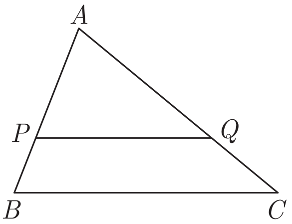
CHAPTER 6

Triangles

1. OBJECTIVE QUESTIONS

1. In the given figure, P and Q are points on the sides AB and AC respectively of a triangle ABC. PQ is parallel to BC and divides the triangle ABC into 2 parts, equal in area. The ratio of $PA:AB=$



(a) 1 : 1 (b) $(\sqrt{2}-1):\sqrt{2}$
(c) $1:\sqrt{2}$ (d) $(\sqrt{2}-1):1$

Ans : (c) $1:\sqrt{2}$

As PQ is parallel to BC

$$\Delta ABC \quad \Delta APQ$$

$$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta APQ} = \frac{2}{1}$$

$$\text{Ratio of sides } = \frac{AB}{AP} = \frac{\sqrt{2}}{1}$$

$$AP:AB = 1:\sqrt{2}$$

$$\text{Ratio of } PB = AB:AP = \sqrt{2}-1:\sqrt{2}$$

2. It is given that $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR}=\frac{1}{3}$. Then $\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)}$ is equal to

(a) 9 (b) 3
(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

Ans : (a) 9

Since, $\Delta ABC \sim \Delta PQR$

$$\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)} = \frac{AR^2}{AC^2}$$

$$= \frac{QR^2}{BC^2} = \frac{9}{1} \qquad \left[\frac{QR}{BC}=\frac{3}{1}\right] = 9$$

3. The area of a right angled isosceles triangle whose hypotenuse is equal to 270 m is-

(a) 19000 m^2 (b) 18225 m^2
(c) 17256 m^2 (d) 18325 m^2

Ans : (b) 18225 m^2

$$\text{Hypotenuse} = 270 \text{ m}$$

$$\text{Hypotenuse}^2 = \text{Side}^2 + \text{Side}^2 = 2\,\text{Side}^2$$

$$\text{Side}^2 = (270)^2/2 = 72900/2 = 36450$$

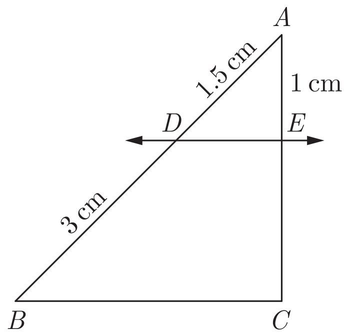
or $\text{side} = 190.91$ m

$$\text{Required Area} = 1/2 \times 190.91 \times 190.91$$

$$= 36446.6/2$$

$$= 18225 \text{ m}^2 \text{ (approx)}$$

4. In the given figure, $DE \parallel BC$. The value of EC is



(a) 1.5 cm (b) 3 cm
(c) 2 cm (d) 1 cm

Ans : (c) 2 cm

Since, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC}$$

$$EC = 2 \text{ cm}$$

5. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If $BC = 4.5$ cm, then the length of QR is

(a) 4 cm (b) 4.5 cm
(c) 3 cm (d) 6 cm

Ans : (d) 6 cm

Since, $\Delta ABC \sim \Delta PQR$

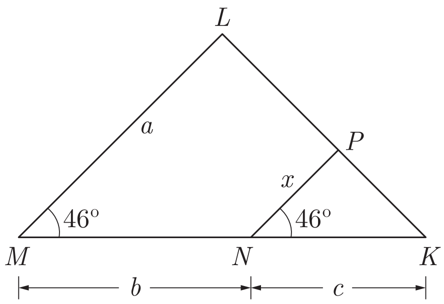
$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\frac{9}{16} = \frac{(4.5)^2}{QR^2}$$

$$QR^2 = \frac{16\times(4.5)^2}{9}$$

$$QR = 6 \text{ cm}$$

6. In the given figure, express x in terms of a, b and c.



(a) $x = \frac{ab}{a+b}$ (b) $x = \frac{ac}{b+c}$

(c) $x = \frac{bc}{b+c}$ (d) $x = \frac{ac}{a+c}$

Ans : (b) $x = \frac{ac}{b+c}$

In ΔKPN and ΔKLM, we have

$$\angle KNP = \angle KML = 46°$$

$$\angle K = \angle K \quad \text{(Common)}$$

$$\Delta KNP \sim \Delta KML$$

(By $A - A$ criterion of similarity)

$$\frac{KN}{KM} = \frac{NP}{ML}$$

$$\frac{c}{b+c} = \frac{x}{a}$$

$$x = \frac{ac}{b+c}$$

7. If $\Delta ABC \sim \Delta APQ$ and ar $(\Delta APQ) = 4$ ar (ΔABC), then the ratio of BC to PQ is

(a) 2 : 1 (b) 1 : 2

(c) 1 : 4 (d) 4 : 1

Ans : (b) 1 : 2

Since, $\Delta ABC \sim \Delta APQ$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta APQ)} = \frac{BC^2}{PQ^2}$$

$$\frac{\text{ar}(\Delta ABC)}{4 \cdot \text{ar}(\Delta ABC)} = \frac{BC^2}{PQ^2}$$

$$\left(\frac{BC}{PQ}\right)^2 = \frac{1}{4}$$

$$\frac{BC}{PQ} = \frac{1}{2}$$

8. The length of the side of a square whose diagonal is 16 cm, is

(a) $8\sqrt{2}$ cm (b) $2\sqrt{8}$ cm

(c) $4\sqrt{2}$ cm (d) $2\sqrt{2}$ cm

Ans : (a) $8\sqrt{2}$ cm

Let side of square $= x$ cm

By Pythagoras theorem,

$$x^2 + x^2 = (16)^2 = 256$$

$$2x^2 = 256$$

$$x^2 = 128$$

$$x = 8\sqrt{2} \text{ cm}$$

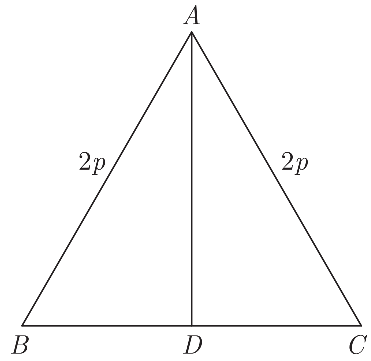
9. ΔABC is an equilateral triangle with each side of length $2p$. If $AD \perp BC$ then the value of AD is

(a) $\sqrt{3}$ (b) $\sqrt{3}\,p$

(c) $2p$ (d) $4p$

Ans : (b) $\sqrt{3}\,p$

Given an equilateral triangle ABC in which,



$$AB = BC = CA = 2p$$

and $AD \perp BC$

In ΔADB, $AB^2 = AD^2 + BD^2$

(By Pythagoras theorem)

$$(2p)^2 = AD^2 + p^2$$

$$AD^2 = \sqrt{3}\,p$$

10. Which of the following statement is false?

(a) All isosceles triangles are similar.

(b) All quadrilateral triangles are similar.

(c) All circles are similar.

(d) None of the above

Ans : (a) All isosceles triangles are similar.

Au is isosceles triangle is a triangle with two side of equal length hence statement given in option (a) is false.

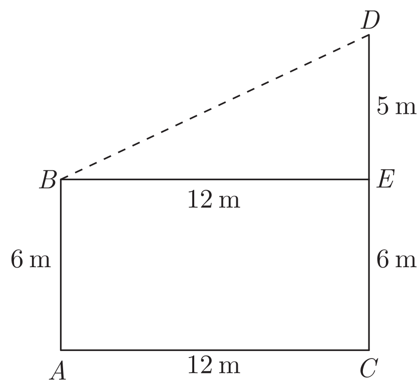
11. Two poles of height 6m and 11m stand vertically upright on a plane ground. If the distance between their foot is 12 m, then distance between their tops is

(a) 12 m (b) 14 m

(c) 13 m (d) 11 m

Ans : (c) 13 m

Let AB and CD be the vertical poles.



$$AB = 6 \text{ m}, \; CD = 11 \text{ m}$$

and $AC = 12$ m

Draw $BE || AC$, $DE = CD - CE$

$= (11 - 6)$ m $= 5$ m

In right angled, ΔBED,

$$BD^2 = BE^2 + DE^2 = 12^2 + 5^2 = 169$$

$$BD = \sqrt{169}\,\text{m} = 13 \text{ m}$$

Hence, distance, between their tops = 13 m.

12. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively, then the ratio of their corresponding medians is

(a) 7 : 9 (b) 9 : 81

(c) 9 : 7 (d) 81 : 7

Ans : (c) 9 : 7

Given, area of two similar triangles,

$$A_1 = 81\,\text{cm}^2$$

$$A_2 = 49\,\text{cm}^2$$

$$\text{Ratio of corresponding medians} = \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{81}{49}} = \frac{9}{7}$$

13. Sides of two similar triangles are in the ratio 4:9. Areas of these triangles are in the ratio.

(a) 2:3 (b) 4:9

(c) 81:16 (d) 16:81

Ans : (d) 16:81

We have two similar triangles such that the ratio of their corresponding sides is 4:9.

The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{\text{ar}(\Delta_1)}{\text{ar}(\Delta_2)} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

$$\text{ar}(\Delta_1) : \text{ar}(\Delta_2) = 16 : 81$$

14. In a right angled ΔABC right angled at B, if P and Q are points on the sides AB and BC respectively, then

(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$

(b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$

(c) $AQ^2 + CP^2 = AC^2 + PQ^2$

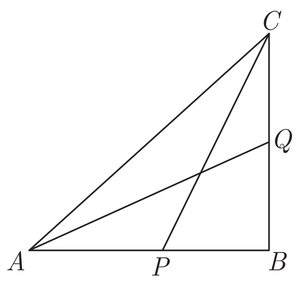
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

Ans : (c) $AQ^2 + CP^2 = AC^2 + PQ^2$

In right angled ΔABQ and ΔCPB,

$$CP^2 = CB^2 + BP^2$$

and $$AQ^2 = AB^2 + BQ^2$$



$$CP^2 + AQ^2 = CB^2 + BP^2 + AB^2 + BQ^2$$

$$= CB^2 + AB^2 + BP^2 + BQ^2$$

$$= AC^2 + PQ^2$$

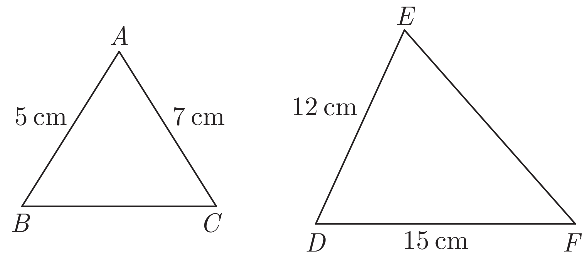
15. It is given that, $\Delta ABC \sim \Delta EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm, then the sum of the remaining sides of the triangles is

(a) 23.05 cm (b) 16.8 cm

(c) 6.25 cm (d) 24 cm

Ans : (a) 23.05 cm

Given, $\Delta ABC \sim \Delta EDF$



Since, $\Delta ABC \sim \Delta EDF$

$$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

On taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5}$$

$$= 16.8 \text{ cm}$$

On taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12}$$

$$= 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle,

$$= EF + BC = 16.8 + 6.25$$

$$= 23.05 \text{ cm}$$

16. The area of a right angled triangle is 40 sq cm and its perimeter is 40 cm. The length of its hypotenuse is

(a) 16 cm (b) 18 cm

(c) 17 cm (d) data insufficient

Ans : (b) 18 cm

Suppose hypotenuse of the triangle is c and other sides are a and b, obviously.

$$c = \sqrt{a^2 + b^2}$$

We have, $a + b + c = 40$ and $\frac{1}{2}ab = 40 \Rightarrow ab = 80$

$$c = 40 - (a + b) \text{ and } ab = 80$$

$$(a + b)^2 - 2 \times 40(a + b) + 1600 = a^2 + b^2$$

$$a^2 + b^2 + 2 \times 80 - 80(a + b) + 1600 = a^2 + b^2$$

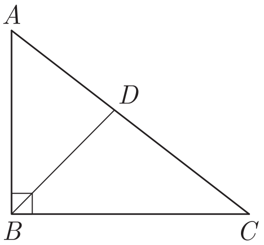
$$80(a + b - 2) = 1600$$

$$a + b = 22$$

$$c = 40 - (a + b)$$

$$= 40 - 22 = 18 \text{ cm}$$

17. In the figure given below, $\angle ABC = 90°$, $AD = 15$ cm and $DC = 20$ cm. If BD is the bisector of $\angle ABC$, What is the perimeter of the triangle ABC?



(a) 74 cm (b) 84 cm

(c) 91 cm (d) 105 cm

Ans : (b) 84 cm

Since BD is the angle bisector of $\angle B$, therefore by angle bisctor theorem, we get $\Delta ABD \sim \Delta CBD$

$$\frac{AB}{BC} = \frac{AD}{DC} = \frac{15}{20} \Rightarrow \frac{AB}{BC} = \frac{3}{4} \quad ...(1)$$

Now, by pythagoras theorem, we get,

$$AC^2 = AB^2 + BC^2$$

$$(AD + DC)^2 = \left(\frac{3}{4}BC\right)^2 + BC^2$$

$$(35)^2 = \frac{25}{16}BC^2 \Rightarrow 1225 \times \frac{16}{25} = BC^2$$

$$BC^2 = 49 \times 16 \Rightarrow BC = 7 \times 4$$

$$= 28 \text{ cm}$$

From Eq. (1), We get,

$$AB = \frac{3}{4} \times BC = \frac{3}{4} \times 28 = 21 \text{ cm}$$

Thus, the perimeter of

$$\Delta ABC = (28 + 21 + 35)\text{ cm} = 84 \text{ cm}$$

18. Diagonal AC of a rectangle $ABCD$ is produced to the point E such that $AC:CE = 2:1$, $AB = 8$ cm and $BC = 6$m. The length of DE is

(a) $2\sqrt{19}$ cm (b) 15 cm

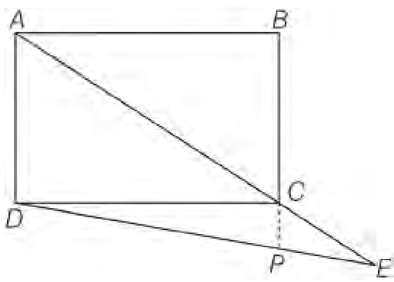
(c) $3\sqrt{17}$ cm (d) 13 cm

Ans : (c) $3\sqrt{17}$ cm

Given, $AB = 8$ cm and $BC = 6$ cm

$$AC = \sqrt{8^2 + 6^2} = 10 \text{ cm}$$

Also, given, $AC:CE = 2:1$



Now, produce BC to meet DE at the point P. As CP is parallel to AD,

$$\Delta ECP \sim \Delta EAD \quad ...(1)$$

$$\frac{CP}{AD} = \frac{CE}{AE} \Rightarrow \frac{CP}{6} = \frac{1}{3}$$

$$CP = 2 \text{ cm}$$

Also, ΔCPD is a right triangle

$$DP = \sqrt{CD^2 + CP^2} = \sqrt{68}$$

$$= 2\sqrt{17} \text{ cm}$$

But, $DP:PE = 2:1$ [From eq. (1)]

$$PE = \sqrt{17}$$

Thus, $DE = DP + PE$

$$= 2\sqrt{17} + \sqrt{17} = 3\sqrt{17} \text{ cm}$$

19. O is the point of intersection of the diagonals AC and BD of a trapezium $ABCD$ with $AB || DC$. Through O, a line segment PQ is drawn parallel to AB meeting AD is P and BC in Q, then $OP =$

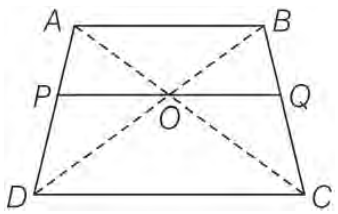
(a) $OP = OQ$ (b) $OP = 2OQ$

(c) $OQ = 2OP$ (d) $OP = \frac{1}{3}OQ$

Ans : (a) $OP = OQ$

Given, $ABCD$ is a trapezium. Diagonals AC and BD intersect at O.

$$PQ || AB || DC$$



To prove, $PO = QO$

Proof: In ΔABD and ΔPOD,

$PO \parallel AB$ [$PQ || AB$]

$\angle ADB \ \angle PDO$ [common angle]

$\angle ABD = \angle POD$ [corresponding angles]

$\Delta ABD \sim \Delta POD$

[by AA similarly criterion]

Then, $$\frac{OP}{AB} = \frac{PD}{AD} \quad ...(1)$$

In ΔABC and ΔOQC, $OQ || AB$ [$PQ || AB$]

$\angle ACB = \angle OCQ$ [common angle]

and $\angle BAC = \angle QOC$ [corresponding angles]

$\Delta ABC \sim \Delta OQC$

[by AA similarity criterion]

Then, $$\frac{OQ}{AB} = \frac{QC}{BC} \quad ...(2)$$

Now, in ΔADC, $OP || DC$

$$\frac{AP}{PD} = \frac{OA}{OC} \quad ...(3)$$

[by basic proportionality theorem]

In ΔABC, $OQ \parallel AB$

$$\frac{BQ}{QC} = \frac{OA}{OC} \quad ...(4)$$

[by basic proportionality theorem]

From Eq. (3) and (4),

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

On adding 1 to both sides, we get

$$\frac{AP}{PD}+1 = \frac{BQ}{QC}+1$$

$$\frac{AP+PD}{PD} = \frac{BQ+QC}{QC}$$

$$\frac{AD}{PD} = \frac{BC}{QC} \Rightarrow \frac{PD}{AD} = \frac{QC}{BC}$$

[on taking reciprocal of the terms]

$$\frac{OP}{AB} = \frac{OQ}{AB} \quad \text{[From Eq. (1) and (2)]}$$

$$OP = OQ$$

Hence proved.

20. The area of the semi-circle drawn on the hypotenuse of a right angled triangle is equal

(a) Sum of the areas of the semi-circles drawn on the other two sides of the triangle.

(b) difference of the areas of semi-circles drawn on the other two sides of the triangle.

(c) product of the areas of semi-circles drown on the other two sides of the triangle

(d) None of these

Ans : (c) product of the areas of semi-circles drown on the other two sides of the triangle

Let ABC be a right angled triangle, right angled at B and $AB = y$, $BC = x$.

Then, three semi-circles are drawn on the sides AB, BC and AC, respectively with diameter AB, BC and AC, respectively. (see figure).

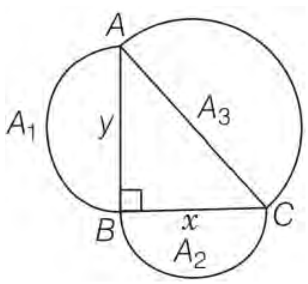
Again, let area of semi-circles with diameters AB, BC and AC are A_1, A_2 and A_3 respectively.

To prove, $A_3 = A_1 + A_2$

Proof: In ΔABC, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = y^2 + x^2$$

$$AC = \sqrt{y^2 + x^2}$$



We know that, area of a semi-circle with radius $r = \frac{\pi r^2}{2}$

Area of semi-circle drawn on AC,

$$A_3 = \frac{\pi}{2}\left(\frac{AC}{2}\right)^2 = \frac{\pi}{2}\left(\frac{\sqrt{y^2+x^2}}{2}\right)^2$$

$$A_3 = \frac{\pi(y^2+x^2)}{8} \quad ...(1)$$

Now, area of semi-circle drawn on AB,

$$A_1 = \frac{\pi}{2}\left(\frac{AB}{2}\right)^2$$

$$A_1 = \frac{\pi}{2}\left(\frac{y}{2}\right)^2$$

$$A_1 = \frac{\pi y^2}{8} \quad ...(2)$$

and area of semi-circle drawn on BC,

$$A_2 = \frac{\pi}{2}\left(\frac{BC}{2}\right)^2 = \frac{\pi}{2}\left(\frac{x}{2}\right)^2$$

$$A_2 = \frac{\pi x^2}{8} \quad ...(3)$$

On adding Eq. (2) and (3), we get

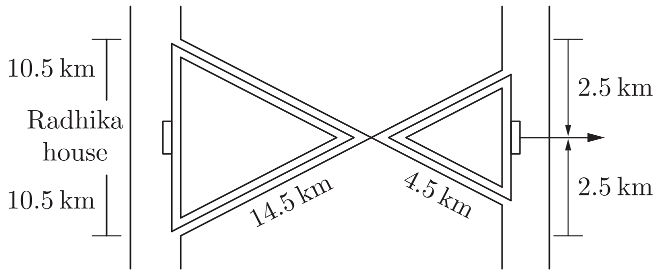
$$A_1 + A_2 = \frac{\pi y^2}{8} + \frac{\pi x^2}{8}$$

$$= \frac{\pi(y^2+x^2)}{8} = A_3 \text{[From eq. (1)]}$$

$$A_1 + A_2 = A_3$$

Hence proved.

21. Radhika wants to visit her friend who recently moved to a new house. The road map between Radhika's home and her friend's as well as the distance known to Radhika are as shown in the figure given below:



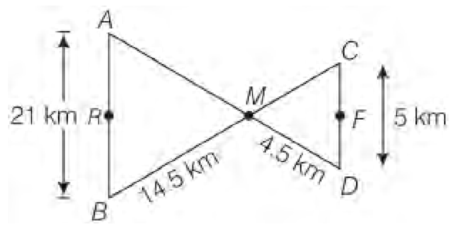
To reach the friend's house, the shortest distance which Radhika has to travel, is

(a) 30.95 km (b) 32.5 km

(c) 28.5 km (d) 35.35 km

Ans : (a) 30.95 km

Given figure can be redrawn as



In ΔBAM and ΔCDM

$$\angle BAM = \angle CDM$$

[alternate interior angles as $AB || CD$]

$$\angle ABM = \angle DCM$$

[alternate interior angles]

and $\angle AMB = \angle DMC$

[vertically opposite angles]

$$\Delta BAM \sim \Delta CDM$$

[by AAA similarly criterion]

So, $\frac{AM}{DM} = \frac{BA}{CD} = \frac{BM}{CM}$

[since, corresponding sides if similar triangles are proportional]

$$\frac{AM}{4.5} = \frac{21}{5} = \frac{14.5}{CM}$$

Now, consider $\frac{AM}{4.5} = \frac{21}{5}$

$$AM = \frac{21 \times 4.5}{5}$$

$$= 21 \times 0.9 = 18.9 \text{ km}$$

There are four routes which Radhika may follows to reach friend's house, that are given below:

Ist Route $R \to A \to M \to C \to F$

$$\text{Total distance} = RA + AM + MC + CF$$
$$= 10.5 + 18.9 + 3.45 + 2.5$$
$$= 35.35 \text{ km}$$

2nd Route $R \to B \to M \to D \to F$

$$\text{Total distance} = RB + BM + MD + DF$$
$$= 10.5 + 14.5 + 4.5 + 2.5$$
$$= 32 \text{ km}$$

3rd Route $R \to A \to M \to D \to F$

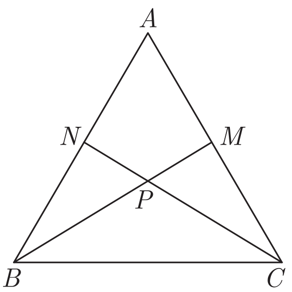
$$\text{Total distance} = RA + AM + MD + DF$$
$$= 10.5 + 18.9 + 4.5 + 2.5$$
$$= 36.4 \text{ km}$$

4th Route $R \to B \to M \to C \to F$

$$\text{Total distance} = RB + BM + MC + CF$$
$$= 10.5 + 14.5 + 3.45 + 2.5$$
$$= 30.95 \text{ km}$$

Hence, the shortest distance which Radhika has to travel is 30.95 km.

22. In the figure given below, $AM : MC = 3:4$, $BP : PM = 3:2$ and $BN = 12$ cm. Then AN equals to



(a) 10 cm (b) 12 cm
(c) 24 cm (d) 16 cm

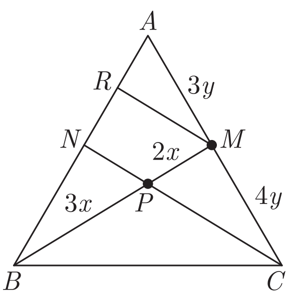
Ans : (c) 24 cm

Given, $AM : MC = 3:4$

$$BP : PM = 3:2$$

and $BN = 12\text{cm}$

Draw MR parallel to CN which meets AB at the point R



Consider ΔBMR. In this, we have $PN || MR$ (By construction)

By BPT,

$$\frac{BN}{NR} = \frac{BP}{PM} \Rightarrow \frac{12}{NR} = \frac{3}{2}$$

$$NR = 8 \text{ cm}$$

Now, consider $\Delta ANC, RM || NC$ [by construction]

By BPT, $\frac{AR}{RN} = \frac{AM}{MC} \Rightarrow \frac{AR}{8}$

$$= \frac{3}{4} \Rightarrow AR = 6 \text{ cm}$$

Thus, $AN = AR + RN = 6 + 18 = 24$ cm

23. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6m towards the wall, then the distance by which the top of the ladder would slide upwards on the wall is

(a) 0.6 m (b) 0.2 m
(c) 0.4 m (d) 0.8 m

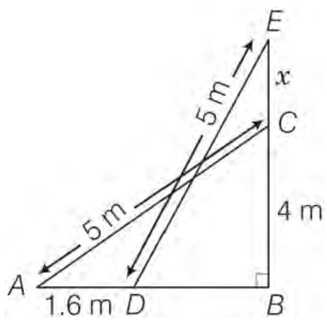
Ans : (d) 0.8 m

Let AC be the ladder of length 5 m and $BC = 4$m be the height of the wall, on which ladder is placed.

If the foot of the ladder is moved 1.6 m towards the wall, ie.. $AD = 1.6$m, then the ladder is slide upward, i.e. $CE = x$m.

In right angled ΔABC,

$$AC^2 = AB^2 + BC^2 \quad \text{[by Pythagoras theorem]}$$
$$(5)^2 = (AB)^2 + (4)^2$$
$$AB^2 = 25 - 16 = 9$$
$$AB = 3 \text{ m}$$
$$DB = AB - AD = 3 - 1.6 = 1.4 \text{ m}$$



In right angled ΔEBD,

$$ED^2 = EB^2 + BD^2 \quad \text{[by Pythagoras theorem]}$$
$$(5)^2 = (EB)^2 + (1.4)^2 \quad [BD = 1.4 \text{ m}]$$
$$25 = (EB)^2 + 1.96$$
$$(EB)^2 = 25 - 1.96 = 23.04$$
$$EB = \sqrt{23.04} = 4.8$$

Now, $EC = EB - BC = 4.8 - 4 = 0.8$

Hence, the top of the ladder would slide upwards on the wall by a distance of 0.8 m.

24. In a ΔPQR, L and M are two points on base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$.

Then, which of the following is/are true:
1. $\Delta PQL \sim \Delta RPM$
2. $QL \times RM = PL \times PM$
3. $PQ^2 = QR \cdot QL$

(a) Both (1) and (2)
(b) Both (2) and (3)
(c) Both (1) and (3)
(d) All the three

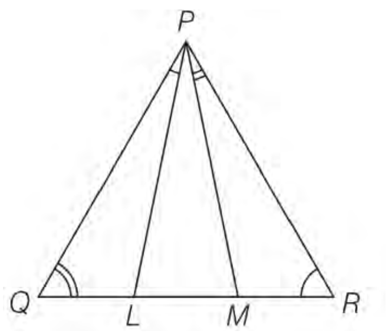
Ans : (d) All the three

Given, $\angle LPQ = \angle QRP$

and $\angle RPM = \angle RQP$

In ΔPQL and ΔRPM,

$\angle LPQ = \angle MRP$ [$\angle LPQ = \angle QRP$, given]



and $\angle LQP = \angle RPM$ [$\angle RQP = \angle RPM$, given]

$\Delta PQL \sim \Delta RPM$ [by AA similarly criterion]

Since, $\Delta PQL \sim \Delta RPM$

$$\frac{QL}{PM} = \frac{PL}{RM}$$

[corresponding of similar triangles are proportional]

$$QL \times RM = PL \times PM$$

In ΔPQL and ΔRQP,

$\angle PQL = \angle RQP$ [common angle]

and $\angle QPL = \angle QRP$ [given]

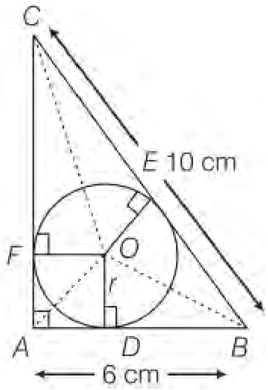
$\Delta PQL \sim \Delta RQP$ [by AA similarity criterion]

Then, $\frac{PQ}{QR} = \frac{QL}{PQ}$

$\Rightarrow$ $PQ^2 = QR \times QL$

25. ΔABC is right angled at A with $AB = 6$ cm, $BC = 10$ cm. A circle with centre O has been inscribed inside the triangle. The radius of the incircle is
(a) 4 cm (b) 3 cm
(c) 2 cm (d) 1 cm

Ans : (c) 2 cm
Given, a right angled ΔABC, in which a circle of centre O is inscribed., the sides of a triangle are $AB = 6$ cm, $BC = 10$ cm and AC.
Join AO, OB and OC.



Now, draw perpendicular from O to AB, BC and CA meeting them at D, E and F, respectively.

$BC^2 = AB^2 + AC^2$ [by Pythagoras theorem]

$(10)^2 = (6)^2 + (AC)^2$ [$BC = 10$ cm, $AB = 6$ cm]

$100 = 36 + AC^2$

$AC^2 = 100 - 36 = 64$

$AC = \sqrt{64} = 8$ cm

Now, $\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 6 \times 8$ ...(1)

$= 24$ sq cm

[area of a triangle $= \frac{1}{2} \times$ base $\times$ height]

Also,

$\text{ar}(\Delta ABC) = \text{ar}(\Delta AOB) + \text{ar}(\Delta BOC) + \text{ar}(\Delta AOC)$

$= \left(\frac{1}{2} \times r \times AB\right) + \left(\frac{1}{2} \times r \times BC\right) + \left(\frac{1}{2} \times r \times AC\right)$

[$OD = OE = OF = r$]

$= \frac{1}{2} \times r \times (AB + BC + AC)$

$= \frac{1}{2} \times r \times (6 + 10 + 8) = 12r$ ...(2)

From Eq. (1) and (2),

$12r = 24$

$r = 2$

Hence, the radius of the incidence of ΔABC is 2 cm.

2. FILL IN THE BLANK

1. A line drawn through the mid-point of one side of a triangle parallel to another side bisects the .... side.
Ans : third

2. .... theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
Ans : Pythagoras

3. Line joining the mid-points of any two sides of a triangle is .... to the third side.
Ans : parallel

4. All squares are ....
Ans : similar

5. Two triangles are said to be .... if corresponding angles of two triangles are equal.
Ans : equiangular

6. All similar figures need not be ....
Ans : congruent

7. The ratio of the areas of two similar triangles is equal to the square of the ratio of their ....
Ans : corresponding sides

8. Two polygons of the same number of sides are similar, if their corresponding angles are .... and their corresponding sides are in the same ....
Ans : equal, ratio

9. All circles are ....
Ans : similar

10. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the .... side.
Ans : third

11. If a line divides any two sides of a triangle in the same ratio, then the line is .... to the third side.
Ans : parallel

12. All congruent figures are similar but the similar figures need .... be congruent.
Ans : not

13. If two polygons are similar then the same ratio of the corresponding sides is referred to as the ....
Ans : scale factor

14. Two polygons of the same number of sides are similar, if all the corresponding angles are ....
Ans : equal

15. Two figures are said to be .... if they have same shape but not necessarily the same size.
Ans : similar

16. All circles are ....
Ans : similar

17. .... theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
Ans : Basic proportionality

18. All .... triangles are similar.
Ans : equilateral

19. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the .... ratio.
Ans : same

20. Two figures having the same shape and size are said to be ....
Ans : congruent

21. The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO} \cdot ABCD$ is a ....
Ans : trapezium

3. TRUE/FALSE

1. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar.
Ans : True

2. Two photographs of the same size of the same person at the age of 20 years and the other at the age of 45 years are not similar.
Ans : True

3. A square and a rectangle are similar figure as each angle of the two quadrilaterals is 90°.
Ans : False

4. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.
Ans : True

5. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar.
Ans : True

6. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
Ans : True

7. All congruent figures need not be similar.
Ans : False

8. All the congruent figures are similar but the converse is not true.
Ans : True

9. A circle of radius 3 cm and a square of side 3 cm are similar figures.
Ans : False

10. In $\triangle PQR$, if X and Y are points on sides PQ and PR such that $\frac{PX}{XQ} = \frac{4}{18}$ and $\frac{PY}{PR} = \frac{6}{27}$, than RQ is not parallel to XY.

Ans : True

11. $A \triangle ABC$ with $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm is a right triangle.

Ans : True

12. Two figures having the same shape but not necessarily the same size are called similar figures.

Ans : True

13. If $\triangle DEF \sim \triangle QRP$, then $\angle D = \angle Q$ and $\angle E = \angle P$.

Ans : False

14. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar.

Ans : True

15. In a right triangle, the square of the hypotenuse is equal to the sum of the other two sides.

Ans : False

16. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Ans : True

17. If $\triangle ABC \sim \triangle XYZ$, than $\frac{AB}{XY} = \frac{AC}{XZ}$.

Ans : True

18. If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Ans : True

19. If $\triangle DEF \sim \triangle PQR$, $ar(\triangle DEF) = 9$ sq. units, then

$ar(\triangle PQR) : ar(\triangle DEF) = 4:3$

Ans : True

20. Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O, $\frac{OA}{OC} = \frac{OB}{OD}$.

Ans : True

4. MATCHING QUESTIONS

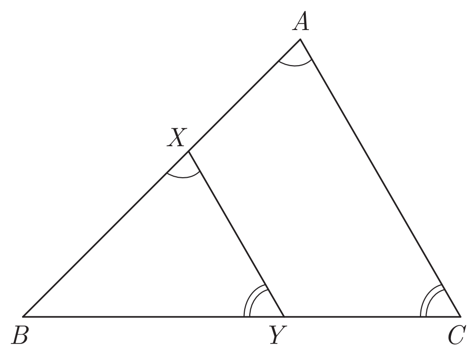
DIRECTION : Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.

1. If in a ΔABC, $DE \parallel BC$ and intersects AB in D and AC in E, then.

	Column-I		Column-II
(A)	$\frac{AD}{DB}$	(p)	$\frac{AC}{AE}$
(B)	$\frac{AB}{AD}$	(q)	$\frac{AE}{EC}$
(C)	$\frac{DB}{AB}$	(r)	$\frac{AE}{AC}$
(D)	$\frac{AD}{AB}$	(s)	$\frac{EC}{AC}$

Ans : (A) – q, (B) – p, (C) – s, (D) – r

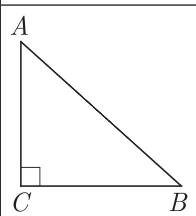
2. In figure, the line segment XY is parallel to the side AC of ΔABC and it divides the triangle into two parts of equal areas, then

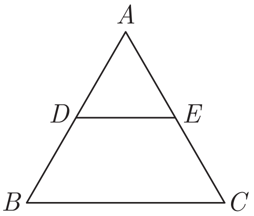


	Column-I		Column-II
(A)	$AB : XB$	(p)	$\sqrt{2} : 1$
(B)	ar (ΔABC) : ar (ΔXBY)	(q)	$2:1$
(C)	$AX : AB$	(r)	$(\sqrt{2} - 1)^2 : \sqrt{2}$
(D)	$\angle X : \angle A$	(s)	$1 : 1$

Ans : (A) – p, (B) – q, (C) – r, (D) – s

3.

	Column-I		Column-II
(A)		(p)	36 : 49
	ABC is an isosceles right angled triangle. $AB^2 = ?$	(q)	$AB^2 = 2AC^2$
(B)	$\Delta ABC \sim \Delta DEF$, Such that and $AB = 1.2$ cm $DE = 1.4$ cm $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = ?$	(r)	$AB^2 = AC^2 + BC^2$

	Column-I		Column-II
(C)	$\Delta ABC \sim \Delta APQ$ and $\frac{\text{area}(\Delta APQ)}{\text{area}(\Delta ABC)} = \frac{36}{49}$ $\frac{BC}{PQ} = ?$	(s)	6 : 7
(D)	 If $DE \parallel BC$ and $\frac{AD}{DB} = \frac{6}{7}$ then, $\frac{AE}{EC} = ?$	(t)	72 : 98

Ans : (A) – (q, r), (B) – (p, t), (C) – s,(D) – s

(A) $$AB^2 = AC^2 + BC^2$$

Since, ΔABC is an isosceles right angled triangle.

$$AC = BC$$

Now, $$AB^2 = AC^2 + AC^2 = 2AC^2$$

(B) $$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{(AB)^2}{(DE)^2} = \frac{(1.2)^2}{(1.4)^2} = \frac{1.44}{1.96}$$

$$= \frac{36}{49} = \frac{(36 \times 2)}{(49 \times 2)} = \frac{72}{98}$$

(C) $$\frac{\text{area}(\Delta APQ)}{\text{area}(\Delta ABC)} = \frac{(BC)^2}{(PQ)^2} = \frac{36}{49}$$

$$\frac{BC}{PQ} = \frac{6}{7}$$

(D) $$DE \parallel BC$$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{6}{7}$$

5. ASSERTION AND REASON

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1. **Assertion :** If in a ΔABC, a line $DE \parallel BC$, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$

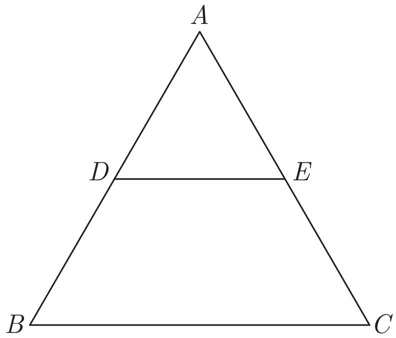
Reason : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Reason is true : [This is Thale's Theorem]

For Assertion

Since, $DE \parallel BC$ by Thale's Theorem



$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{DB}{AD} = \frac{EC}{AE}$$

$$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

Assertion (a) is true

Since, reason gives Assertion.

2. **Assertion :** In ΔABC, $DE || BC$ such that $AD = (7x-4)$cm, $AE = (5x-2)$cm, $DB = (3x+4)$cm and $EC = 3x$ cm than x equal to 5.

Reason : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distant point, than the other two sides are divided in the same ratio.

Ans : (d) Assertion (A) is false but reason (R) is true.

We have, $$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$$21x^2 - 12x = 15x^2 + 20x - 6x - 8$$

$$6x^2 - 26x + 8 = 0$$

$$3x^2 - 13x + 4 = 0$$

$$3x^2 - 12x - x + 4 = 0$$

$$3x(x-4) - 1(x-4) = 0$$

$$(x-4)(3x-1) = 0$$

$$x = 4, \frac{1}{3}$$

So, A is incorrect but R is correct.

3. **Assertion :** $\Delta ABC \sim \Delta DEF$ such that $ar(\Delta ABC) = 36\text{cm}^2$ and $ar(\Delta DEF) = 49\text{cm}^2$ then, $AB : DE = 6 : 7$.

Reason : If $\Delta ABC \sim \Delta DEF$, then $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

$$\frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{AB^2}{DE^2}$$

$$\frac{36}{49} = \frac{AB^2}{DE^2}$$

$$\frac{AB}{DE} = \frac{6}{7}$$

$$AB : DE = 6 : 7$$

So, both A and R are correct and R explain A.

4. **Assertion :** $\triangle ABC$ is an isosceles triangle right angled of C, then $AB^2 = 2AC^2$.
Reason : In right $\triangle ABC$, right angled at B, $AC^2 = AB^2 + BC^2$.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

In an isosceles $\triangle ABC$, right angled at C is

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = AC^2 + AC^2$$

$$AB^2 = 2AC^2 \qquad (AC = BC)$$

So, both A and R are correct and R explains A.

5. **Assertion :** Two similar triangle are always congruent.
Reason : If the areas of two similar triangles are equal then the triangles are congruent.

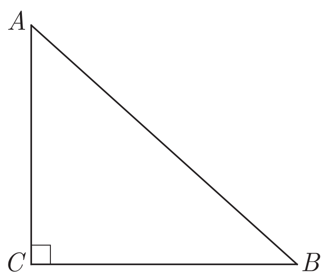
Ans : (d) Assertion (A) is false but reason (R) is true.

Two similar triangles are not congruent generally.
So, A is incorrect but R is correct.

6. **Assertion :** ABC is an isosceles, right triangle, right angled at C. Then $AB^2 = 3AC^2$
Reason : In an isosceles triangle ABC if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C = 90°$

Ans : (d) If Assertion is incorrect, but Reason is correct.

In right angled ΔABC,



$$AB^2 = AC^2 + BC^2$$

(By Pythagorus Theorem)

$$= AC^2 + AC^2 \qquad [BC = AC]$$

$$= 2AC^2$$

$$AB^2 = 2AC^2$$

Assertion is false.

Again since, $AB^2 = 2AC^2 = AC^2 + AC^2$

$$= AC^2 + BC^2 (AC = BC \text{ given})$$

$$\angle C = 90°$$

(By converse of Pythagoras Theorem)

Reason is true.

7. **Assertion :** ABC and DEF are two similar triangles such that $BC = 4$ cm, $EF = 5$ cm and area of $\Delta ABC = 64$ cm^2, then area of $\Delta DEF = 100$ cm^2.
Reason : The areas of two similar triangles are in the ratio of the squares of teh corresponding altitudes.

Ans : (b) It both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

Reason is true. [standard result]

For Assertion, since $\Delta ABC \sim \Delta DEF$

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{(4)^2}{(5)^2} = \frac{16}{25}$$

(ratio of areas of two similar Δs is equal to the ratio of the squares of corresponding sides)

$$\frac{64}{\text{area}(\Delta DEF)} = \frac{16}{25}$$

$$\text{area}(\Delta DEF) = \frac{64 \times 25}{16}$$

$$= 4 \times 25 = 100 \text{ cm}^2$$

Assertion is true. But Reason is not correct explanation for Assertion.

8. **Assertion :** In the $\triangle ABC$, $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm, then $\triangle ABC$ is a right angle triangle.
Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

We have, $AB^2 + BC^2 = (24)^2 + (10)^2$

$$= 576 + 100 = 676 = AC^2$$

$$AB^2 + BC^2 = AC^2$$

ABC is a right angled triangle.
Also, two triangle are similar if their corresponding angles are equal.
So, both A and R are correct but R does not explain A.