

11. Solve the following pair of linear equations graphically:
$x-y=1,\ 2x+y=8$
Also find the co-ordinates of the points where the lines represented by the above equation intersect $y-axis$.

Ans : [Board Term-1, 2012, Set-56]

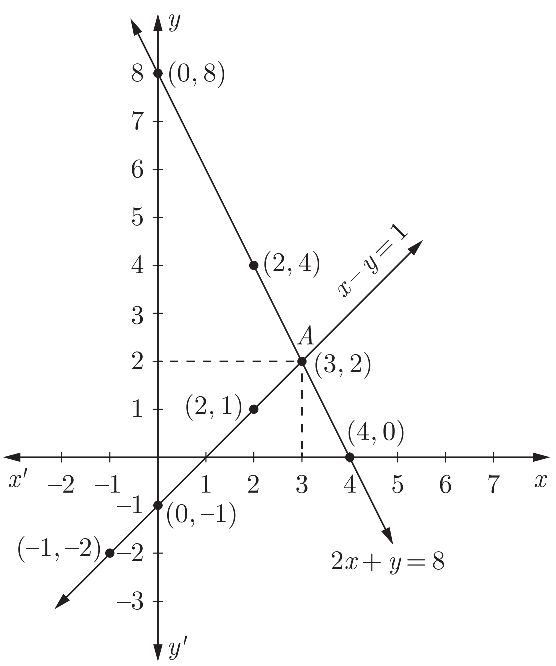
We have $x-y=1 \Rightarrow y=x-1$

x	2	3	-1
y	1	2	-2

and $2x+y=8 \Rightarrow y=8-2x$

x	2	4	0
y	4	0	8

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $A(3,2)$. Thus solution of given equations is $x=3, y=2$.

Again, $x-y=1$ intersects $y-axis$ at $(0,-1)$
and $2x+y=8\ y-axis$ at $(0,8)$.

12. Draw the graph of the following equations:

$$2x-y=1,\ x+2y=13$$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

Ans : [Board Term-1, 2012 Set-52]

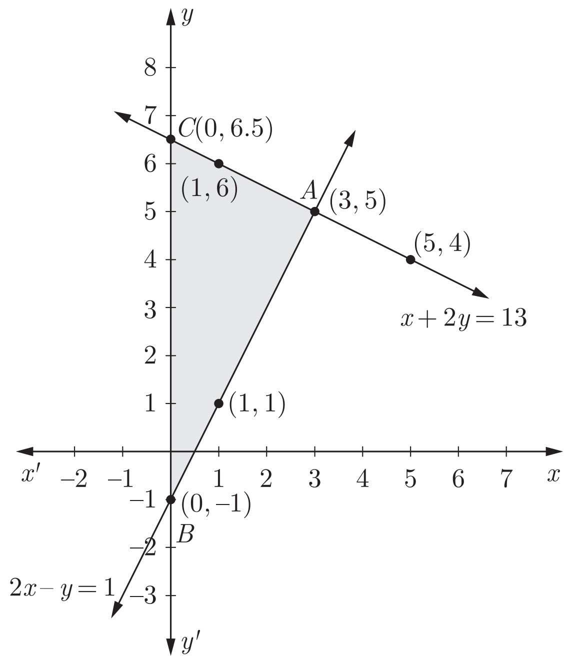
We have $2x-y=1 \Rightarrow y=2x-1$

x	0	1	3
y	-1	1	5

and $x+2y=13 \Rightarrow y=\dfrac{13-x}{2}$

x	1	3	5
y	6	5	4

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two obtained lines intersect at point $A(3,5)$.

Hence, $x=3$ and $y=5$
ABC is the triangular shaded region formed by the obtained lines with the y-axis. 1

13. Solve the following pair of equations graphically:

$$2x+3y=12,\ x-y-1=0.$$

Shade the region between the two lines represented by the above equations and the X-axis.

Ans : [Board Term-1, 2012, Set-48]

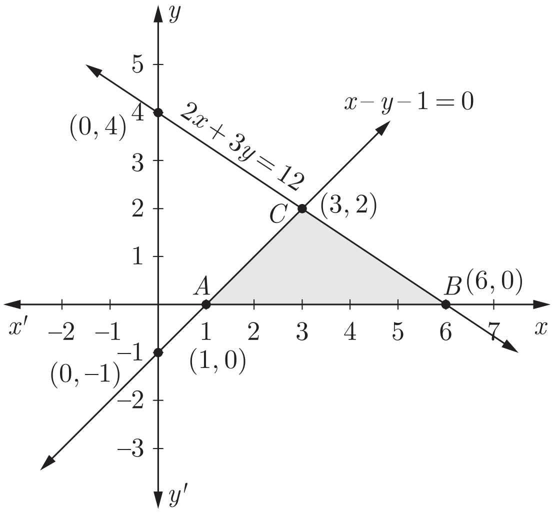
We have $2x+3y=12 \Rightarrow y=\dfrac{12-2x}{3}$

x	0	6	3
y	4	0	2

also $x - y = 1 \Rightarrow y = x - 1$

x	0	1	3
y	-1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $(3,2)$, Hence, $x = 3$ and $y = 2$.
ΔABC is the region between the two lines represented by the given equations and the X-axis.

SHORT ANSWER TYPE QUESTIONS - I

1. Solve the following pair of linear equations by cross multiplication method:

$$x + 2y = 2$$
$$x - 3y = 7$$

Ans : [Board Term-1, 2016, Set-O4YP6G7]

We have $x + 2y - 2 = 0$

$$x - 3y - 7 = 0$$

Using the formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

we have $$\frac{x}{-14-6} = \frac{y}{-2+7} = \frac{1}{-3-2}$$

$$\frac{x}{-20} = \frac{y}{5} = \frac{-1}{5}$$

$$\frac{x}{-20} = \frac{-1}{5} \Rightarrow x = 4$$

$$\frac{y}{5} = \frac{-1}{5} \Rightarrow y = -1$$

2. Solve the following pair of linear equations by substitution method:

$$3x + 2y - 7 = 0$$
$$4x + y - 6 = 0$$

Ans : [Board Term-1, 2015, CJTOQ]

We have $3x + 2y - 7 = 0$ ...(1)

$4x + y - 6 = 0$ ...(2)

From equation (2), $y \; 6 - 4x$ ...(3)

Putting this value of y in equation (1) we have

$$3x + 2(6 - 4x) - 7 = 0$$
$$3x + 12 - 8x - 7 = 0$$
$$5 - 5x = 0$$
$$5x = 5$$

Thus $x = 1$

Substituting this value of x in (2), we obtain,

$$y = 6 - 4 \times 1 = 2$$

Hence, values of x and y are 1 and 2 respectively.

3. Solve the following pairs of linear equations by the substitution method:

(a) $x + y = 14$

$x - y = 4$

(b) $s - t = 3$

$\frac{s}{3} + \frac{t}{2} = 6$

(c) $3x - y = 3$

$9x - 3y = 9$

(d) $0.2x + 0.3y = 2.3$

$0.4x + 0.5y = 2.3$

(e) $\sqrt{2x} + \sqrt{3y} = 0$

$\sqrt{3x} - \sqrt{8y} = 0$

(f) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Ans : [NCERT]

Given,

(a) We have $x + y = 14$ ...(1)

and $x - y = 4$ ...(2)

or, $y = x - 4$ ...(3)

Substituting the value of y from equation (3) in equation (1), we get

$$x + x - 4 = 14$$
$$2x = 18$$

Thus $x = 9$

Substituting this value of x in equation (3), we get

$$y = 9 - 4$$

i.e., $y = 5$

Hence, $s = 9$ and $y = 5$

(b) We have, $s - t = 3$ ...(1)

and $\frac{s}{3} + \frac{t}{2} = 6$ ...(2)

From equation (1), $x = t + 3$ ...(3)

Substituting $x = t + 3$ in equation (2), we get

$$\frac{t+3}{3} + \frac{t}{2} = 6$$

$$\begin{aligned} 2(t+3)+3t &= 36 \\ 5t+6 &= 36 \\ 5t &= 30 \\ t &= 6 \end{aligned}$$

Substituting this value of t in (1) we get

$$s = 6+3 = 9$$

Hence, $s = 9, t = 6$

(c) We have $3x - y = 3$ ...(1)

and $9x - 3y = 9$ ...(2)

From equation (1) $y = 3x - 3$ ...(3)

Substituting this value of y in equation (3),

$$\begin{aligned} 9x - 3(3x-3) &= 9 \\ 9 &= 9 \end{aligned}$$

Hence, x and y both have infinitely many solutions

(d)We have $0.2x + 0.3y = 1.3$ ...(1)

and $0.4x + 0.5y = 2.3$ ...(2)

From equation (1) we have

$$\begin{aligned} 3y &= 13 - 2x \\ y &= \frac{13-2x}{3} \quad ...(3) \end{aligned}$$

Substituting this value of y in equation (2),

$$\begin{aligned} \frac{4}{10}x + \frac{5}{10} \times \frac{(13-2x)}{3} &= \frac{23}{10} \\ 4x + \frac{5}{3}(13-2x) &= 23 \\ 12x + 5(13-2x) &= 3 \times 23 \\ 12x + 65 - 10x &= 69 \\ 2x &= 69 - 65 = 4 \\ x &= 2 \end{aligned}$$

Substituting $x = 2$ in equation (3), we get

$$\begin{aligned} y &= \frac{13 - 2 \times 2}{3} = \frac{9}{3} \\ y &= 3 \end{aligned}$$

Hence, $x = 2,\ y = 3$

(d) We have $\sqrt{2}\,x + \sqrt{3}\,y = 0$ ...(1)

and $\sqrt{3}\,x - \sqrt{8}\,y = 0$ ...(2)

or $y = \frac{\sqrt{3}\,x}{\sqrt{8}}$ ...(3)

Substituting y from equation (3) in equation (1),

$$\begin{aligned} \sqrt{2}\,x + \sqrt{3} \times \left[\frac{\sqrt{3}\,x}{\sqrt{8}}\right] &= 0 \\ \sqrt{2}\,x + \frac{3x}{\sqrt{8}} &= 0 \\ \sqrt{2}\,x \times \sqrt{8} + 3x &= 0 \\ \sqrt{16x} + 3x &= 0 \\ 4x + 3x &= 0 \\ 7x &= 0 \end{aligned}$$

Thus $x = 0$

Substituting $x = 0$ in equation (3), we have

$$\begin{aligned} y &= \frac{\sqrt{3} \times 0}{\sqrt{8}} = 0 \\ y &= 0 \end{aligned}$$

Hence, $x = 0,\ y = 0$

(e) We have $\frac{3x}{2} - \frac{5y}{3} = -2$ ...(1)

and $\frac{x}{3} + \frac{y}{3} = \frac{13}{6}$ ...(2)

From equation (2), $\frac{y}{52} = \frac{13}{6} - \frac{x}{3} = \frac{13-2x}{6}$

$$y = 2 \times \frac{(13-2x)}{6} = \frac{(13-2x)}{3} \quad (3)$$

Substituting this value of y in equation (1),

$$\begin{aligned} \frac{3x}{2} - \frac{5}{3} \times \frac{(13-2x)}{3} &= -2 \\ \frac{3x}{2} - \frac{5}{9}(13-2x) &= -2 \\ 27x - 10(13-2x) &= -36 \\ 27x - 130 + 20x &= -36 \\ 27x + 20x &= 130 - 36 \\ 47x &= 94 \\ x &= 2 \end{aligned}$$

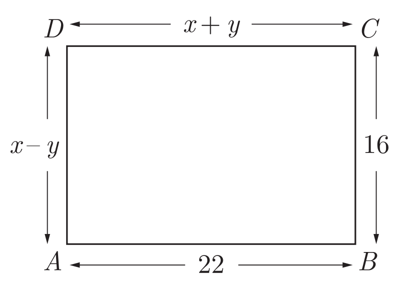
Substituting $x = 2$ in equation (3), we have

$$\begin{aligned} y &= \frac{13 - 2 \times 2}{3} = \frac{9}{3} \\ y &= 3 \end{aligned}$$

Hence, $x = 2, y = 3$

4. In the figure given below, $ABCD$ is a rectangle. Find the values of x and y.

Ans : [Board Term-1, 2012, Set-30]



From given figure we have

$x + y = 22$ ...(1)

and $x - y = 16$ ...(2)

Adding (1) and (2), we have

$$\begin{aligned} 2x &= 38 \\ x &= 19 \end{aligned}$$

Substituting the value of x in equation (1), we get

$$\begin{aligned} 19 + y &= 22 \\ y &= 22 - 19 = 3 \end{aligned}$$

Hence, $x = 19$ and $y = 3$.

5. Solve : $99x + 101y = 499$, $101x + 99y = 501$

Ans : [Board Term-1, 2012, Set-55]

We have
$$99x + 101y = 499 \quad ...(1)$$
$$101x + 99y = 501 \quad ...(2)$$

Adding equation (1) and (3), we have
$$200x + 200y = 1000$$
or,
$$x + y = 5 \quad ...(3)$$

Subtracting equation (2) from equation (3), we get
$$-2x + 2y = -2$$
or,
$$x - y = 1 \quad ...(4)$$

Adding equations (3) and (4), we have
$$2x = 6$$
$$x = 3$$

Substituting the value of x in equation (3), we get
$$3 + x = 5$$
$$y = 2$$

6. Solve the following system of linear equations by substitution method:
$$2x - y = 2$$
$$x + 3y = 15$$

Ans : [Board Term-1, 2012, Set-50]

We have
$$2x - y = 2 \quad ...(1)$$
$$x + 3y = 15 \quad ...(2)$$

From equation (1), we get $y = 2x - 2$ ...(3)

Substituting the value of y in equation (2),
$$x + 6x - 6 = 15$$
or,
$$7x = 21$$
$$x = 3$$

Substituting this value of x in (3), we get

From equation (1), we have
$$y = 2 \times 3 - 2 = 4$$
$$x = 3 \text{ and } y = 4$$

7. Find the value(s) of k for which the pair of Linear equations $kx + y = d^2$ and $x + ky = 1$ have infinitely many solutions.

Ans : [Sample Paper 2017]

We have
$$kx + y = k^2$$
and
$$x + ky = 1$$
$$\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$$

For infinitely many solution
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{k}{1} = \frac{1}{k} = \frac{k^2}{1} = k^2 = 1$$
$$k = \pm 1$$

SHORT ANSWER TYPE QUESTIONS - II

1. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

Ans : [CBSE Marking Scheme, 2015]

Let age of father and son be x and y respectively.
$$x = y = 40 \quad ...(1)$$
$$x = 3y \quad ...(2)$$

By solving equations (1) and (2), we get
$$x = 30 \text{ and } y = 10$$

Ages are 30 years and 10 years.

2. Solve using cross multiplication method:
$$5x + 4y - 4 = 0$$
$$x - 12y - 20 = 0$$

Ans : Board Term-1, 2015, Set-FHN8MGD]

We have
$$5x + 4y - 4 = 0 \quad ...(1)$$
$$x - 12y - 20 = 0 \quad ...(2)$$

By cross-multiplication method,
$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{b_1b_2 - a_2b_1}$$
$$\frac{x}{-80-48} = \frac{y}{-4+100} = \frac{1}{-60-4}$$
$$\frac{x}{-128} = \frac{y}{96} = \frac{1}{64}$$
$$\frac{x}{-128} = \frac{1}{-64} \Rightarrow x = 2$$
and
$$\frac{y}{96} = \frac{1}{-64} \Rightarrow y = \frac{-3}{2}$$

Hence, $x = 2$ and $y = \frac{-3}{2}$

3. A part of monthly hostel charge is fixed and the remaining depends on the number of days on e has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs. 3,000 as hostel charges whereas Mansi who takes food for 25 days Rs. 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

Ans : [Term-1, 2016, MV98HN3, Term-1, 2015, FHN8MGD]

Ans :

Let fixed charge be x and per day food cost be y
$$x + 20y = 3000 \quad ...(1)$$
$$x + 25y = 3500 \quad ...(2)$$

Subtracting (1) from (2) we have
$$5y = 500 \Rightarrow y = 100$$

Substituting this value of y in (1), we get
$$x + 20(100) = 3000$$
$$x = 1000$$

Thus $x = 1000$ and $y = 100$

Fixed charge and cost of food per day are Rs. 1,000 and Rs. 100.

4. Solve for x and y :

$$\frac{x}{2}+\frac{2y}{3}=-1$$

$$x-\frac{y}{3}=3$$

Ans : [Board Term-1, 2015, CJTOQ, NCERT]

We have $\frac{x}{2}+\frac{2y}{3}=-1$

or $3x+4y=-6$ ...(1)

and $\frac{x}{1}-\frac{y}{3}=3$

or $3x+y=9$ ...(2)

Subtracting equation (2) from equation (1), we have

$$5y=-15 \Rightarrow y=-1$$

Substituting $y=-3$ in eq (1), we get

$$3x+4(-3)=-6$$

$$3x-12=-6$$

$$3x=12-6$$

Thus $x=2$

Hence $x=-2$ and $y=-3$.

5. From the pair of linear equations for the following problems and find their solution by substitution method:

(a) The difference between two numbers is 26 and one number is three times the other. Find the numbers.

(b) The larger of two supplementary angles exceeds the smaller by 18 degree. Find the angles.

(c) The coach of cricket team buys 7 bats and 6 balls for Rs. 3,800. Later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball.

(d) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10km, the charge paid is Rs. 105 and for a journey of 15 km the charge paid is Rs. 155. What are the fixed charges and the charge per km?

How much does a person have to pay for travelling a distance of 25 km?

(e) A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator. If 3 is added to bothe the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(f) Five year hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Ans : [NCERT]

(a) Let the two number be x and $y(x>y)$

We have $x-y=26$ ...(1)

and $x=3y$ ...(2)

Substituting the value x from equation (2) in equation (1), we get

$$3y-y=26$$

$$2y=26$$

Thus $y=13$

Substituting $y=13$ in equation (2), we get

$$x=3\times 13=39$$

Hence, the two number are 39 and 13.

(b) Let the supplementary angles be x and $y(x>y)$

Now, $x+y=180°$ ...(1)

and $x-y=18°$ ...(2)

From equation (2) we have

, $y=x-18°$ ...(3)

Substituting the value y from equation (3) in equation (1),

$$x+x-18°=180°$$

$$2x=198°$$

$$x=99°$$

Substituting $x=99°$ in equation (3) in equation(1),

$$y=99°-18°=81°$$

Thus $y=81°$

Hence, the angles are 99° and 81°

(c) Let, cost one bat and a ball be Rs. x and Rs. y respectively

$7x+6y=3,800$ ...(1)

and $3x+5y=1750$ ...(2)

From equation (2), we have

$$5y=1750-3x$$

$y=\frac{1750-3x}{5}$ ...(3)

Substituting y from equation (3) in equation (1),

$$7x+6\times\frac{1750-3x}{5}=3,800$$

$$35x+6\times(1750-3x)=5\times 3,800$$

$$35x+10500-18x=19000$$

$$35-18x=19000-10500$$

$$17x=8,500$$

Thus $x=500$

Substituting $x=500$ in equation (3) we have,

$$y=\frac{1750-3\times 500}{5}$$

$$=\frac{1750-1500}{5}=\frac{250}{5}=50$$

Thus $y=50$

Hence, cost of one bat $=500$ Rs

and cost of one ball $=50$ Rs

(d) Let fixed charge be Rs. x and charge per km be Rs. y.

$x+10y=105$ ...(1)

$x+15y=155$ ...(2)

From equation (1) we have

$x=105-10Y$ ...(3)

Substituting x from equation (3) in equation (2),

$$105-10y+15y = 155$$
$$5y = 155-105=50$$
$$y = 10$$

Substituting $=10y$ in equation (3),

$$x = 105-10\times 10$$
$$= 105-100=5$$

Thus $x=5$

Hence, fixed charges $= Rs.5$

Rate per km $= Rs.10$

Amount to be paid for travelling 25km

$$= 5+10\times 25$$
$$= 5+250=255 \text{ Rs}$$

(e) Let $\frac{x}{y}$ be the fraction, where x and y are positive integers.

We have $\frac{x+2}{y+2}=\frac{9}{11}$ and $\frac{x+3}{y+3}=\frac{5}{6}$

From 1st equation we have

$$11\times(x+2) = 9\times(y+2)$$
$$11x+22 = 9y+18$$
$$11-9y+4 = 0$$

From 2nd equation we get

$$6\times(x+3) = 5\times(y+3)$$
$$6x+18 = 5y+15$$
$$6x-5y+3 = 0$$
$$11x-9y+4 = 0 \quad ...(1)$$

and $$6x-5y+3 = 0 \quad ...(2)$$

From equation (2),

$$5y = 6x+3$$

or, $$y = \frac{6x+3}{5} \quad ...(3)$$

Substituting y from equation (3) in equation (1),

$$11x-9\times\left(\frac{6x+3}{5}\right)+4 = 0$$

$$55x-9\times(6x+3)+20 = 0$$

Thus $$x = 7$$

Substituting $x=7$ in equation (3),

$$y = \frac{6\times 7+3}{5}$$

$$y = 9$$

Hence, the required fraction$=\frac{7}{9}$

(f) Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob. 5years hence, it has relation :

$$(y+5) = 3(x+5)$$
$$y+5 = 3x+15$$
$$3x-y+10 = 0 \quad ...(1)$$

5 years ago, it has relation,

$$(y-5) = 7(x-5)$$
$$7x-y-30 = 0 \quad ...(2)$$

From equation (1), we have

$$y = 3x+10 \quad ...(3)$$

Substituting the value of y in equation (2) we get

$$7x-(3x+10)-30 = 0$$
$$4x-40 = 0$$
$$x = 10$$

Substituting $x=10$ in equation (3), we get

$$y = 3\times 10+10$$

Thus $$y = 40$$

Hence, the present age of Jacob = 40 years and son's age = 10 years

6. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(a) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. it becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

(b) Five years ago, Nuri was thrice as old as Sonu. Ten years leter, Juri will bw twice as old as Sonu. How old are Nuri and sonu ?

(c) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(d) Meena went to a bank to withdraw Rs. 2,000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs 100 she received.

(e) A lending library has fixed charge for the first three days and as additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Ans : [NCERT]

(a) Let the fraction be $\frac{x}{y}$

According to the given conditions we have

$$\frac{x+1}{y-1} = 1 \text{ and } \frac{x}{y+1}=\frac{1}{2}$$

$$x+1 = y-1 \text{ and } 2x=y+1$$

or, $$x-y = -2 \quad ...(1)$$

and $$2x-y = 1 \quad ...(2)$$

Subtracting equation (1) from equation (2), we have

$$(2x-y)-(x-y) = 1+2$$
$$x = 3$$

Substituting $x=3$ in equation (1), we have

$$3-y = -2$$
$$y = 5$$

Hence, the fraction is $\frac{3}{5}$.

(b) Let the present age of Nuri be x and Present age of Sonu be y.

According to the given conditions,

5 years ago, $$x-5 = 3(y-5)$$

$$x - 3y = -10 \quad ...(1)$$

10 year later, $x + 10 = 2(y + 10)$

$$x - 2y = 10 \quad ...(2)$$

Subtracting equation (1) from equation (2), we have

$$(x - 2y)(x - 3y) = 10 + 10$$

or, $-2y + 3y = 20$

$$y = 20$$

Substituting $y = 20$ in equation (2), we have

$$x = 2 \times 20 + 10 = 50$$

Thus $x = 50$

Hence, Present age of Nuri is 50 years and present age of Sonu is 20 years.

(c) Let x be the digit at unit's place and y be the digit at ten's place.

According to the given condition

$$x + y = 9 \quad ...(1)$$

Value of the number $= x + 10y$

When the order of the digits is reversed, the value of the new number $= y + 10x$

We have,

$$9 \times (x + 10y) = 2 \times (y + 10x)$$

$$9x + 90y = 2y + 20x$$

$$88y = 11x$$

$$x = 8y \quad ...(2)$$

Substituting this value of x in equation (1), we have

$$8y + y = 9$$

$$9y = 9$$

$$y = \frac{9}{9} = 1$$

and $x = 8 + 1 = 9$

Hence, the number.

$$10y + x = 10 \times 1 + 8 = 18$$

(d) let number of Rs. 50 notes be x

Number of Rs. 100 notes be y

According to the given condition,

$$x + y = 25$$

and $50 \times x + 100 \times y = 2,000$

or, $x + 2y = 40 \quad ...(2)$

Substituting equation (1) from equation (2),

$$(x + 2y) - (x + y) = 40 - 25$$

$$y = 15$$

Substituting $y = 15$ in equation (1),

$$x + 15 = 25$$

$$x = 10$$

Hence number of Rs.50 notes $= 10$

and number of Rs. 100 notes$= 15$

(e) Let the fixed charges for the first three days be Rs. x. Let the additional change per day be Rs. y.

According to the given conditions,

Sarita paid for 7 days = Rs.27

$$x + 4 \times y = 27$$

$$x + 4y = 27 \quad ...(1)$$

[$\therefore$ Rs. $4y$ are to be paid for extra 4 days]

In the case of Susy,

$$x + 2y = 21 \quad ...(2)$$

[$\because$ Rs. $2y$ are to be paid for extra 2 days]

Subtracting equation (2) from equation (1),

$$2y = 27 - 21$$

or, $2y = 6$

Thus $y = 3$

Substituting this value of y in equation (1),

$$x + 4 \times 3 = 27$$

$$x = 27 - 12$$

$$x = 15$$

Hence, fixed charges for first three dauys = Rs 15 Additional charges per extra day = Rs 3. 1

7. Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions ? In case there is a unique solution, find it by using cross-multiplication method.

(a) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

(b) $2x + y = 5$

$3x + 2y = 8$

(c) $3x - 5y = 20$

$6x - 10y = 40$

(d) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Ans : [NCERT]

(a) We have $x - 3y - 3 = 0 \quad ...(1)$

and $3x - 3y - 2 = 0 \quad ...(2)$

Comparing equation (1) and (2) with $ax + by + c = 0$

$$a_1 = 1, b_1 = -3, c_1 = -3$$

and $a_2 = 3, b_2 = -9, c_2 = -2$

Thus $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{3}{2}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, no solution exists.

(b) We have $2x + y = 5$

or, $2x + y - 5 = 0 \quad ...(1)$

and $3x + 2y = 8$

or, $3x + 2y - 8 = 0 \quad (2)$

Comparing equation (1) and (2) with $ax + by + c = 0$,

$$a_1 = 2, b_1 = 1, c_1 = -5$$

and $a_2 = 3, b_2 = 2, c_2 = -8$

Now, $\frac{a_1}{a_2} = \frac{2}{3}$ and $\frac{b_1}{b_2} = \frac{1}{2}$

Since $\frac{2}{3} \neq \frac{1}{2}$ $\quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, a unique solution exists.

By cross multiplication method,

$$\frac{x}{b_2c_1-b_1c_2}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\frac{x}{\{(1)(-8)-(2)(-5)\}}=\frac{x}{\{(-5)(3)-(-8)(2)\}}$$

$$=\frac{1}{\{(2)(2)-(3)(1)\}}$$

$$\frac{x}{(-8+10)}=\frac{y}{(-15+16)}=\frac{1}{(4-3)}$$

$$\frac{x}{2}=\frac{y}{1}=\frac{1}{1}$$

Thus $\frac{x}{2}=\frac{1}{1}$ and $\frac{y}{1}=\frac{1}{1}$

Hence $x=2$ and $y=1$ 1

(c) We have $3x-5y=20$

or, $3x-5y-20=0$ ...(1)

and $6x-10y=40$

or, $6x-10y-40=0$ ...(2)

Comparing equation (1) and (2) with $ax+by+c=0$,

$$a_1=3, b_1=-5, c_1=-20$$

and $a_2=6, b_2=-10, c_2=-40$

Now, $\frac{a_1}{a_2}=\frac{3}{6}, \frac{b_1}{b_2}=\frac{-5}{-10}$ and $\frac{c_1}{c_2}=\frac{-20}{-40}$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{2}$$

Hence, infinitely many solutions exist.

(d) We have $x-3y-7=0$ ...(1)

and $3x-3y-15=0$ ...(2)

Comparing equation(1) and (2) with $ax+by+c=0$,

$$a_1=1, b_1=-3, c_1=-7$$

and $a_2=3, b_2=-3, c_2=-15$

Here $\frac{a_1}{a_2}=\frac{1}{3}$ and $\frac{b_1}{b_2}=\frac{1}{3}=1$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

Hence, a unique solution exists.

By cross multiplication,

$$\frac{x}{\{(-3)(-15)-(-3)(-7)\}}=\frac{y}{\{(-7)(3)-(-15)(1)\}}$$

$$=\frac{1}{\{(1)(-3)-(3)(-3)\}}$$

$$\frac{x}{(45-21)}=\frac{y}{(-21+15)}=\frac{1}{(-3+9)}$$

$$\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}$$

$$\frac{x}{24}=\frac{1}{6} \text{ and } \frac{y}{-6}=\frac{1}{6}$$

$$x=\frac{24}{6} \text{ and } y=\frac{-6}{6}$$

Hence, $x=4$ and $y=-1$

8. Solve the following pair of linear equations by the substitution and cross - multiplication method :

$$8x+5y=9$$
$$3x+2y=4$$

Ans : [NCERT]

We have $8x+5y=9$

or, $8x+5y-9=0$ ...(1)

and $3x+2y=4$

or, $3x+2y-4=0$ ...(2)

Comparing equation (1) and (2) with $ax+by+c=0$,

$$a_1=8, b_1=5, c_1=-9$$

and $a_2=3, b_2=2, c_2=-4$

By cross-multiplication method,

$$\frac{x}{b_2c_1-b_1c_2}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-b_2b_1}$$

$$\frac{x}{\{(5)(-4)-(2)(-9)\}}=\frac{y}{\{(-9)(3)-(-4)(8)\}}$$

$$=\frac{1}{\{8\times2-3\times5\}}$$

or, $\frac{x}{-2}=\frac{1}{1}$ and $\frac{y}{5}=\frac{1}{1}$

$$x=-2 \text{ and } y=5$$

We use substitution method.

From equation (2), we have

$$3x=4-2y$$

or, $x=\frac{4-2y}{3}$ ...(3)

Substituting this value of y in equation (3) in (1), we get

$$8\left(\frac{4-2y}{3}\right)+5y=9$$

$$32-16y+15y=27$$

$$-y=27-32$$

Thus $y=5$

Substituting this value of y in equation (3)

$$x=\frac{4-2(5)}{3}=\frac{4-10}{3}=-2$$

Hence, $x=-2$ and $y=5$.

9. From the pair of linear equations in the following problems and find their solutions (if they exist) any algebraic method :

(a) A part of monthly lostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1,000 as hostel charges whereas a student B, who takes food for 26 days, pay Rs 1,180 as hostel sharges. Find the fixed charges and the cost of food per day.

(b) A fraction becomes $\frac{1}{3}$,when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$, when 8 is added to its denomiantor. find the fraction.

(c) Yesh scored 40 marks in a test, getting 3 marks for each right answer and losing 1 marks for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

(d) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the some time. If the cars travel in the same direction at different speeds, they meet in 5 hr. If they travel towards each other, they meet in 1 hr. What are the speeds of the two cars ?

Ans :

(a) Let fixed part of monthly hostel charges be y and cost of food for one day be x.

In the case of student A,

$$20x + y = 1{,}000 \qquad ...(1) \ 1$$

In the case of student B,

$$26x + y = 1{,}180 \qquad ...(2)$$

Subtracting equation (1) from equation (2),

$$x = 30$$

Substituting $x = 30$ in equation (1),

$$y = 1{,}000 - 20 \times 30$$

$$= 1{,}000 - 600 = 400$$

Hence, monthly dixed charges = Rs 400

Cost of food per day = Rs 30

(2) Let the fraction be $\frac{x}{y}$,

According to the given conditions,

$$\frac{x-1}{y} = \frac{1}{3} \text{ and } \frac{x}{y+8} = \frac{1}{4}$$

We get, $\quad 3x - y = 3 \qquad ...(1)$

and $\quad 4x - y = 8 \qquad ...(2)$

Subtracting equation (1) from equation (2), we get

$$x = 5$$

Substituting $x = 5$ in equation (1), we have

$$3 \times 5 - y = 3$$

$$y = 12$$

Hence, the fraction $= \frac{5}{12}$

(c) Let the number of right answer be x and the number of wrong answers be y.

Total number of question $= x + y$

In first case,

Marks awarded for x right answer $= 3x$

Marks lost for y wrong answer $= y \times 1 = y$

$$3x - y = 40 \qquad ...(1)$$

In second case,

Marks awarded for x right answers $= 4x$

Marks lost for y wrong answers $= 2y$

$$4x - 2y = 50 \qquad ...(2)$$

From equation (1), $y = 3x - 40 \qquad ...(3)$

Substituting the value of y from equation (3) in equation (2),

$$4x - 2(3x - 40) = 50$$

$$4x - 6x + 80 = 50$$

$$2x = 30$$

$$x = 15$$

Substituting the value of x in equation (3),

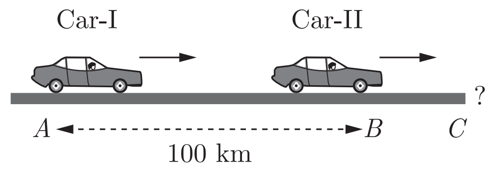
$$y = 3 \times 15 - 40 = 5$$

Total number of questions $= x + y = 15 = 5 = 20$ 1

(d) Let speed of car I be x km/hr and speed of car II be y km/hr.

Car I starts from point A and car II starts from point B

First Case :



Two cars meet at C after 5 hr.

AC = Distance travelled by car I in 5 hr $= 5x$ km

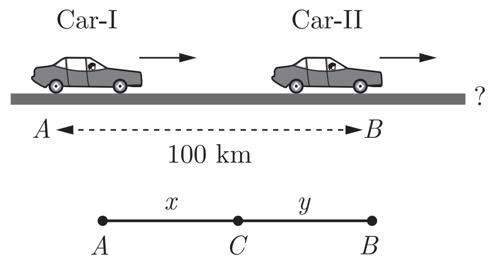
BC = Distance travelled by car II in 5 hr $= 5y$ km

Since, $\quad AC - BC = AB$

So, $\quad 5x - 5y = 100$

or, $\quad x - y = 20$

Second Case :



Two cars meet at C after one hour, thus

$$x + y = 100 \qquad ...(2)$$

Adding equation (1) and (2), we have

$$2x = 120$$

$$x = 60$$

Substituting $x = 60$ in equation (2), we have

$$60 + y = 100$$

$$y = 40$$

10. 2 man and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it ?

Ans : [Board Term-1, 2013, LK-59]

Let the man can finish the work in x days and the boy can finish work in y days.

Work done by one man in one day $= \frac{1}{x}$

And work done by one boy in one day $= \frac{1}{y}$

$$\frac{2}{x} + \frac{7}{y} = \frac{1}{4} \qquad ...(1)$$

and $$\frac{4}{x}+\frac{4}{y} = \frac{1}{3} \qquad ...(2)$$

Let $\frac{1}{x}$ be a and $\frac{1}{y}$ be b, then we have

$$2a+7b = \frac{1}{4} \qquad ...(3)$$

and $$4a+4b = \frac{1}{3} \qquad ...(4)$$

Multiplying equation (3) by 2 and subtract equation (4) from it

$$10b = \frac{1}{6}$$

$$b = \frac{1}{60} = \frac{1}{y}$$

Thus $y = 60$ days.

Substituting $b = \frac{1}{60}$ in equation (3), we have

$$2a+\frac{7}{60} = \frac{1}{4}$$

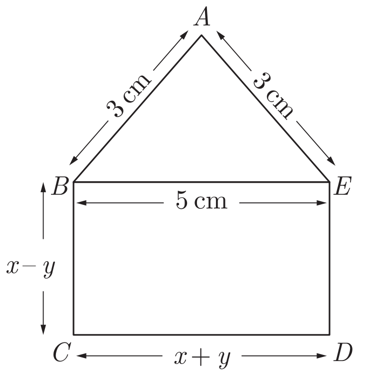
$$2a = \frac{1}{4}-\frac{7}{60}$$

$$a = \frac{1}{15}$$

Now $$\frac{1}{15} = \frac{1}{x}$$

Thus $x = 15$ days.

11. In the figure below $ABCDE$ is a pentagon with $BE || CD$ and $BC || DE$. BC is perpendicular to DC. If the perimeter of $ABCDE$ is 21 cm, find the values of x and y.



Ans : [Board Term-I, 2011, Set-45, 53]

Since $BC || DE$ and $BE || CD$ with $BC \perp DC$, $BCDE$ is a rectangle.

$$BE = CD,$$

$$x+y = 5 \qquad ...(1)$$

and $$DE = BE = x-y$$

Since perimeter of $ABCDE$ is 21,

$$AB+BC+CD+DE+EA = 21$$

$$3+x-y+x+y+x-y+3 = 21$$

$$6+3x-y = 21$$

$$3x-y = 15$$

Adding equations (1) and (2), we get

$$4x = 20 \qquad ...(2)$$

$$x = 5$$

Substituting the value of x in (1), we get

$$y = 0$$

Thus $x = 5$ and $y = 0$.

12. Solve for x and y :

$$\frac{x+1}{2}+\frac{y-1}{3} = 9;\ \frac{x-1}{3}+\frac{y+1}{2} = 8.$$

Ans : [Board Term-1, 2011, Set-52]

We have $$\frac{x+1}{2}+\frac{y-1}{3} = 9$$

$$3(x+1)+2(y-1) = 54$$

$$3x+2y = 53 \qquad (1)$$

and $$\frac{x+1}{3}+\frac{y+1}{2} = 8$$

$$2(x-1)+3(y+1) = 48$$

$$2x+3y = 47 \qquad (2)$$

Multiplying equation (1)by 3 we have

$$9x+6y = 159 \qquad (3)$$

Multiplying equation (2)by 2 we have

$$4x+6y = 94 \qquad (4)$$

Subtracting equation (4) from (3) we have

$$5x = 65$$

or $$x = 13$$

Substitute the value of x in equation (2),

$$2(13)+3y = 47$$

$$3y = 47-26 = 21$$

$$y = \frac{21}{3} = 7$$

Hence, $x = 13$ and $y = 7$

13. Solve for x and y :

$$\frac{6}{x-1}-\frac{3}{y-2} = 1$$

$$\frac{5}{x-1}-\frac{1}{y-2} = 2, \text{ where } x \neq 1,\ y \neq 2.$$

Ans : [Board Term-1, 2011, Set-21]

We have $$\frac{6}{x-1}-\frac{3}{y-2} = 1 \qquad (1)$$

$$\frac{5}{x-1}-\frac{1}{y-2} = 2, \qquad (2)$$

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$. then given equations become

$$6p-3q = 1 \qquad ...(3)$$

and $$5p+q = 2 \qquad ...(4)$$

Multiplying equation (2) by 3 and adding in equation (1),

$$21p = 7$$

$$p = \frac{7}{21} = \frac{1}{3}$$

Substituting this value of p in equation (1),

$$6\left(\frac{1}{3}\right) - 3q = 1$$

$$2 - 3q = 1 \Rightarrow q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3}$

or, $x - 1 = 3 \Rightarrow x = 4$

and $\frac{1}{y-2} = q = \frac{1}{3}$

or, $y - 2 = 3 \Rightarrow y = 5$

Hence $x = 4$ and, $y = 5$.

14. Solve the following pair of equations for x and y :

$$\frac{a^2}{x} - \frac{b^2}{y} = 0, \frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b, \qquad x \neq 0; y \neq 0.$$

Ans : [Board Term-1, Set-39]

We have $\frac{a^2}{x} - \frac{b^2}{y} = 0$ (1)

$$\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b = a + b \quad (2)$$

Substituting $p = \frac{1}{x}$ and $q = \frac{1}{y}$ in the given equations,

$$a^2 p - b^2 q = 0 \quad ...(1)$$
$$a^2 bp + b^2 aq = a + b \quad ...(2)$$

Multiplying equation (1), by a

$$a^3 p - b^2 aq = 0 \quad ...(3)$$

Adding equation (2) and equation (3),

$$(a^3 + a^2 b)p = a + b$$

or, $p = \frac{(a+b)}{a^2(a+b)} = \frac{1}{a^2}$

Substituting the value of p in equation (1),

$$a^2\left(\frac{1}{a^2}\right) - b^2 q = 0 \Rightarrow q = \frac{1}{b^2}$$

Now, $p = \frac{1}{x} = \frac{1}{a^2} \Rightarrow x = a^2$

and $q = \frac{1}{y} = \frac{1}{b^2} \Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$

15. Solve for x and y :

$$ax + by = \frac{a+b}{2}$$
$$3x + 5y = 4$$

Ans : [Board Term-1, 2011, Set-44]

We have $ax + by = \frac{a+b}{2}$

or $2ax + 2by = a + b$ ...(1)

and $3x + 5y = 4$ ...(2)

Multiplying equation (1) by 5 we have

$$10ax + 10by = 5a + 5b \quad (3)$$

Multiplying equation (2) by $2b$, we have

$$6bx + 10by \ 4b \quad (4)$$

Subtracting (4) from (3) we have

$$(10a - 6b)x = 5a - 3b$$

or $x = \frac{5a - 3b}{10a - 6b} = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in equation (2), we get

$$3 \times \frac{1}{2} + 5y = 4$$

$$5y = 4 - \frac{3}{2} = \frac{5}{2}$$

$$y = \frac{5}{2 \times 5} = \frac{1}{2}$$

Hence $x = \frac{1}{2}$ and $y = \frac{1}{2}$.

16. Solve the following pair of equations for x and y :

$4x + \frac{6}{y} = 15, 6x - \frac{8}{y} = 14$

and also find the value of p such that $y = px - 2$.

Ans : [Board Term-1, 2011, Set-60]

We have $4x + \frac{6}{y} = 15$ (1)

$$6x - \frac{8}{y} = 14, \quad (2)$$

Let $\frac{1}{y} = z$, the given equations become

$$4x + 6z = 15 \quad ...(3)$$
$$6x - 8z = 14 \quad ...(4)$$

Multiply equation (3) by 4 we have

$$16x + 24z = 60 \quad (5)$$

Multiply equation (4) by 3 we have

$$18x - 24z = 24 \quad (6)$$

Adding equation (5) and (6) we have

$$34x = 102$$

$$x = \frac{102}{34} = 3$$

Substitute the value of x in equation (3),

$$4(3) + 6z = 15$$
$$6z = 15 - 12 = 3$$
$$z = \frac{3}{6} = \frac{1}{2}$$

Now $z = \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$

Hence $x = 3$ and $y = 2$.

Again $y = px - 2$

$$2 = p(3) - 2$$
$$3p = 4$$

Thus $p = \frac{4}{3}$

17. Find whether the following pair of linear equations has a unique solutions. If yes, find the solution :

$$7x - 4y = 49, 5x - 6y = 57.$$

Ans : [Board Term-1, 2011, Set-39]

We have $7x - 4y = 49$ (1)

$$5x - 6y = 57 \quad (2)$$

Comparing with the equation $a_1 x + b_1 y = c_1$,

$$a_1 = 7, b_1 = -4, c_1 = 49$$
$$a_2 = 5, b_2 = -6, c_2 = 57$$

Since, $\frac{a_1}{a_2} = \frac{7}{5}$ and $\frac{b_1}{b_2} = \frac{4}{6}$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, system has a unique solution.

Multiply equation (1) by 5 we get

$$35x - 20y = 245 \qquad (3)$$

Multiply equation (2) by 7 we get

$$35x - 42y = 399 \qquad (4)$$

Subtracting (4) by (3) we have

$$22y = -154$$
$$y = -7$$

Putting the value of y in equation (2),

$$5x - 6(-7) = 57$$
$$5x = 57 - 42 = 15$$
$$x = 3$$

Hence $x = 3$ and $y = -7$

LONG ANSWER TYPE QUESTIONS

1. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of none chair and one tabel separetely.

Ans : [Boared Term-1,2015, Set -WJQWQBN]

Let cost of 1 chair be Rs x and cost of 1 table be Rs y According to the question,

$$4x + 3y = 2100 \qquad ...(1)$$
$$5x + 2y = 1750 \qquad ...(2)$$

Multiplying equation (1) by 2 and equation (2) by 3,

$$8x + 6y = 4200 \qquad ...(3)$$
$$15x + 6y = 5250 \qquad ...(iv)$$

Subtracting equation (3) from (4) we have

$$7x = 1050$$
$$x = 150$$

Substituting the value of x in (1), $y = 500$

Thus cost of chair and tabel is Rs 150, Rs 500 respectively.

2. Solve the following pair of equations :

$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ and $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Ans : [Board Term-1, Set-WJQZQBN]

We have $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Substitute $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

$$2X + 3Y = 2 \qquad ...(1)$$
$$4X - 9Y = -1 \qquad ...(2)$$

Multiplying equation (1) by 3, and adding in (2) we get

$$10X = 5 \Rightarrow X = \frac{5}{10} = \frac{1}{2}$$

Thus $\frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$

Putting the value of X in equation (1), we get

$$2 \times \frac{1}{2} + 3y = 2$$
$$3Y = 2 - 1$$
$$Y = \frac{1}{3}$$

Now $Y = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$

Hence $x = 4$, $y = 9$.

3. Solve for x and y :

$$2x - y + 3 = 0$$
$$3x - 5y + 1 = 0$$

Ans : [Board Term-1, DDE-M]

We have $2x - y + 3 = 0 \qquad ...(1)$

$$3x - 5y + 1 = 0 \qquad ...(2)$$

Multiplying equation (1) by 5,and subtracting (2) from it we have

$$7x = -14$$
$$x = \frac{-14}{7} = -2$$

Substituting the value of x in equation (1) we get

$$2x - y + 3 = 0$$
$$2(-2)y + 3 = 0$$
$$-4 - y + 3 = 0$$
$$-y - 1 = 0$$
$$y = -1$$

Hence, $x = -2$ and $y = -1$.

4. Solve the following pairs of linear equations by elimination method and the substitution method.

(a) $x + y = 5$ and $2x - 3y = 4$

(b) $3x + 4y = 10$ and $2x - 2y = 2$

(c) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

Ans : [NCERT]

(a) By Elimination Method :

We have, $x + y = 5 \qquad ...(1)$

and $2x - 3y = 4 \qquad ...(2)$

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x + y) + (2x - 3y) = 3 \times 5 + 4$$

or, $3x + 3y + 2x - 3y = 15 + 4$

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

Substituting $x = \frac{19}{5}$ in equation (1),

$$\frac{19}{5} + y = 5$$
$$y = 5 - \frac{19}{5} = \frac{25 - 19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

By Substituting Method :

We have, $x + y = 5$ ...(1)

and $2x - 3y = 4$ ...(2)

From equation (1), $y = 5 - x$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$2x - 3y(5 - x) = 4$$
$$2x - 15 + 3x = 4$$
$$5x = 19$$
$$x = \frac{19}{5}$$

Substituting this value of x in equation (3), we get

$$y = 5 - \frac{19}{5} = \frac{6}{5}$$

Hence $x = \frac{19}{5}$ and $y = \frac{6}{5}$

(b) By Elimination Method :

We have, $3x + 4y = 10$ ...(1)

and $2x - 2y = 2$ ...(2)

Multiplying equation (2) by 2 and adding in (1),

$$(3x + 4y) + 2(2x - 2y) = 10 + 2 \times 2$$

or, $3x + 4y + 4x - 4y = 10 + 4$

or, $7x = 14$

$$y = 1$$

Hence, $x = 2$ and $y = 1$.

By Substitution Method :

We have $3x + 4y = 10$ ...(1)

and $2x - 2y = 2$ ...(2)

From equation (2) $2y = 2x - 2$

or, $y = x - 1$ ...(3)

Substituting this value of y in equation (1),

$$3x + 4(x - 1) = 10$$
$$7x = 14$$
$$x = 2$$

From equation (3), $y = 2 - 1 = 1$

Hence, $x = 2$ and $y = 1$ 1

(3) By Elimination Method :

We have, $3x - 5y = 4$ ...(1)

and $9x = 2y + 7$ ...(2)

Multiplying equation (1) by 3 and rewriting equation (2) we have

$$9x - 15y = 12 \quad ...(3)$$
$$9x - 2y = 7 \quad ...(4)$$

Subtracting equation (4) from equation (3),

$$-13y = 5$$
$$y = -\frac{5}{13}$$

Substituting value of y in equation (1),

$$3x - 5\left(\frac{-5}{13}\right) = 4$$
$$3x = 4 - \frac{25}{13}$$
$$x = \frac{27}{13 \times 3} = \frac{9}{13}$$

Hence $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

By Substituting Method :

We have $3x - 5y = 4$ ...(1)

and $9x = 2y + 7$ ...(2)

$$y = \frac{9x - 7}{2} \quad ...(3)$$

Substituting this value of y (3) in equation (1),

$$3x - 5 \times \left(\frac{9x - 7}{2}\right) = 4$$
$$6x - 45x + 35 = 8$$
$$-39x = -27$$
$$x = \frac{9}{13}$$

Substituting $x = \frac{9}{13}$ in equation (3),

$$y = \frac{9 \times \frac{9}{13} - 7}{2} = \frac{81 - 91}{2 \times 13}$$
$$= -\frac{10}{26} = -\frac{5}{13}$$

Hence, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

5. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Ans : [NCERT]

Let the actual speed of the train be x km/hr and actual time taken y hr.

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$= xy \text{ km}$$

According to the given condition, we have

$$xy = (x + 10)(y - 2)$$
$$xy = xy - 2x + 10y - 20$$
$$2x - 10 + 20 = 0$$
$$x - 5y = -10 \quad (1)$$

and $xy = (x - 10)(y + 3)$

$$xy = xy + 3x - 10y - 30$$
$$3x - 10y = 30 \quad ...(2)$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$3 \times (x - 5y) - (3x - 10y) = -3 \times 10 - 30$$
$$-5y = -60$$
$$y = 12$$

Substituting value of y equation (1),

$$x - 5 \times 12 = -10$$

or, $x = -10 + 60$

or, $x = 50$

Hence, the distance covered by the train

$$= 50 \times 12 = 600 \text{ km.}$$

6. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

Ans : [Board Term-1, 2012, Set-38]

Let the incomes of two persons be $11x$ and $7x$.
Also the expenditures of two persons be $9y$ and $5y$.

$$11x - 9y = 400 \quad ...(1)$$

and $$7x - 5y = 400 \quad ...(2)$$

Multiplying equation (1) by 5 and equation (2) by 9 we have

$$55x - 45y = 2000 \quad ...(3)$$

and $$63x - 45y = 3600 \quad ...(4)$$

Subtracting, above equation we have

$$-8x = -1600$$

or, $$x = \frac{-1,600}{-8} = 200$$

Hence Their monthly incomes are $11 \times 200 =$ Rs 2200 and $7 \times 200 =$ Rs 1400.

7. A and B are two points 150 km apart on a highway. Two cars start A and B at the same time. If they move in the same direction they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hours. Find their speeds.

Ans : [Board Term-1, 2012, Set-62, 44]

Let the speed of the car I from A be x km/hr. Speed of the car II from B be y km/hr.

Same Direction :

Distance covered by car I= 150 +(distance covered by car II)

$$15x = 150 + 15y$$

$$15x - 15y = 150$$

$$x - y = 10 \quad ...(1)$$

Opposite Direction :

Distance covered by car I + distance covered by car II = 150 km

$$x + y = 150 \quad ...(2)$$

Adding equation (1) and (2), we have

$$y = 70$$

Speed of the car I from A = 80 km/hr and speed of the car II from B = 70 km/hr.

8. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction.

Ans : [Board Term-1, 2012, Set-48]

Let the fraction be $\frac{x}{y}$ then we have

$$\frac{x-2}{y+1} = \frac{1}{2}$$

$$2x - 4 = y + 1$$

$$2x - y = 5 \quad ...(1) 1$$

Also, $$\frac{x+4}{y-3} = \frac{3}{2}$$

$$2x + 8 = 3y - 9]$$

$$2x - 3y = -17 \quad ...(2) 1$$

Subtracting equation (2) from equation (1),

$$2y = 22 \Rightarrow y = 11$$

Substituting this value of y in equation (1) we have,

$$2x - 11 = 5$$

$$x = 8$$

Hence, Fraction $= \frac{8}{11}$

9. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Ans : [Board Term-1, 2012, Set-55]

Let the number of red balls be x and white balls be y. According to the question,

$$\frac{1}{2y} = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad ...(1)$$

and $$3(x + y) - 7y = 6$$

or $$3x - 4y = 6 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) we have

$$6x - 9y = 0 \quad ...(3)$$

$$6x - 8y = 12 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

Substituting $y = 12$ in equation (1),

$$2x - 36 = 0$$

$$x = 18$$

Hence, number of red balls = 18

and number of white balls = 12

10. A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number.

Ans : [Board Term-1, 2012, Set-48]

Let the digits of number be x and y, then number will $10x + y$

According to the question, we have

$$8(x + y) - 5 = 10x + y$$

$$2x - 7y + 5 = 0 \quad ...(1)$$

also $$16(x - y) + 3 = 10x + y$$

$$6x - 17y + 3 = 0 \quad ...(2)$$

Comparing the equation with $ax + by + c = 0$ we get

$$a_1 = 2, b_1 = -1, c_1 = 5$$

$$a_2 = 6, b_2 = -17, c_2 = 3$$

Now $$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{c_1b_2 - a_2b_1}$$

$$\frac{x}{(-7)(3) - (-17)(5)} = \frac{y}{(5)(6) - (2)(3)}$$

$$= \frac{1}{(2)(-17) - (6)(-7)}$$

$$\frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\frac{x}{8} = \frac{y}{3} = 1$$

Hence, $x = 8, y = 3$

So required number$= 10 \times 8 + 3 = 83$.

11. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. find the perimeter of the rectangle.

Ans : [Board Term-1, 2012, Set-48]

Let length of given rectangle be x and breadth be y, then area of rectangle will be xy

According to the first condition we have

$$(x-5)(y+3) = xy - 9$$

or, $3x - 5y = 6$ ...(1)

According to the second condition, we have

$$(x+3)(y+3) = xy - 67$$

or, $2x + 5y = 61$ ...(2)

Multiplying equation (1) by 3 and equation (2) by 5 and then adding,

$$9x - 15y = 18$$

$$10x + 15y = 305$$

$$x = \frac{323}{19} = 17$$

Substituting this value of x in equation (1),

$$3(17) - 5y = 6$$

$$5y = 51 - 6$$

$$y = 9$$

Hence, perimeter $= 2(x+y) = 2(17+9) = 52$ units.

12. Solve for x and y : $2(3x-y) = 5xy, 2(x+3y) = 5xy$.

Ans : [Board Term-1, 2012, Set-25]

We have $2(3x-y) = 5xy$ ...(1)

$2(x+3y) = 5xy$ ...(2)

Divide equation (1) and (2) by xy,

$\frac{6}{y} - \frac{2}{x} = 5$ ...(3)

and $\frac{2}{y} + \frac{6}{x} = 5$ ...(4)

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$, then equations (3) and (4) become

$6a - 2b = 5$ ...(5)

$2a + 6b = 5$ ...(6)

Multiplying equation (5) by 3 and then adding with equation (6),

$$20a = 20$$

$$a = 1$$

Substituting this value of a in equation (5),

$$b = \frac{1}{2}$$

Now $\frac{1}{y} = a = 1 \Rightarrow y = 1$

and $\frac{1}{x} = b = \frac{1}{2} \Rightarrow x = 2$

Hence, $x = 2, y = 1$

13. The Present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Ans : [Board Term-1, 2012, Set-39]

Let the sum of the ages of the 2 children be x and the age of the father be y years.

Now $y = 2x$

$2x - y = 0$ ...(1)

and $20 + y = x + 40$

$x - y = -20$ ...(2)

Subtracting (2) from (1), we get

$$x = 20$$

From(1), $y = 2x = 2 \times 20 = 40$

Hence, the age of the father is 40 years.

14. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans : [NCERT]

Let the number of students in a row be x and the number of rows be y. Thus total will be xy.

Now $(x+3)(y-1) = xy$

$xy + 3y - x - 3 = xy$

$-x + 3y - 3 = 0$ ...(1)

and $(x-3)(y+2) = xy$

$xy - 3y + 2x - 6 = xy$

$2x - 3y - 6 = 0$ ...(2)

Multiply equation (1) 2 we have

$-2x + 6y - 6 = 0$ ...(3)

Adding equation (2) and (3) we have

$$3y - 12 = 0$$

$$y = 4$$

Substitute $y = 4$ in equation (1)

$$-x + 12 - 3 = 0$$

$$x = 9$$

Total students $xy = 9 \times 4 = 36$

Total students in the class is 36.

15. The ages of two friends ani and Biju differ by 3 years. Ani's father Dharam is twice as old as ani and Biju is twice as old as his sister Cathy. The ages of Cathy

and Dharam differ by 30 year. Find the ages of Ani and Biju.

Ans : [NCERT]

Let the ages of Ani and Biju be x and y, respectively. According to the given condition,

$$x-y=\pm 3 \qquad ...(1)$$

Also, age of Ani's father Dharam $=2x$ years

And age of Biju's sister $=\frac{y}{2}$ years

According to the given condition,

$$2x-\frac{y}{2}=30$$

$$4x-y=60 \qquad ...(2)$$

Case I : When $x-y=3$ ...(3)

Subtracting equation (3) from equation (2),

$$3x=57$$

$$x=19 \text{ years}$$

Putting $x=19$ in equation (3),

$$19-y=3$$

$$y=16 \text{ years}$$

Case II : When $x-y=-3$ ...(4)

Subtracting equation (iv) from equation (2),

$$3x=60+3$$

$$3x=63$$

$$x=21 \text{ years}$$

Subtracting equation (4), we get

$$21-y=-3$$

$$y=24 \text{ years}$$

Hence, Ani's age $=19$ years or 21 years Biju age $=16$ years or 24 years.

16. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital.

Ans : [NCERT]

Let the amount of their respective capitals be x and y.

According to the given condition,

$$x+100=2(y-100)$$

$$x-2y=-300 \qquad ...(1)$$

and $$6(x-10)=y+10$$

$$6x-y=70 \qquad ...(2)$$

Multiplying equation (2) by 2 we have

$$12x-2y=140 \qquad ...(3)$$

Subtracting (1) from equation (3) we have

$$11x=440$$

$$x=40$$

Substituting $x=40$ in equation (1),

$$40-2y=-300$$

or, $$2y=340$$

$$y=170$$

Hence, the amount of their respective capitals are 40 and 170.

17. A fraction become $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

Ans : [Board Term-1, 2012, Set-60]

Let the fraction be $\frac{x}{y}$, then according to the question,

$$\frac{x+2}{y+2}=\frac{9}{11}$$

$$11x+22=9y+18$$

or, $$11x-9y+4=0 \qquad ...(1)$$

and $$\frac{x+3}{y+3}=\frac{5}{6}$$

or, $$6x-5y+3=0 \qquad ...(2)$$

Comparing with $ax+by+c=0$

we get $$a_1=11, b_1=9, c_1=4,$$

$$a_2=6, b_2=-5, \text{ and } c_2=3$$

Now, $$\frac{x}{b_2c_1-b_1c_2}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-b_2b_1}$$

$$\frac{x}{(-9)(3)-(-5)(4)}=\frac{y}{(4)(6)-(11)(3)}$$

$$=\frac{1}{(11)(-5)-(9)(-9)}$$

1

or, $$\frac{x}{-27+20}=\frac{y}{24-33}=\frac{1}{-55+54}$$

$$\frac{x}{-7}=\frac{y}{-9}=\frac{1}{-1}$$

Hence, $x=7, y=9$

Thus fraction is $\frac{7}{9}$

18. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of speed of the boat in still water and the speed of the stream.

Ans : [Board Term-1, 2012, Set-48]

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Speed of boat up stream $=(x-y)$ km/hr.

Speed of boat down stream $=(x+y)$ km/hr.

$$\frac{30}{x-y}+\frac{28}{x+y}=7$$

and $$\frac{21}{x-y}+\frac{21}{x+y}=5$$

Let $\frac{1}{x-y}$ be a and $\frac{1}{x+y}$ be b, then we have

$$30a+28b=7 \qquad ...(1)$$

$$21a+21b=5 \qquad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by 4 we have

$$90a+84b=21 \qquad ...(3)$$

$$84a+84b=20 \qquad ...(4)$$

Subtracting (4) from (3) we have,

$$6a = 1$$

$$a = \frac{1}{6}$$

Putting this value of a in equation (1),

$$30 \times \frac{1}{6} + 28b = 7$$

$$28b = 7 - 30 \times \frac{1}{6} = 2$$

$$b = \frac{1}{14}$$

Thus $\quad x + y = 14 \quad ...(5)$

Now, $\quad a = \frac{1}{x-y} = \frac{1}{6}$

or, $\quad x - y = 6 \quad ...(6)$

and $\quad x + y = 14$

Solving equation (5) and (6), we get

$$x = 10, y = 4$$

Hence, speed of the boat in still water = 10km/hr

and speed of the stream = 4 km/hr.

19. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

Ans : [Board Term-1, 2012, Set-48]

Let the speed of the boat be x km/hr and the speed of the stream be y km/hr.

According to the question,

$$\frac{32}{x-y} + \frac{36}{x+y} = 7$$

and $\quad \frac{40}{x-y} + \frac{48}{x+y} = 9$

Let $\frac{1}{x-y} = A, \frac{1}{x+y} = B$, then we have

$$32A + 36B = 7 \quad ...(1)$$

and $\quad 40A + 48B = 9 \quad ...(2)$

Multiplying equation (1) by 5 and (2) by 4, we have

$$160A + 180B = 35 \quad ...(3)$$

and $\quad 160A + 192B = 36 \quad ...(4$

Subtracting (4) from (3) we have

$$-12B = -1$$

$$B = \frac{1}{12}$$

Substituting the value of B in (2) we get

$$40A + 48\left(\frac{1}{12}\right) = 9$$

$$40A + 4 = 9$$

$$40A = 5$$

$$A = \frac{1}{8}$$

Thus $\quad A = \frac{1}{8}$ and $B = \frac{1}{12}$

Hence $\quad A = \frac{1}{8} = \frac{1}{x-y}$

$$x - y = 8 \quad ...(5)$$

and $\quad B = \frac{1}{12} = \frac{1}{x+y}$

$$x + y = 12 \quad ...(6)$$

Adding equations (5) and (6) we have,

$$2x = 20$$

$$x = 10$$

Substituting this value of x in equation (1),

$$y = x - 8 = 10 - 8 = 2$$

Hence, the speed of the boat in still water = 10 km/hr and speed of the stream = 2 km/hr.

20. For what values of a and b does the following pair of linear equations have infinite number of solution ?

$2x + 3y = 7, a(x + y) - b(x - y) = 3a + b - 2$

Ans : [Board Term-1, 2015, CJTOQ]

We have $\quad 2x + 3y - 7 = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

and $\quad a(x+y) - b(x-y) = 3a + b - 2$

$$ax + ay - bx + by = 3a + b - 2$$

$$(a-b)x + (a+b)y - (3a+b-2) = 0$$

Here $a_2 = a - b, b_2 = a + b, c_2 = -(3a + b - 2)$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{(3a+b-2)}$$

From $\frac{2}{a-b} = \frac{7}{3a+b-2}$ we have

$$2(3a+b-2) = 7(a-b)$$

$$6a + 2a - 4 = 7a - 7b$$

$$a - 9b = -4 \quad ...(1)$$

From $\frac{3}{a+b} = \frac{7}{3a+b-2}$ we have

$$3(3a+b-2) = 7(a+b)$$

$$9a + 3b - 6 = 7a + 7b$$

$$2a - 4b = 6$$

$$a - 2b = 3 \quad ...(2)$$

Subtracting equation (1) from (2),

$$-7b = -7$$

$$b = 1$$

Substituting the value of b in equation (1),

$$a = 5$$

Hence, $a = 5, b = 1$.

21. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

Ans : [Sample Question Paper 2017]

Let the ten's and unit digit by y and x respectively, So the number is $10y + x$

The number when digits are reversed becomes $10x+y$

Thus
$$7(10y+x) = 4(10x+y)$$
$$70y+7x = 40x+4y$$
$$70y-4y = 40x-7x$$
$$2y = x \quad ...(1)$$
or
$$x-y = 3 \quad ...(2)$$
From (1) and (2) we get
$$y = 3 \text{ and } x = 6$$
Hence the number is 36.

HOTS QUESTIONS

1. At a certain time in a deer, the number of heads and the number of legs of deer and human visitors were counted and it was found that there were 39 heads and 132 legs.

Find the number of deer and human visitors in the park.

Ans :

Let the no. of deer be x and no. of human be y.
According to the question,
$$x+y = 39 \quad ...(1)$$
and
$$4x+2y = 132 \quad ...(2)$$
Multiply equation (1) from by 2,
$$2x+2y = 78 \quad ...(3)$$
Subtract equation (3) from (2),
$$2x = 54$$
$$x = 27$$
Substituting this value of x in equation (1)
$$27+y = 39$$
$$y = 12$$
So, No. of deer $= 27$ and No. of human $= 12$

2. Find the value of p and q for which the system of equations represent coincident lines $2x+3y=7$, $(p+q+1)x+(p+2q+2)y=4(p+q)+1$

Ans :

We have
$$2x+3y = 7$$
$$(p+q+1)x+(p+2q+2)y = 4(p+q)+1$$
Comparing given equation to $ab+by+c=0$ we have
$a_1=2, b_1=3, c_1=-7$
$a_2=p+q+1, b_2=p+2q+2, c_2=-4(p+q)-1$
For coincident lines,
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{2}{p+q+1} = \frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$$
From $\frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$ we have
$$7p+14q+14 = 12p+12q+3$$
$$5p-2q-11 = 0 \quad ...(1)$$
From $\frac{2}{p+q+1} = \frac{7}{4(p+q)+1}$ we have
$$8(p+q)+2 = 7p+7q+7$$
$$8p+8q+2 = 7p+7q+7$$
$$p+q-5 = 0 \quad ...(2)$$
Multiplying equation (2) by 5 we have
$$5p+5q-25 = 0 \quad ...(3)$$
Subtracting equation (1) from (3) we get
$$7q = 14$$
$$q = 2$$
Hence, $p = 3$ and $q=2$.

3. A chemist has one solution which is 50 % acid and a second which is 25 % acid. How much of each should be mixed to make 10 litre of 40 % acid solution.

Ans :

Let 50 % acids in the solution be x and 25 % of other solution be y.
Total volume in the mixture
$$x+y = 10 \quad ...(1)\ 1$$
and
$$\frac{50}{100}x+\frac{25}{100}y = \frac{40}{100}\times 10$$
$$2x+y = 16 \quad ...(2)\ 1$$
Subtracting equation (1) from (2) we have
$$x = 6$$
Substituting this value of x in equation (1) we get
$$6+y = 10$$
$$y = 4$$
Hence, $x = 6$ and $y=4$.

4. The length of the sides of a triangle are $2x+\frac{y}{2}$, $\frac{5x}{3}+y+\frac{1}{2}$ and $\frac{2}{3}x+2y+\frac{5}{2}$. If the triangle is equilateral , find its perimeter.

Ans :

For an equilateral Δ,
$$2x+\frac{y}{2} = \frac{5x}{3}+y+\frac{1}{2} = \frac{1}{2}x+2y+\frac{5}{2}$$
Now
$$\frac{4x+y}{2} = \frac{10x+6y+3}{6}$$
$$12x+3y = 10x+6y+3$$
$$2x-3y = 3 \quad ...(1)$$
Again,
$$2x+\frac{y}{2} = \frac{2}{3}x+2y+\frac{5}{2}$$
$$\frac{4x+y}{2} = \frac{4x+12y+15}{6}$$
$$12x+3y = 4x+12y+15$$
$$8x-9y = 15 \quad ...(2)$$
Multiplying equation (1) by 3 we have
$$6x-9y = 9 \quad ...(1)$$
Subtracting it from (2) we get
$$2x = 6 \Rightarrow x = 3$$
Substituting this value of x into (1), we get

$$2 \times 3 - 3y = 3$$

or, $$3y = 3 \Rightarrow y = 1$$

Now substituting these value of x and y

$$2x + \frac{y}{2} = 2 \times 3 + \frac{1}{2} = 6.5$$

The perimeter of equilateral triangle $=$ side $\times 3$

$$= 6.5 \times 3 = 19.5 \text{ cm}$$

Hence, the perimeter of $\Delta = 19.5$ m

5. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 1,8000 persons on the electoral roll. How many votes for B.

Ans :

Let x and y be the no. of votes for A and B respectively.
The no. of persons who did not vote

$$= (18000 - x - y)$$

We have $$x = 2(18000 - x - y)$$

or $$3x + 2y = 36000 \quad ...(1)$$

and $$(18000 - x - y) = 2(x - y)$$

or $$3x - y = 18000 \quad ...(2)$$

Subtracting equation (2) from equation (1),

$$3y = 18000$$

$$y = 6000$$

Hence vote for $B = 6000$

6. When 6 boys were admitted and 6 girls left, the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class. 2

Ans :

Let the no. of boys be x and no. of girls be y.
No. of students $= x + y$

Now $$\frac{x}{x+y} = \frac{60}{100} \quad ...(1)$$

and $$\frac{x+6}{(x+6)+(y-6)} = \frac{75}{100} \quad ...(2)$$

From (1), we have

$$100x = 60x + 60y$$

$$40x - 60y = 0$$

$$2x - 3y = 0$$

$$2x = 3y \quad (3)$$

From (2) we have

$$100x + 600 = 75x + 75y$$

$$25x - 75y = -600$$

$$x - 3y = -24 \quad ...(4)$$

Substituting the value of $3y$ from (3) in to (4) we have,

$$x - 2x = -24 \Rightarrow x = 24$$

$$3y = 24 \times 2$$

$$y = 16$$

Hence, no. of boys is 24 and no. of girls is 16.

7. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30 km at half speed in 5 hours. If the breakdown had occurred 10 km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.

Ans :

Let x km be the distance of the place where breakdown occurred and y be the original speed

$$\frac{x}{y} + \frac{30-x}{\frac{y}{2}} = 5$$

or $$\frac{x}{y} + \frac{60-2x}{y} = 5 \quad ...(1)$$

and $$\frac{x+10}{y} + \frac{30-(x+10)y}{} = 4 \quad ...(2)$$

From (1), we have

$$x + 60 - 2x = 5y$$

$$x + 5y = 60 \quad ...(3)$$

From (3) we have

$$x + 10 + 60 - 2x - 20 = 4y$$

$$x + 4y = 50 \quad (4)$$

Subtract equation (4) from (3), $y = 10$ km/hr.

Now from (4), $$x + 40 = 50$$

$$x = 10 \text{ km}$$

Break down occurred at 10 km. Hence original speed was 10 km/hr.

8. The population of a village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Ans :

Let the number of males be x and females be y

Now $$x + y = 5{,}000 \quad ...(1)$$

and $$x + \frac{5}{100}x + y + \frac{3y}{100} = 5202$$

$$\frac{5x+3y}{100} + 5000 = 5202$$

$$5x + 3y = (5202 - 5000) \times 100$$

$$5x + 3y = 20200 \quad (2)$$

Multiply (1) by 3 we have

$$3x + 3y = 15{,}000 \quad ...(3)$$

Subtracting (2) from (3) we have

$$2x = 5200 \Rightarrow x = 2600$$

Substituting value of x in (1) we have

$$2600 - y = 5000$$

$$y = 2400$$

Thus no. of males is 2600 and no. of females is 2400..