CHAPTER 2

Polynomials

VERY SHORT ANSWER TYPE QUESTIONS

1. If α and β are the roots of $ax^2 - bx + c = 0\,(a \neq 0)$, then calculate $\alpha + \beta$.

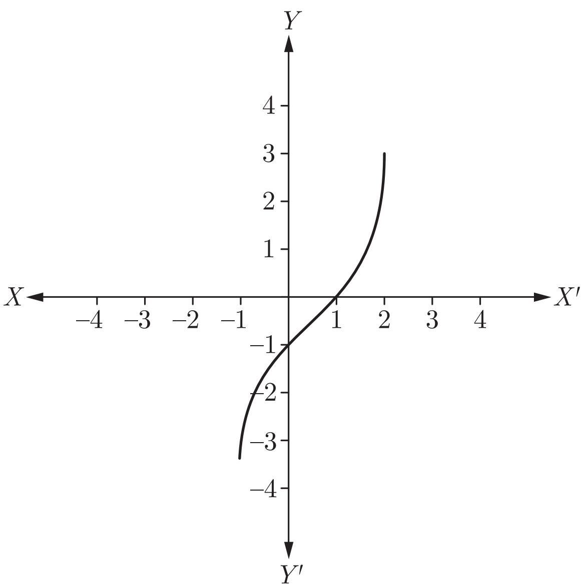
Ans : [Board Term-1, 2014]

We know that

$$\text{Sum of the roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $$\alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}$$

2. In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.



Ans :

The graph intersects x-axis at one point $x = 1$. Thus the number of zeroes of $p(x)$ is 1.

3. Calculate the zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$.

Ans :

We have
$$\begin{aligned} p(x) &= 4x^2 - 12x + 9 \\ &= 4x^2 - 6x - 6x + 9 \\ &= 2x(2x-3) - 3(2x-3) \\ &= (2x-3)(2x-3) \end{aligned}$$

Substituting $p(x) = 0$, and solving we get $x = \frac{3}{2}, \frac{3}{2}$

$x = \frac{3}{2}, \frac{3}{2}$

Hence, zeroes of the polynomial are $\frac{3}{2}, \frac{3}{2}$. 1

4. If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Ans :

We have $$p(x) = 3x^2 - kx - 6$$

$$\text{Sum of the zeroes} = 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $$3 = -\frac{(-k)}{3} \Rightarrow k = 9$$

5. If -1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

Ans :

We have $$f(x) = x^2 - 7x - 8$$

Let other zero be k, then we have

Sum of zeroes, $$-1 + k = -\left(\frac{-7}{1}\right) = 7$$

or $$k = 8$$

SHORT ANSWER TYPE QUESTIONS - I

1. If zeroes of the polynomial $x^2 + 4x + 2a$ are a and $\frac{2}{a}$, then find the value of a.

Ans : [Board Term-1, 2016 Set-O4YP6G7]

Product of (zeroes) roots,

$$\frac{c}{a} = \frac{2a}{1} = \alpha \times \frac{2}{\alpha} = 2$$

or, $$2a = 2$$

Thus $$a = 1$$

2. Find all the zeroes of $f(x) = x^2 - 2x$.

Ans : [Board Term-1, 2013, LK-59]

We have
$$\begin{aligned} f(x) &= x^2 - 2x \\ &= x(x-2) \end{aligned}$$

Substituting $f(x) = 0$, and solving we get $x = 0, 2$

Hence, zeroes are 0 and 2.

3. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

Ans : [Board Term-1, 2013, LK-59]

$$\begin{aligned} p(x) &= \sqrt{3}x^2 - 8x + 4\sqrt{3} = 0 \\ &= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3} = 0 \\ &= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3}) = 0 \\ &= (\sqrt{3}x - 2)(x - 2\sqrt{3}) = 0 \end{aligned}$$

Substituting $p(x) = 0$, and solving we get $x = \frac{2}{\sqrt{3}},\ 2\sqrt{3}$

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

4. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

Ans : [Board Term-1, 2016 Set- LGRKEGO]

Sum of zeroes, $\alpha + \beta = 6$

Product of zeroes $\alpha\beta = 9$

Now $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

Thus $= x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $p(x) = x^2 - 6x + 9$

$= (x - 3)(x - 3)$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3

5. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

Ans : [Board Term-1, 2012, Set-35]

Sum of zeroes, $\alpha + \beta = \frac{21}{8}$

Product of zeroes $\alpha\beta = \frac{5}{16}$

Now $p(x)\ x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \frac{21}{8}x + \frac{5}{16}$

or $p(x) = \frac{1}{16}(16x^2 - 42x + 5)$

6. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes, respectively.

Ans : [Board Term-1, 2012, Set-64]

Sum of zeroes, $\alpha + \beta = 3$

Product of zeroes $\alpha\beta = -\frac{2}{5}$

Now $p(x)\ x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 3x - \frac{2}{5}$

$= \frac{1}{5}(5x^2 - 15x - 2)$

The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$

7. What should added to the polynomial $x^3 - 3x^2 + 6x - 15$ so that it is completely divisible by $x - 3$.

Ans : [Board Term-1, 2016 Set-ORDAWEZ]

We divide $x^3 - 3x^2 + 6x - 15$ by $x - 3$ as follows.

$$\begin{array}{r|l} & x^2 + 6 \\ x - 3 & x^3 - 3x^2 + 6x - 15 \\ & x^3 - 3x^2 \\ \hline & \qquad 6x - 15 \\ & \qquad 6x - 18 \\ \hline & \qquad\quad 3 \end{array}$$

Here remainder is 3, hence -3 must be added so that there is no remainder.

8. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

Ans : [Board Term-1, 2012, Set-40]

We have $\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m + n)^2 - 2mn}{mn}$ (1)

Sum of zeroes $m + n = -\frac{11}{3}$

Product of zeroes $mn = \frac{-4}{3}$

Substituting in (1) we have

$$\begin{aligned} \frac{m}{n} + \frac{n}{m} &= \frac{(m + n)^2 - 2mn}{mn} \\ &= \frac{(-\frac{11}{3})^2 - \frac{-4}{3} \times 2}{\frac{-4}{3}} \\ &= \frac{121 + 4 \times 3 \times 2}{-4 \times 3} \end{aligned}$$

or $\frac{m}{n} + \frac{n}{m} = \frac{-145}{12}$

9. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

Ans : [Board Term-1, 2012, Set-21]

We have $f(x) = 2x^2 - 7x + 3$

Sum of zeroes $p + q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

Since, $(p + q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p + q)^2 - 2pq$

$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$

Hence $p^2 + q^2 = \frac{37}{4}$.

10. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

Ans : [Board Term-1, 2012, Set-50]

We have $p(x) = ax^2 + bx + c$

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

Product of zeroes,

$$\frac{c}{a} = \alpha \times \frac{1}{\alpha} = 1 \text{ or } \frac{c}{a} = 1$$

So, required condition is, $c = a$

11. Find the value of k if -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.

Ans : [Board Term-1, 2012, Set-62]

We have $p(x) = kx^2 - 4x + k$

Since, -1 is a zero of the polynomial, then

$$p(-1) = 0$$

$$k(-1)^2 - 4(-1) + k = 0$$

or, $k + 4 + k = 0$

or, $2k + 4 = 0$

or, $2k = -4$

Hence, $k = -2$

12. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

Ans : [Board Term-1, 2015, Set-DDE-M]

We have $p(x) = x^2 - 4\sqrt{3}\,x + 3$

If α and β are the zeroes of $x^2 - 4\sqrt{3}\,x + 3$, then

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{(-4\sqrt{3})}{1}$

or, $\alpha + \beta = 4\sqrt{3}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{3}{1}$

or, $\alpha\beta = 3$

Now $\alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$.

13. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$.

Ans : [Board Term-1, 2013, FFC],

We have $p(x) = x^2 + ax + b$

Since a and b, are the zeroes of polynomial, we get,

Product of zeroes, $ab = b \Rightarrow a = 1$

Sum of zeroes, $a + b = -a \Rightarrow b = -2a = -2$

14. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

We have $f(x) = x^2 - 6x + k$

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-6)}{1} = 6$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$$(6)^2 - 2k = 40$$

$$36 - 2k = 40$$

$$-2k = 4$$

Thus $k = -2$

15. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

Ans : [Board Term-1, 2012, Set-48]

We have $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $= \alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

Sum of zeroes $0 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

16. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

Ans : [Board Term-1, 2012, Set-71]

Let, the zero of $2x^2 + 3x + \lambda$ be $\frac{1}{2}$ and β.

Product of zeroes $\frac{1}{2}\beta = \frac{\lambda}{2}$ $\qquad \frac{c}{a}$

or, $\beta = \lambda$

and sum of zeroes, $\frac{1}{2} + \beta = -\frac{3}{2}$ $\qquad -\frac{b}{a}$

or $\beta = -\frac{3}{2} - \frac{1}{2} = -2$

Hence $\lambda = \beta = -2$

Thus other zero is -2.

17. If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

Ans : [Board Term-1, 2013, Set FFC]

We have $f(x) = x^2 - x - k$

Since α and β are the zeroes of the polynomial, then

Sum of zeroes, $\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\left(\frac{-1}{1}\right) = 1$$

$\alpha + \beta = 1$ ...(i)

Given $\alpha - \beta = 9$ ...(ii) 1

Solving (i) and (ii) we get $\alpha = 5$ and $\beta = -4$

$$\alpha\beta = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

or $\alpha\beta = -k$

Substituting $\alpha = 5$ and $\beta = -4$ we have

$$(5)(-4) = -k$$

Thus $k = 20$

18. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Ans : [Board Term-1, 2012, Set-39]

We have $f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial be α and β, then

Sum of zeroes $\alpha + \beta = \frac{5}{2}$

Product of zeroes $\alpha\beta = -\frac{3}{2}$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2β.

Sum of zeros, $2\alpha + 2\beta = \frac{-p}{1}$

$$2(\alpha + \beta) = -p$$

Substituting $\alpha + \beta = \frac{5}{2}$ we have

$$2 \times \frac{5}{2} = -p$$

or $p = -5$

Product of zeroes, $2\alpha 2\beta = \frac{q}{1}$

$$4\alpha\beta = q$$

Substituting $\alpha\beta = -\frac{3}{2}$ we have

$$4 \times \frac{-3}{2} = q$$

$$-6 = q$$

Thus $p = -5$ and $q = -6$.

19. If α and β are zeroes of $x^2 - (k-6)x + 2(2k-1)$, find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

Ans : [KVS Practice Test 2017]

We have $p(x) = x^2 - (k-6)x + 2(2k-1)$

Since α, β are the zeroes of polynomial $p(x)$, we get

$$\alpha + \beta = -[-(k-6)] = k - 6$$

$$\alpha\beta = 2(2k-1)$$

Now $\alpha + \beta = \frac{1}{2}\alpha\beta$

Thus $k + 6 = \frac{2(2k-1)}{2}$

or, $k - 6 = 2k - 1$

$$k = -5$$

Hence the value of k is -5.

SHORT ANSWER TYPE QUESTIONS - II

1. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

Ans : [Board Term-1, 2013, LK-59]

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

(1) 2, $p(x) = 2x^2 - 11x^2 + 17x - 6$

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$$

$$= 16 - 44 + 34 - 6$$

$$= 50 - 50$$

or $p(2) = 0$

(2) 3, $p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$$= 54 - 99 + 51 - 6$$

$$= 105 - 105$$

or $p(3) = 0$

(3) $\frac{1}{2}$ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

2. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

Ans : [Board Term-1, 2011, Set-25]

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha + \beta = -\frac{-5}{a} = \frac{5}{a}$

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

3. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Ans : [Board Term-1, 2011, Set-40]

We have $f(x) = 3x^2 - 8x + 2k + 1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha + \beta = -\left(-\frac{8}{3}\right)$

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $\alpha = \frac{1}{3}$

Product of zeroes, $\alpha \times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7 \times \frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3} - 1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

4. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Ans : [Board Term-2, 2015, Set-DDE-E]

We have $f(x) = 2x^2 - 3x + 1$

If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$p(x) = x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$
$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$
$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$
$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$
$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

5. If α and β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Ans : [Board Term-1, 2011, Set-39]

We have $p(y) = 6y^2 - 7y + 2$

Sum of zeroes $\alpha + \beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$

Product of zeroes $\alpha\beta = \frac{2}{6} = \frac{1}{3}$

Sum of zeroes of new polynomial $g(y)$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2}$$

and product of zeroes of new polynomial $g(y)$,

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3$$

The required polynomial is

$$g(x) = y^2 - \frac{7}{2}y + 3$$
$$= \frac{1}{2}[2y^2 - 7y + 6]$$

6. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify relationship between zeroes and coefficients of the polynomial.

Ans : [Board Term-1, 2011, Set-21]

We have $p(x) = 4x^2 + 4x - 3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$$
$$= 1 + 2 - 3 = 0$$

and $$p\left(-\frac{3}{2}\right) = 4\left(\frac{9}{2}\right) + 4\left(-\frac{3}{2}\right) - 3$$
$$= 9 - 6 - 3 = 0$$

Thus $\frac{1}{2}, -\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$.

Sum of zeroes $= \frac{1}{2} - \frac{3}{2} = -1 = \frac{-4}{4}$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes $= \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$

$$= \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$ Verified

7. Find the zeroes of the quadratic polynomial $x^2 - 2\sqrt{2}x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board Term-1, 2015, Set-FHN8MG0]

We have $p(x)x^2 - 2\sqrt{2}x = 0$

$$x(x - 2\sqrt{2}) = 0$$

Thus zeroes are 0 and $2\sqrt{2}$.

Sum of zeroes $2\sqrt{2} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and product of zeroes $0 = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$

Hence verified

8. Find the zeroes of the quadratic polynomial $5x^2 + 8x - 4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Ans : [Board Term-1, 2013, Set LK-59]

We have $p(x) = 5x^2 + 8x - 4 = 0$

$$= 5x^2 + 10x - 2x - 4 = 0$$

$$= 5x(x+2) - 2(x+2) = 0$$
$$= (x+2)(5x-2)$$

Substituting $p(x) = 0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes} = -2 + \frac{2}{5} = \frac{-8}{5}$$

$$\text{Product of zeroes} = (-2) \times \left(\frac{2}{5}\right) = \frac{-4}{5}$$

Now from polynomial we have

$$\text{Sum of zeroes} \quad -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-8}{5}$$

$$\text{Product of zeroes} \quad \frac{c}{a} = \frac{\text{Constan term}}{\text{Coefficient of } x^2} = -\frac{4}{5}$$

Hence Verified.

9. If α and β are the zeroes of a quadratic polynomial such that $\alpha + \beta = 0$ and $\alpha - \beta = 8$. Find the quadratic polynomial having α and β as its zeroes.

Ans : [Board Term-1, 2011, Set-44]

We have $\alpha + \beta = 24$ ...(1)

$\alpha - \beta = 8$ ...(2)

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

Subtracting (1) from (2) we have

$$2\beta = 24 \Rightarrow \beta = 12$$

Hence, the quadratic polynomial

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (16+8)x + (16)(8)$$
$$= x^2 - 24x + 128$$

10. If α, β and γ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

Ans : [KVS practice Test 2017, CBSE Board 2010]

We have $p(x) = 6x^3 + 3x^2 - 5x + 1$

Since α, β and γ are zeroes polynomial $p(x)$, we have

$$\alpha + \beta + \gamma = -\frac{b}{c} = -\frac{3}{6} = -\frac{1}{2}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = -\frac{5}{6}$$

and $$\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$$

Now $$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{-5/6}{-1/6} = \frac{-5}{6} \times \frac{6}{-1} = 5$$

Hence $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$.

LONG ANSWER TYPE QUESTIONS

1. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q.

Ans : [Board Term-1, 2015, Set-DDE-M]

We have $f(x) = x^4 + 7x^3 + 7x^2 + px + q$

Now $x^2 + 7x + 12 = 0$

$$x^2 + 4x + 3x + 12 = 0$$
$$x(x+4) + 3(x+4) = 0$$
$$(x+4)(x+3) = 0$$
$$x = -4, -3 \quad \text{...(i)}$$

Since $f(x) = x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ must be its zeroes and these must satisfy $f(x) = 0$

So putting $x = -4$ and $x = -3$ in $f(x)$ and equating to zero we get

$$f(-4): \ (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q = 0$$
$$256 - 448 + 112 - 4p + q = 0$$
$$-4p + q - 80 = 0$$
$$4p - q = -80 \text{...(1)}$$

$$p(-3): \ (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q = 0$$
$$81 - 189 + 63 - 3p + q = 0$$
$$-3p + q - 45 = 0$$
$$3p - q = -45 \text{ ...(2)}$$

Subtracting eq, (2) from (1) we have

$$p = -35$$

On putting the value of p in eq. (1) we have

$$4(-35) - q = -80$$
$$-140 - q = -80$$
$$-q = 140 - 80$$

or $$-q = 60$$
$$q = -60$$

Hence, $p = -35$ and $q = -60$.

2. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

Ans : [Board Term-1, 2012, Set-50]

We have $p(x) = 2x^2 + 5x + k$

Sum of zeroes, $$\alpha + \beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$$

Product of zeroes $$\alpha\beta = \frac{c}{a} = \frac{k}{2}$$

According to the question,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - \alpha\beta = \frac{21}{4}$$

$$(\alpha + b)^2 - \alpha\beta = \frac{21}{4}$$

Substituting values we have

$$\left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = \frac{25}{4} - \frac{21}{4}$$

$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

3. If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Ans : [Board Term-1, 2012, Set-45, 62, 2010, Set-15]

We have $p(x) = 3x^2 + 2x + 1$

Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$, we have

$$\alpha + \beta = -\frac{2}{3}$$

and $$\alpha\beta = \frac{1}{3}$$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1 + \beta_1 = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$

$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$= \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$

$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$

Hence, Required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)2x + \alpha_1\beta_1$$
$$= x^2 - 2x + 3$$

4. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

Ans : [Board Term-1, 2013 LK-59]

We have $p(x) = x^2 + 4x + 3$

Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$,

So, $\alpha + \beta = -4$

and $\alpha\beta = 3$ 1

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1 + \beta_1 = 1 + \frac{\alpha}{\beta} + 1 + \frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)}{3} = \frac{16}{3}$$ 1

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$$

$$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

Hence, Required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)2x + \alpha_1\beta_1$$

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}(3x^2 - 16x + 16)$$

5. If α and β are zeroes of the polynomial $p(x) = 6x - 5x + k$ such that $\alpha - \beta = \frac{1}{6}$, Find the value of k.

Ans :

We have $p(x) = 6x - 5x + k$

Since α and β are zeroes of $p(x) = 6x - 5x + k$,

Sum of zeroes, $\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha - \beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$$

Hence, $k = 1$.

6. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2 + a)x^2 + 61x + 6a$. Find the value of β and α.

Ans :

We have $p(x) = (a^2 + a)x^2 + 61x + 6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta + \frac{1}{\beta} = -\frac{61}{a^2 + a}$

or, $\frac{\beta^2 + 1}{\beta} = \frac{-61}{a^2 + a}$ ...(1)

Product of zeroes $\beta\frac{1}{\beta} = \frac{6a}{a^2 + a}$

or, $1 = \frac{6}{a + 1}$

$a + 1 = 6$

$a = 5$

Substituting this value of a in (1) we get

$$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$$

$$30\beta^2 + 30 = -61\beta$$

$$30\beta^2 + 61\beta + 30 = 0$$

Now $\beta \frac{-61 \pm \sqrt{(-61)^2 \times 4 \times 30 \times 30}}{2 \times 30}$

$$= \frac{-61 \pm \sqrt{3721 - 3600}}{60}$$

$$\frac{-61 \mp 11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

SHORT ANSWER TYPE QUESTIONS - I

1. On dividing $x^3 - 5x^2 + 6x + 4$ by a polynomial $g(x)$, the quotient and the remainder were $x - 3$ and 4 respectively. Find $g(x)$.

Ans : [Board Term-1, 2012, Set-55]

We have $x^3 - 5x^2 + 6x + 4 = g(x)(x - 3) + 4$

$$g(x) = \frac{x^3 - 5x^2 + 6x + 4 - 4}{x - 3}$$

or, $g(x) = \frac{x^3 - 5x^2 + 6x}{x - 3}$

Now we divide $x^3 - 5x^2 + 6x$ by $x - 3$ as follows.

$$\begin{array}{r|l} & x^2 - 2x \\ x - 3 & x^3 - 5x^2 + 6x \\ & \underline{x^3 - 3x^2} \\ & -2x^2 + 6x \\ & \underline{2x^2 + 6x} \\ & 0 \end{array}$$

Hence $g(x) = x^2 - 2x$.

2. Find the quotient and remainder on dividing $p(x)$ by $g(x)$:

$p(x) = 4x^3 + 8x^2 + 8x + 7; g(x) = 2x^2 - x + 1$

Ans : [Board Term-1, 2012, Set-55]

$$\begin{array}{r|l} & 2x + 5 \\ 2x^2 - x + 1 & 4x^3 + 8x^2 + 8x + 7 \\ & \underline{4x^3 - 2x^2 + 2x} \\ & 10x^2 + 6x + 7 \\ & \underline{10x^2 - 5x + 7} \\ & 11x + 2 \end{array}$$

Thus, Quotient $= 2x + 5$

and Remainder $= 11x + 2$

3. Check whether the polynomial $g(x) = x^2 + 3x + 1$ is a factor of the polynomial $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$.

Ans : [Board Term-1, 2012, Set-48]

$$\begin{array}{r|l} & 3x^2 - 4x + 2 \\ x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 4 \\ & \underline{3x^4 + 9x^3 + 3x^2} \\ & -4x^3 - 10x^2 + 2x \\ & \underline{-4x^3 - 12x^2 - 4x} \\ & 2x^2 + 6x + 4 \\ & \underline{2x^2 + 6x + 2} \\ & 2 \end{array}$$

Since remainder is not zero, polynomial $g(x) = x^2 + 3x + 1$ is not a factor of the polynomial $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$.

4. What should be added in the polynomial $x^3 - 6x^2 + 11x + 8$ so that is completely divisible by $x^2 - 3x + 2$?

Ans : [Board Term-1, Set, 2015]

$$\begin{array}{r|l} & x - 3 \\ x^2 - 3x + 2 & x^3 - 6x^2 + 11x + 8 \\ & \underline{x^3 - 3x^2 + 2x} \\ & -3x^2 + 9x + 8 \\ & \underline{-3x^2 + 9x - 6} \\ & 14 \end{array}$$

Since Remainder $= 14$ to make it$0 - 14$ should be added.

5. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find the values of a and b.

Ans : [Board Term-1, Set FHN8MGI, 2015]

$$\begin{array}{r|l} & 2x^2 + 5 \\ 3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\ & \underline{6x^4 + 8x^3 + 2x^2} \\ & 15x^2 + 21x + 7 \\ & \underline{15x^2 + 20x + 5} \\ & x + 2 \end{array}$$

Comparing both the sides we get $a = 1$ and $b = 2$

6. If x^3-6x^2+6x+k is completely divisible by $x-3$, then find the value of k.

Ans : [Board Term-1, Set-WJQZQBN]

$$\begin{array}{r|l} & x^2-3x-3 \\ x-3 & x^3-6x^2+6x+k \\ & \underline{x^3-3x^2} \\ & -3x^2+6x+k \\ & \underline{-3x^2+9x} \\ & -3x+k \\ & \underline{-3x+9} \\ & k-9 \end{array}$$

Remainder should be zero

$$k-9=0$$

So, $k=0$

7. Divide the polynomial $p(x)=x^3-4x+6$ by the polynomial $g(x)=2-x^2$ and find the quotient and the remainder.

Ans : [Board Term-1, 2015, Set-1E]

$$\begin{array}{r|l} & -x \\ -x^2-2 & x^3-4x+6 \\ & \underline{x^3-2x} \\ & -2x+6 \end{array}$$

Thus, Quotient $=-x$

and Remainder $=6-2x$

8. Divide the polynomial $p(x)=x^2-5x+16$ by the polynomial $g(x)=x-2$ and find the quotient and the remainder.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

$$\begin{array}{r|l} & x-3 \\ x-2 & x^2-5x+16 \\ & \underline{x^2-2x} \\ & -3x+16 \\ & \underline{-3x+6} \\ & 10 \end{array}$$

Quotient $=x-3$, Remainder $=10$

SHORT ANSWER TYPE QUESTIONS - II

1. What should be added to x^3+5x^2+7x+3 so that it is completely divisible by x^2+2x.

Ans : [Board Term-1, 2016 Set-MV98HN3]

$$\begin{array}{r|l} & x+3 \\ x^2+2x & x^3+5x^2+7x+3 \\ & \underline{x^3+2x^2} \\ & 3x^2+7x+3 \\ & \underline{3x^2+6x} \\ & x+3 \end{array}$$

2. Divided $6x^3+2x^2-4x+3$ by $3x^2-2x+1$ and verify the division algorithm.

Ans : [Board Term-1, 2011, Set-74]

$$\begin{array}{r|l} & 2x+2 \\ 3x^2-2x+1 & 6x^3+2x^2-4x+3 \\ & \underline{6x^3-4x^2+2x} \\ & 6x^2-6x+3 \\ & \underline{6x^2-4x+2} \\ & -2x+1 \end{array}$$

Quotient $=2x+2$; Remainder $=-2x+1$

$$\begin{aligned} p(x) &= g(x)q(x)+r(x) \\ &= (3x^2-2x+1)(2x+2)+(-2x+1) \\ &= 6x^3-4x^2+2x+6x^2-4x+2-2x+1 \\ &= 6x^3+2x^2-4x+3 \end{aligned}$$

Verified

3. Find the value of a and b so that $8x^2+14x^3-2x^2+ax+b$ is exactly divisible by $4x^2+3x-2$.

Ans : [Board Term-1, 2011, Set-66]

$$\begin{array}{r|l} & 2x^2+2x-1 \\ 4x^2+3x-2 & 8x^4+14x^3-2x^2+ax+b \\ & \underline{8x^4+6x^3-4x^2} \\ & 8x^3+2x^2+ax \\ & \underline{8x^3+6x^2-4x} \\ & -4x^2+(a+4)x+b \\ & \underline{-4x^2-3x+2} \\ & (a+7)x+b \end{array}$$

For exact division, remainder is zero, so

$$(a+7)x+b-2=0$$

or $a+7=0, b-2=0$

$\Rightarrow$ $a=-7, b=2$

4. On dividing a polynomial $3x^3+4x^2+5x-13$ by a polynomial $g(x)$, the quotient and the remainder are $(3x+10)$ and $(16x-43)$ respectively. Find $g(x)$.

Ans : [Board Term-1, 2011, Set-40]

$$\begin{array}{r|l} & x^2-2x+3 \\ 3x+10 & 3x^3+4x^2-11x+30 \\ & \underline{3^3+10x^2} \\ & -6x^2-11x \\ & \underline{-6x^2-20x} \\ & 9x+30 \\ & \underline{9x+30} \\ & 0 \end{array}$$

$$3x^3+4x^2+5x-13=(3x+10)g(x)+(16x-43)$$
$$g(x)(3x+10)=(3x^3+4x^2+5x-13)-(16x-43)$$

Hence, $g(x)=x^2-2x+3$

5. When $p(x)=x^2+7x+9$ is divisible by $g(x)$, we get $(x+2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

Ans : [Board Term-1, 2011, Set-74]

We have $p(x)=x^2+7x+9$

$q(x)=x+2$

$r(x)=-1$

Now $p(x)=g(x)q(x)+r(x)$

$$x^2+7x+9 = g(x)(x+2)-1$$

or,
$$g(x) = \frac{x^2+7x+10}{x+2}$$
$$= \frac{(x+2)(x+5)}{(x+2)} = x+5$$

Thus $g(x) = x+5$

6. Check by divisible, algorithm whether x^2-2 is a factor of $x^4+x^3+x^2-2x-3$.

Ans : [Board Term-1, 2011, Set-39]

Quotient: x^2+x+3

$x^2-2\,)\,x^4+x^3+x^2-2x-3$

$x^4 \quad -2x^2$

x^3+3x^2-2x

$x^3 \quad -2x$

$3x^2 - 3$

$3x^2 - 6$

3

Since Remainder $= 3 \neq 0$, hence x^2-2 is not a factor of the given polynomial.

7. On dividing $x^4-x^3-3x^2+3x+2$ by a polynomial $g(x)$, the quotient and the remainder are x^2-x-2 and $2x$ respectively. Find $g(x)$.

Ans : [Board Term-1, 2015, Set-CJTOQ]

Quotient: x^2-1

$x^2-x-2\,)\,x^4-x^3-3x^2+x+2$

$x^4-x^3-2x^2$

$-x^2+x+2$

$-x^2+x+2$

0

$$x^4-x^3-3x^2+3x+2 = (x^2-x-2)g(x)+2x$$
$$g(x)(x^2-x-2) = (x^4-x^3-3x^2+3x+2)-2x$$
$$g(x) = \frac{x^4-x^3-3x^2+x+2}{x^2-x-2}$$

Hence, $g(x) = x^2-1$

8. What should be added in the polynomial x^3+2x^2-9x+1 so that it is completely divisible by $x+47$.

Ans : [Board Term-1, 2015, Set-DDE-M]

Let k be added.

Quotient: x^2-2x-1

$x+4\,)\,x^3+2x^2-9x+1$

x^3+4x^2

$-2x^2-9x+1$

$-2x^2-8x$

$-x+1$

$-x-4$

5

Remainder should be zero

$$5+k = 0$$

Hence –5 should be added.

9. If the polynomial $f(x) = 3x^4+3x^3-11x^2-5x+10$ is completely divisible by $3x^2-5$, find all its zeroes.

Ans : [Board Term-1, 2013, FFC; 2011, Set-13]

Ans :

Since $3x^2-5$ divides $f(x)$ completely, $(3x^2-5)$ is a factor of $f(x)$.

Thus
$$3x^2-5 = 0$$
$$x^2 = \frac{5}{3}$$
$$x = \pm\sqrt{\frac{5}{3}}$$

Quotient: x^2+x-2

$3x^2-5\,)\,3x^4+3x^3-11x^2-5x+10$

$3x^4 \quad -5x^2$

$3x^3-6x^2-5x+10$

$3x^3 \quad -5x$

$-6x^2 \quad +10$

$-6x^2 \quad +10$

0

Since (x^2+x-2) is a factor of $p(x.)$

$$x^2+2-2 = 0$$

Factorising it, we get

$$x = -2 \text{ and } 1$$

Thus -2 and 1 are zeroes or $p(x)$.

All the zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -2 and 1.

LONG ANSWER TYPE QUESTIONS

1. If two zeroes of a polynomial x^3+5x^2+7x+3 are -1 and -3, then find the third zero.

Ans : [Board Term-1, 2016 Set MV98HN3]

Quotient: $x+1$

$x^2+4x+3\,)\,x^3+5x^2+7x+3$

x^3+4x^2+3x

x^2+4x+3

x^2+4x+3

0

$x = -1$ and $x = -3$ are zeroes.

2. Given that $x-\sqrt{5}$ is a factor of the polynomial $x^3-3\sqrt{5}\,x^2-5x+15\sqrt{5}$, find all the zeroes of the polynomial.

Ans : [Board Term-1, 2014] [Board Term-1, 2012, Set-39]

$$\begin{array}{r|l} & x^2-2\sqrt{5}\,x-15 \\ \hline x-\sqrt{5} & x^3-3\sqrt{5}\,x^2-5x+15\sqrt{5} \\ & x^3-\sqrt{5}\,x^2 \\ \hline & -2\sqrt{5}\,x^2-5x+15\sqrt{5} \\ & -2\sqrt{5}\,x^2+10x \\ \hline & -15x+15\sqrt{5} \\ & -15x+15\sqrt{5} \\ \hline & 0 \end{array}$$

Factorising the quotient we get

$$\begin{aligned} x^2-2\sqrt{5}\,x-15 &= x^2-3\sqrt{5}\,x+\sqrt{5}\,x-15 \\ &= x(x-3\sqrt{5})+\sqrt{5}(x-3\sqrt{5}) \\ &= (x+\sqrt{5})(x-3\sqrt{5}) \end{aligned}$$

$$(x+\sqrt{5})(x-3\sqrt{5}) = 0 \quad \Rightarrow \quad x = \sqrt{5},\ 3\sqrt{5}$$

All the zeroes are $\sqrt{5}$, $-\sqrt{5}$ and $3\sqrt{5}$.

3. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by (x^2-2x+k), the remainder comes out to be $x+a$, find k and a.

Ans : [Board Term-1, 2012, Set-35]

$$\begin{array}{r|l} & x^2-4x+(8-k) \\ \hline x^2-2x+k & x^4-6x^3+16x^2-25x+10 \\ & x^4-2x^3+kx^2 \\ \hline & -4x^3+(16-k)x^2-25x+10 \\ & -4x^3+8x^2-4kx \\ \hline & (8-k)x^2-(25-4k)x+10 \\ & (8-k)x-(16-2k)x+(8k-k^2) \\ \hline & (2k-9)x+(10-8k+k^2) \end{array}$$

Given, remainder $= x+a$

Comparing the multiples of x

$$\begin{aligned} (2k-9)x &= 1\times x \\ 2k-9 &= 1 \\ k &= \frac{10}{2} = 5 \end{aligned}$$

Substituting this value of k into other portion of remainder, we get

and $\quad a = 10-8k+k^2 = 10-40+25 = -5$

4. Find the other zeroes of the polynomial $x^4-5x^3+2x^2+10x-8$ if it is given that two zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

Ans : [Board Term-1, 2012, Set-35]

We have two zeroes $\sqrt{2}$ and $-\sqrt{2}$.

Two factors are $(x+\sqrt{2})$ and $(x-\sqrt{2})$

$g(x)=(x+\sqrt{2})(x-\sqrt{2})=x^2-2$ is a factor of the given polynomial

$$\begin{array}{r|l} & x^2-5x+4 \\ \hline x^2-2 & x^4-5x^3+2x^2+10x-8 \\ & x^4-2x^2 \\ \hline & -5x^3+4x^2+10x-8 \\ & -5x^3-10x \\ \hline & 4x^2-8 \\ & 4x^2-8 \\ \hline & 0 \end{array}$$

Quotient $= x^2-5x+4 = (x-4)(x-1)$

Hence other zeroes are 4 and 1.

5. Show that 3 is a zero of the polynomial $2x^2-x^2-13x-6$. Hence find all the zeroes of this polynomial.

Ans :

$$\begin{array}{r|l} & 2x^2+5x+2 \\ \hline x-3 & 2x^3-x^2-13x-6 \\ & 2x^3-6x^2 \\ \hline & 5x^2-13x-6 \\ & 5x^2-15x \\ \hline & 2x-6 \\ & 2x-6 \\ \hline & 0 \end{array}$$

$$\begin{aligned} p(x) &= 2x^2-x^2-13x-6 \\ &= 2(3)^3-(3)^2-13(3)-6 \\ &= 2(27)-9-39-6 \\ &= 54-54 = 0 \end{aligned}$$

So, $x-3$ is a factor of $p(x)$.

by long division

Factorising the quotient, we get

$$\begin{aligned} &= 3x^2+4x+x+2 \\ &= (2x+1)(x+2) \\ x &= -\frac{1}{2}, -2 \end{aligned}$$

Hence, All the zeroes of $p(x)$ are $-\frac{1}{2}, -2, 3$

6. Obtain all other zeroes of the polynomial $x^4+6x^3+x^2-24x-20$, if two of its zeroes are $+2$ and -5.

Ans : [Board Term-1, 2015, Set-DDE-E] [NCERT]

$$\begin{array}{r|l} & x^2+3x+2 \\ \hline x^2+3x-10 & x^4+6x^3+x^2-24x-20 \\ & x^4+3x^3-10x^2 \\ \hline & 3x^3+11x^2-24x-20 \\ & 3x^3+9x^2-30x \\ \hline & 2x^2+6x-20 \\ & 2x^2+6x-20 \\ \hline & 0 \end{array}$$

As $x=2$ and -5 are the zeroes of $x^4+6x^3+x^2-24x-20$.

So $(x-2)$ and $(x+5)$ are two factors of $x^4+6x^3+x^2-24x-20$ and the product of factors is

$$(x-2)(x+5) = x^2+3x-10 = 0$$

Dividing $x^4+6x^3+x^2-24x-20$ by $x^2+3x-10$

$$\begin{aligned} & x^4+6x^3+x^2-24x-20 \\ &= (x^2+3x-10)(x^2+3x+2) \\ &= (x-2)(x+5)(x+2)(x+1) \end{aligned}$$

Hence other two zeroes are -2 and 1.

7. Obtain all other zeroes of the polynomial $4x^4+x^3-72x^2-18x$, if two of its zeroes are $3\sqrt{2}$ and $-3\sqrt{2}$.

Ans : [Board Term-1, 2015, Set-C3TOQ]

Ans :

As $3\sqrt{2}$ and $-3\sqrt{2}$ are the zeroes of $4x^4+x^3-72x^2-18x$, So $(x-3\sqrt{2})$ and $(x+3\sqrt{2})$ are its two factors

Now, $(x-3\sqrt{2})(x+3\sqrt{2}) = 0$

or, $x^2-18 = 0$

On Factorising quotient $4x^2+2$

We get, $x = 0$ and $\frac{1}{4}$

$$= (x^2-18)x(4x+1)$$
$$= (x-3\sqrt{2})(x+3\sqrt{2})(x)(4x+1)$$

Hence, other two zeroes are 0 and $\frac{-1}{4}$.

8. Obtain all other zeroes of the polynomial $9x^4-6x^3-35x^2+24x-4$, if two of its zeroes are 2 and -2.

Ans : [Board Term-1, 2015, Set -DDE -M]

As 2 and -2 are the zeroes of $9x^4-6x^3-35x^2+24x-4$, So $(x-2)$ and $(x+2)$ are its two factors

and $(x-2)(x+2) = x^2-4$

Dividing $9x^4-6x^3-35x^2+24x-4$ by x^2-4

$$x^2-4\,\overline{)\,9x^4-6x^3-35x^2+24x-4\,}\quad 9x^2-6x+1$$
$$9x^4 \quad -36x^2$$
$$-6x^3+x^2+24x-4$$
$$-6x^3 \quad +24x$$
$$x^2 \quad -4$$
$$x^2 \quad -4$$
$$0$$

Factorising this quotient

$$= [9x^2-3x-3x+1]$$
$$= [3x(3x-1)-1(3x-1)]$$
$$= [(3x-1)(3x-1)]$$
$$= (3x-1)(3x-1)$$

Hence, other two zeroes are $\frac{1}{3},\frac{1}{3}$. 1

9. Find all the zeros of the polynomial $3x^4+6x^3-2x^2-10x-5$ it two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans : [Sample Paper 2017]

$$3x^2-5\,\overline{)\,3x^4+6x^3-2x^2-10x-5\,}\quad x^2+2x+1$$
$$3x^4 \quad -5x^2$$
$$6x^3+3x^2-10x-5$$
$$-6x^3 \quad -10x$$
$$3x^2 \quad -5$$
$$3x^2 \quad -5$$
$$0$$

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of the given polynomial.

So, $\left(x-\sqrt{\frac{5}{3}}\right),\left(x+\sqrt{\frac{5}{3}}\right)$ will be its two factors

$$\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2-5)$$

is a factor of given polynomial

Now, dividing it by $3x^2-5$.

$$x^2+2x+1 = (x+1)^2 = (x+1)(x+1)$$

two other zeroes $=-1$ and -1

Hence all the zeroes of given polynomial

$$= \sqrt{\frac{5}{3}},\sqrt{\frac{5}{3}},-1 \text{ and } -1$$

HOTS QUESTIONS

1. Find the value for k for which $x^4+10x^3+25x^2+15x+k$ is exactly divisible by $x+7$.

Ans :

We have $f(x) = x^4+10x^3+25x^2+15x+k$

If $x+7$ is a factor then -7 is a zero of $f(x)$ and $x=-7$ satisfy $f(x)=0$.

Thus substituting $x=-7$ in $f(x)$ and equating to zero we have,

$$(-7)^4+10(-7)^3+25(-7)^2+15(-7)+k=0$$
$$2401-3430+1225-105+k = 0$$
$$3626-3535+k = 0$$
$$91+k = 0$$
$$k = -91$$

2. If two zeroes of the polynomial $p(x)=x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$. Find the other zeroes.

Ans :

We have

As $2\pm\sqrt{3}$ are the zeroes of $p(x)$, so $x-(2\pm\sqrt{3})$ are the factor of $p(x)$. 1

and the product of zeros,

$$\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\}$$
$$= \{(x-2)-\sqrt{3}\}\{(x-2)+\sqrt{3}\}$$
$$= (x-2)^2-(\sqrt{3})^2$$
$$= x^2-4x+1$$

Dividing $p(x)$ by x^2-4x+1

$$x^2-4x+1\,\overline{)\,x^4-6x^3-26x^2+138x-35\,}\quad x^2-2x-35$$
$$x^4-4x^3+x^2$$
$$-2x^3-27x^2+138x-35$$
$$-2x^3+8x^2-2x$$
$$-35x^2+140x-35$$
$$-35x^2+140x-35$$
$$0$$

Factorising $(x^2-2x-35)$ we get

$$= (x+5)(x-7)$$

$$x = -5, 7$$

Hence, other two zeroes of $p(x)$ are -5 and 7. 1

3. If α and β are the zeroes the polynomial $2x^2 - 4x + 5$, find the values of

(i) $\alpha^2 + \beta^2$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iii) $(\alpha - \beta)^2$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(v) $\alpha^2 + \beta^2$

Ans :

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{a}{b} = \frac{-(-4)}{2} = 2$$

and $$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= 2^2 - 2 \times \frac{5}{2}$
$= 4 - 5 = -1$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$

(iii) $(\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$
$= 2^2 - \frac{4 \times 5}{2}$
$4 - 10 = -6$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{-1}{(\frac{5}{2})^2} = \frac{-4}{25}$

(v) $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
$= 2^2 - 3 \times \frac{5}{2} \times 2 = 8 - 15 = -7$

4. On dividing the polynomial $4x^2 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial $g(x)$, the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$. Find the polynomial $g(x)$.

Ans :

$Dividend = (Divisor \times Quotient) + Remainder$

$4x^4 - 5x^3 - 39x^3 - 46x - 2$
$= g(x)(x^2 + 3x - 5) + (-5x + 8)$

or, $4x^2 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8$
$= g(x)(x^2 - 3x - 5)$

or, $4x^4 - 5x^3 - 39x^2 - 41x - 10$
$= g(x)(x^2 - 3x - 5)$

$$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$$

Hence, $g(x) = 4x^2 + 7x + 2$

5. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Ans :

We have $f(x) = x^2 + px + 45$

Let α and β be the zeroes of the given quadratic polynomial.

Sum of zeroes, $\alpha + \beta = -p$

Product of zeroes $\alpha\beta = 45$

Given, $(\alpha - \beta)^2 = 144$

$(\alpha + \beta)^2 - 4\alpha\beta = 144$

Substituting value of $\alpha + \beta$ and $\alpha\beta$ we get

$(-p)^2 - 4 \times 45 = 144$

$p^2 - 180 = 144$

$p^2 = 144 + 180 = 324$

Thus $p = \pm\sqrt{324} = \pm 18$

Hence, the value of p is ± 18.