CHAPTER 1

Real Numbers

1. OBJECTIVE QUESTIONS

1. If $X = 28 + (1 \times 2 \times 3 \times 4 \times ... \times 16 \times 28)$ and $Y = 17 + (1 \times 2 \times 3 \times ... \times 17)$, then which of the following is/are true?

1. X is a composite number
2. Y is a prime number
3. $X - Y$ is a prime number
4. $X + Y$ is a composite number.

(a) Both (1) and (4) (b) Both (2) and (3)
(c) Both (2) and (4) (d) Both (1) and (2)

Ans : (a) Both (1) and (4)

We have, $X = 28 + (1 \times 2 \times 3 \times ... \times 16 \times 28)$

$X = 28[1 + (1 \times 2 \times 3 \times ... \times 16)]$

Hence, X is a composite number.
Also, we have

$Y = 17 + (1 \times 2 \times 3 \times ... \times 17)$

$= 17[1 + (1 \times 2 \times 3 \times ... \times 16)]$

Hence, Y is a composite number.

Now, $X - Y = [1 + (1 \times 2 \times ... \times 16)](28 - 17)$

$[1 + (1 \times 2 \times 3 ... \times 16)](45)$

$= [1 + (1 \times 2 \times ... \times 16)](11)$

Hence, $X - Y$ is a composite number.

and, $X + Y = [1 + (1 \times 2 \times 3 \times ... \times 16)](28 + 17)$

$= [1 + (1 \times 2 \times 3 \times ...16) \times 45]$

Hence, $X + Y$ is a composite number.

2. Two positive numbers have their HCF as 12 and their product as 6336. The number of pairs possible for the numbers, is

(a) 2 (b) 3
(c) 4 (d) 5

Ans : (a) 2

Let the numbers be $12x$ and $12y$ where x and y are co-primes.

Product of these numbers $= 144xy$

Hence, $144xy = 6336 \Rightarrow xy = 44$

Since, 44 can be written as the product of two factors in three ways. i.e. 1×44, 2×22, 4×11. As x and y are coprime, so (x, y) can be (1,44) or (4,11) but not (2,22).
Hence, two possible pairs exist.

3. The value of $(12)^{3^x} + (18)^{3^x}$, $x \in N$, end with the digit.

(a) 2
(b) 8
(c) 0
(d) Cannot be determined

Ans : (c) 0

For all $x \in N$, $(12)^{3^x}$ ends with either 8 or 2 and $(18)^{3^x}$ ends with either 2 or 8.
If $(12)^{3^x}$ ends with 8, then $(18)^{3^x}$ ends with 2.
If $(12)^{3^x}$ ends with 2, then $(18)^{3^x}$ ends with 8.
Thus, $(12)^{3^x} + (18)^{3^x}$ ends with 0 only.

4. If n is an even natural number, then the largest natural number by which $n(n+1)(n+2)$ is divisible, is

(a) 6 (b) 8
(c) 12 (d) 24

Ans : (d) 24

Since n is divisible by 2 therefore $(n+2)$ is divisible by 4, and hence $n(n+2)$ is divisible by 8.
Also, $n, n+1, n+2$ are three consecutive numbers.
So, one of them is divisible by 3.
Hence, $n(n+1)(n+2)$ must be divisible by 24.

5. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

(a) an even number (b) an odd number
(c) an odd prime number (d) a prime number

Ans : (a) an even number

$p_1^2 - p_2^2$ is an even number.

Let us take $p_1 = 5$

and $p_2 = 3$

Then, $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

6. The rational form of $0.2\overline{54}$ is in the form of $\frac{p}{q}$ then $(p+q)$ is

(a) 14 (b) 55

(c) 69 (d) 79

Ans : (c) 69

Let, $x = 0.2\overline{54}$, then

$x = 0.2545454 \text{ ....}$ ...(1)

Multiplying Eq. (1) by 100, we get

$100x = 25.4545. \text{ ....}$ ...(2)

Subtracting Eq. (1) from Eq. (2), we get

$$99x = 25.2 \Rightarrow x = \frac{252}{990} = \frac{14}{55}$$

Comparing with $\frac{p}{q}$, we get

$p = 14$

and $q = 55$

Hence, $p + q = 14 + 55 = 69$

7. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM (a, b, c) $= 2^3 \times 3^2 \times 5$, then $n =$

(a) 1 (b) 2

(c) 3 (d) 4

Ans : (b) 2

Value of $n = 2$

8. There sets of Mathematics, Science and Biology books have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. The number of Mathematics books is 240, the number of Science books is 960 and the number of Biology books is 1024. The number of stack of Mathematics, Science and Biology books, assuming that the books are of the same thickness are respectively.

(a) 15, 60, 64 (b) 60, 15, 64

(c) 64, 15, 60 (d) None of these

Ans : (a) 15, 60, 64

The prime factorisation of 240, 960 and 1024 is given below:

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 \times 3 \times 5$$

$$960 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^6 \times 3 \times 5$$

and $1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$

HCF of 240, 960 and 1024 $= 2^4 = 16$

Hence, there must be 16 books in each stack.

Now, number of stacks of Mathematics books

$$= \frac{240}{16} = 15$$

Number of stacks of Science books

$$= \frac{960}{16} = 60$$

and, number of stacks of Biology books

$$= \frac{1024}{16} = 64$$

9. If $a + bp^{1/3} + cp^{2/3} = 0$, where a, b, c, p are rational numbers and p is not perfect cube, then

(a) $\neq b = c$ (b) $a = b \neq c$

(c) $a \neq b \neq c$ (d) $a = b = c$

Ans : (d) $a = b = c$

We have, $a + bp^{1/3} + cp^{2/3} = 0$ ...(1)

On multiplying both sides by $p^{1/3}$, we get

$$ap^{1/3} + bp^{1/3} \times p^{1/3} + cp^{2/3} \times p^{1/3} = 0$$

$$ap^{1/3} + bp^{2/3} + cp^{3/5} = 0$$

$$ap^{1/3} + bp^{2/3} + cp = 0 \quad ...(2)$$

On multiplying Eq. (1) by b and Eq. (2) by c, then subtracting Eq. (2) from Eq. (1), we get

$$(ab + b^2p^{1/3} + bcp^{2/3}) - (acp^{1/3} + bcp^{2/3} + c^2p) = 0$$

$$(b^2 - ac)p^{1/3} + ab - c^2p = 0$$

[Here, $p^{1/3}$ is an irrational number and $ab - c^2p$ is a rational number]

Note that, sum of rational and irrational numbers cannot be zero.

So, $(b^2 - ac)p^{1/3} + ab - c^2p = 0$, only if

$(b^2 - ac) = 0$ and $ab - c^2p = 0$

$b^2 = ac$ ...(3)

and $ab = c^2p$

$a^2b^2 = c^4p^2$ ...(4)

[squaring both sides]

From Eq. (4),

$a^2(ac) = c^4p^2$ $[\sin ce, b^2 = ac]$

$a^3c - c^4p^2 = 0 \Rightarrow c(a^3 - c^3p^2) = 0$

$c = 0$ or $a^3 - c^3p^2 = 0 \Rightarrow c = 0$ or $p^2c^3 = a^3$

On taking, $p^2 = \frac{a^3}{c^3}$

We get, $(p^2)^{1/3} = \frac{a}{c}$ which is not possible as $p^{2/3}$ is an irrational number and $\frac{a}{c}$ is a rational number.

Hence, $c = 0$

On putting $c = 0$ in Eq. (3), we get

$b = 0$

On putting $b = 0$, $c = 0$ in Eq. (1), we get

$a = 0$

Hence, $a = b = c = 0$

10. The rational number of the form $\frac{p}{q}$, $q \neq 0$, p and q are positive integers, which represents $0.1\overline{34}$ i.e., (0.1343434 ....) is

(a) $\frac{134}{999}$ (b) $\frac{134}{990}$

(c) $\frac{133}{999}$ (d) $\frac{133}{990}$

Ans : (d) $\frac{133}{990}$

$$0.1\overline{34} = \frac{134 - 1}{990} = \frac{133}{990}$$

11. If x and y are odd positive integers, then $x^2 + y^2$ is

(a) even and divisible by 4

(b) even and not divisible by 4

(c) odd and divisible by 4

(d) odd and not divisible by 4

Ans : (b) even and not divisible by 4

We know that, any odd positive integer is of the form $2q+1$, where q is any integer. So, $x=2m+1$ and $y=2n+1$ for some integers m and n.

Now,
$$\begin{aligned} x^2+y^2 &= (2m+1)^2+(2n+1)^2 \\ &= 4m^2+1^2+4m+4n^2+1+4n \\ &= 4(m^2+n^2)+4(m+n)+2 \\ &= 4\left[(m^2+n^2)+(m+n)\right]+2 \\ &= 4r+2, \end{aligned}$$

Where, $r = m^2+n^2+m+n$

Clearly, $4r+2$ is an even number and not divisible by 4.

Hence, x^2+y^2 is even but not divisible by 4.

12. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600

(c) 2400 (d) 3600

Ans : (d) 3600

The L.C.M. of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect number.

13. Which of the following rational number have non-terminating repeating decimal expansion?

(a) $\frac{31}{3125}$ (b) $\frac{71}{512}$

(c) $\frac{23}{200}$ (d) None of these

Ans : (d) None of these

3125, 512 and 200 has factorization of the form $2^m \times 5^n$ (where m and n are whole numbers). So given fractions has terminating decimal expansion.

14. When 2^{256} is divided by 17 the remanider would be

(a) 1 (b) 16

(c) 14 (d) None of these

Ans : (a) 1

When 2^{256} is divided by 17 then,

$$\frac{2^{256}}{2^4+1} = \frac{(2^2)^{64}}{(2^4+1)}$$

By remainder theorem when $f(x)$ is divided by $x+a$ the remainder $=f(-a)$

Here, $f(a) = (2^2)^{64}$ and $x=2^4$ and $a=1$

Hence, Remainder $= f(-1) = (-1)^{64} = 1$

15. The least number which when divided by 15, leaves a remainder of 5, when divided by 25, leaves a remainder of 15 and when divided by 35 leaves a remainder of 25, is

(a) 515 (b) 525

(c) 1040 (d) 1050

Ans : (a) 515

The number is short by 10 for complete division by 15, 25 or 35.

16. Without Actually performing the long division, the terminating decimal expansion of $\frac{51}{1500}$ is in the form of $\frac{17}{2^n \times 5^m}$, then $(m+n)$ is equal to

(a) 2 (b) 3

(c) 5 (d) 8

Ans : (c) 5

We have, $\frac{51}{1500} = \frac{17}{500}$

Prime factorization of 500

2	500
2	250
5	125
5	25
5	5
	1

$= 2\times2\times5\times5\times5 = 2^2\times5^3$

which is in the form $2^n \times 5^m$

So, it has a terminating decimal expansion.

Now, $\frac{51}{1500} = \frac{17}{2^2\times5^3}$

By comparing, we get $n=2$ and $m=3$

$$m+n = 2+3 = 5$$

17. The sum of three non-zero prime numbers is 100. One of them exceeds the other by 36. Then the largest number is

(a) 73 (b) 91

(c) 67 (d) 57

Ans : (c) 67

Let X, $X+36$ and y are the three prime numbers.

According to the given condition,

$$x+x+36+y = 100$$

$$2x+y = 64 \quad ...(1)$$

Since X is a prime number, then $2x$ will be an even number

Also addition of two even numbers results in an even number,

Hence, from equation (1), we can conclude that y must be an even prime number.

$y = 2$ as 2 is the only even prime number.

Put $y = 2$ in equation (1), we get

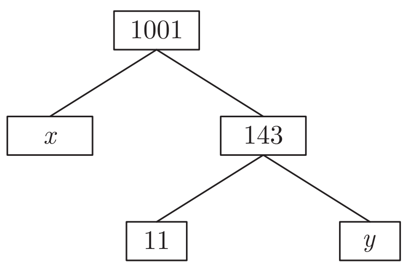
$$2x+2 = 64$$

$$x = 31$$

So the required prime numbers are 31 31+36, 2 or 31, 67 and 2

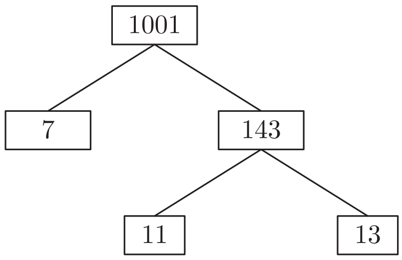
Hence, Largest number is 67.

18. The values of x and y is the given figure are



(a) 7, 13 (b) 13, 7
(c) 9, 12 (d) 12, 9

Ans : (a) 7, 13
Given number is 10001. Then, the factor tree of 1001 is given as below



$$1001 = 7 \times 11 \times 13$$

By comparing with given factor tree, we get

$$x = 7,\ y = 13$$

19. If $P = (2)(4)(6)...(20)$ and $Q = (1)(3)(5)...(19)$, then the HCF of P and Q is

(a) $(3^3)(5)(7)$ (b) $(3^4)(5)$
(c) $(3^4)(5^2)(7)$ (d) $(3^3)(5^2)$

Ans : (c) $(3^4)(5^2)(7)$
In P, the primes that occurs are 2, 3, 5, 7.
In Q, there is no. 2
So, the HCF of P, Q has powers of only 3, 5 and 7.
In P,3 comes from 6, 12, 18. So, 3^4 is the greatest power of 3 in prime factorisation of P. While in Q, 3 comes from 3, 9, 15.
So, 3^4 is also in prime factorisation of Q.
Similarly 5^2 is the greatest power of 5 for both P and Q and 7^1 is the greatest power of 7 for both P and Q.
Hence, HCF of P and Q is $(3^4)(5)^2(7)$.

20. The number $3^{13} - 3^{10}$ is divisible by
(a) 2 and 3 (b) 3 and 10
(c) 2, 3 and 10 (d) 2, 3 and 13

Ans : (d) 2, 3 and 13

$$3^{13} - 3^{10} = 3^{10}(3^3 - 1) = 3^{10}(26)$$
$$= 2 \times 13 \times 3^{10}$$

Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.

21. Which of the following will have a terminating decimal expansion?
(a) $\frac{77}{210}$ (b) $\frac{23}{30}$
(c) $\frac{125}{441}$ (d) $\frac{23}{8}$

Ans : (d) $\frac{23}{8}$

For terminating decimal expansion, denominator must have only 2 or only 5 or 2 and 5 as factor.

Here, $\frac{23}{8} = \frac{23}{(2)^3}$

(only 2 as factor of denominator so terminating)

22. For any natural number $n, 9^n$ cannot end with the digit.
(a) 1 (b) 2
(c) 9 (d) None of these

Ans : (b) 2
For $n = 1$, $9^n = 9^1 = 9$, so 9^n can end with digit 9.
For $n = 2$, $9^n = 9^2 = 81$, so 9^n can also end with digit 1.
Now, let if possible 9^n ends with 2 for some natural numbers n.
Then, 9^n is divisible by 2.
But prime factors of 9 are 3×3.

$$9^n = (3 \times 3)^n = 3^{2n}$$

Thus, prime factorisation of 9^n does not contain 2 as a factor.
By the fundamental theorem of arithmetic, there are no other primes in factorisation of 9^n.
Therefore, 9^n is not divisible by 2. So, our assumption is wrong.
Hence, there is no natural number n for which 9^n ends in the digit 2.

23. A number lies between 300 and 400. If the number is added to the number formed by reversing the digits, the sum is 888 and if the unit's digit and the ten's digit change places, the new number exceeds the original number by 9. Then the number is
(a) 339 (b) 341
(c) 378 (d) 345

Ans : (d) 345
Sum is 888 $\Rightarrow$ unit's digit should add up to 8. This is possible only for option (d) as "3"+"5" = "8".

24. 1. The L.C.M. of x and 18 is 36.
2. The H.C.F. of x and 18 is 2.
What is the number x?
(a) 1 (b) 2
(c) 3 (d) 4

Ans : (d) 4
L.C.M. $\times$ H.C.F. = First number $\times$ second number

Hence, required number $= \frac{36 \times 2}{18} = 4$

25. A circular field has a circumference of 360 km. Two cyclists Sumeet and John start together and can cycle at speeds of 12 km/h and 15 km/h respectively, round the circular field. They will meet again at the starting point after
(a) 40 h (b) 30 h
(c) 180 h (d) 120 h

Ans : (d) 120 h

Given, Total distance = 360 km

and, Speed of Sumeet = 12 km/h

Number of hours taken by Sumeet to complete 1 round.

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{12}$$
$$= 30 \text{ h}$$

and, Speed of John = 15 km/h

Number of hours taken by John to complete 1 round

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{15}$$
$$= 24 \text{ h}$$

Thus, Sumeet and John complete 1 round in 30 h and 24 h, respectively.
Now, to find required hours, we find the LCM of 30 and 24.

$$30 = 2 \times 3 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3$$

Then, $$\text{LCM }(30, 24) = 2 \times 2 \times 2 \times 3 \times 5 = 120$$

Hence, Sumeet and John will meet each other again after 120 h.

26. If n is an even natural number, then the largest natural number by which $n(n+1)(n+2)$ is divisible is

(a) 6 (b) 8
(c) 12 (d) 24

Ans : (d) 24

Out of n and $n+2$, one is divisible by 2 and the other by 4, hence $n(n+2)$ is divisible by 8. Also n, $n+1$, $n+2$ are three consecutive numbers, hence one of them is divisible by 3. Hence, $n(n+1)(n+2)$ must be divisible by 24. This will be true for any even number n.

27. The remainder on dividing given integers a and b by 7 are respectively 5 and 4. Then, the remainder when ab is divided by 7 is

(a) 5 (b) 4
(c) 0 (d) 6

Ans : (d) 6

By using Euclid's division lemma we get

$$a = 7p+5$$

and $$b = 7q+4$$

where p and q are integers

Hence,
$$\begin{aligned} ab &= (7p+5)(7q+4) \\ &= 49pq + (4p+5q)7 + 20 \\ &= 7(7pq+4p+5q) + 7 \times 2 + 6 \\ &= 7(7pq+4p+5q+2) + 6 \end{aligned}$$

Hence, when ab is divided by 7, we get the remainder 6.0.

2. FILL IN THE BLANK

1. H.C.F. of 6, 72 and 120 is ....
Ans : 6

2. 156 as a product of its prime factors ....
Ans : $2^2 \times 3 \times 13$

3. If $a = bq + r$, least value of r is ....
Ans : Zero

4. If every positive even integer is of the form $2q$, then every positive odd integer is of the form ...., where q is some integer.
Ans : $2q+1$

5. The exponent of 2 in the prime factorisation of 144, is ....
Ans : 4

6. $\sqrt{2}, \sqrt{3}, \sqrt{7}$, etc. are .... numbers.
Ans : Irrational

7. $7\sqrt{5}$ is a/an .... number.
Ans : irrational

8. An algorithm which is used to find HCF of two positive numbers is ....
Ans : Euclid's division algorithm

9. $6+\sqrt{2}$ is a/an .... number.
Ans : irrational

10. Every point on the number line corresponds to a .... number.
Ans : Real

11. A .... is a proven statement used for proving another statement.
Ans : lemma

12. The product of three numbers is .... to the product of their HCF and LCM.
Ans : Not equal

13. L.C.M. of 96 and 404 is ....
Ans : 9696

14. If p is a prime number and it divides a^2 then it also divides ...., where a is a positive integer.
Ans : a

15. $\frac{35}{50}$ is a .... decimal expansion.
Ans : terminating

16. An .... is a series of well defined steps which gives a procedure for solving a type of problem.
Ans : algorithm

17. Every real number is either a .... number or an .... number.
Ans : Rational, irrational

18. Euclid's Division Lemma is a restatement of ....
Ans : Long division process

19. $\frac{1}{\sqrt{2}}$ is a/an .... number.
Ans : irrational

20. Numbers having non-terminating, non-repeating decimal expansion are known as ....
Ans : Irrational numbers

21. $\sqrt{5}$ is a/an .... number.
Ans : irrational

3. TRUE/FALSE

1. Given positive integers a and b, there exist whole numbers q and r satisfying $a = bq + r$, $0 \leq r < b$.
Ans : True

2. HCF of two numbers is always a factor of their LCM.
Ans : True

3. Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
Ans : True

4. Sum of two prime numbers is always a prime number.
Ans : False

5. The number zero is irrational.
Ans : False

6. $\sqrt{2}$ and $\sqrt{3}$ are irrationals numbers.
Ans : True

7. π is an irrational number.
Ans : True

8. Some irrational numbers are negative.
Ans : True

9. If $x = p/q$ be a rational number, such that the prime factorisation of q is not of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which is terminates.
Ans : False

10. All real numbers are rational numbers.
Ans : False

11. Any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.
Ans : True

12. Sum of two irrational numbers is an irrational number.
Ans : False

13. The quotient of two integers is always a rational number
Ans : False

14. Two numbers can have 12 as their LCM and 350 as their HCF.
Ans : False

15. 1/0 is not rational.
Ans : True

16. The product of any three consecutive natural numbers is divisible by 6.
Ans : True

17. If $x = p/q$ be a rational number, such that the prime factorisation of q is of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.
Ans : True

18. All integers are real numbers.
Ans : True

19. The number of irrational numbers between 15 and 18 is infinite.
Ans : True

20. Every fraction is a rational number.
Ans : True

4. MATCHING QUESTIONS

DIRECTION : Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.

1.

	Column-I		Column-II
(A)	Irrational number is always	(p)	rational number
(B)	Rational number is always	(q)	irrational number
(C)	$\sqrt[3]{6}$ is not a	(r)	non-terminating non-repeating
(D)	$2+\sqrt{2}$ is an	(s)	terminating decimal

Ans : (A) – r, (B) – s, (C) – p, (D) – q

(A) – (r) [12 = 3 × 4; it is a composite number]
(B) – (s) [Greatest common divisor (G.C.D.) between 2 and 7 = 1]
(C) – (p) [2 is a prime number]
(D) – (q) [$\sqrt{2}$ is not a rational number]

2.

	Column-I		Column-II
(A)	H.C.F. of the smallest composite number and the smallest prime number	(p)	6
(B)	H.C.F. of 336 and 54	(q)	5
(C)	H.C.F. of 475 and 495	(r)	2

Ans : (A) – r, (B) – p, (C) – q

3.

	Column-I		Column-II
(A)	$\frac{551}{2^3 \times 5^6 \times 7^9}$	(p)	a prime number
(B)	Product of $(\sqrt{5}-\sqrt{3})$ and $(\sqrt{5}+\sqrt{3})$ is	(q)	is an irrational number
(C)	$\sqrt{5}-4$	(r)	is a terminating decimal representation
(D)	$\frac{422}{2^3 \times 5^4}$	(s)	a rational nubmer
		(t)	is a non-terminating but repeating decimal representation
		(u)	is a non-terminating and non recurring decimal representation

Ans : (A) – (t, s), (B) – (p, s), (C) – (q, u), (D) – (r, s)

4.

	Column I		Column II
(A)	$3-\sqrt{2}$ is	(p)	a rational number between 1 and 2
(B)	$\frac{\sqrt{50}}{\sqrt{80}}$ is	(q)	an irrational number
(C)	3 and 11 are	(r)	co-prime numbers
(D)	2	(s)	neither composite nor prime
(E)	1	(t)	the only even prime number

Ans : (A) – q, (B) – p, (C) – r, (D) – t, (E) – s

5. ASSERTION AND REASON

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1. **Assertion :** $\frac{13}{3125}$ is a terminating decimal fraction.

Reason : If $q = 2^n \cdot 5^m$ where n, m are non-negative integers, then $\frac{p}{q}$ is a terminating decimal fraction.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Since the factors of the denominator 3125 is of the form $2^0 \times 5^5$.

$\frac{13}{3125}$ is a terminating decimal

Since, assertion follows from reason.

2. **Assertion :** A number N when divided by 15 gives the reminder 2. Then the remainder is same when N is divided by 5.

Reason : $\sqrt{3}$ is an irrational number.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Clearly, both A and R are correct but R does not explain A.

3. **Assertion :** Denominator of 34.12345. When expressed in the form $\frac{p}{q}$, $q \neq 0$, is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Reason : 34.12345 is a terminating decimal fraction.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Reason is clearly true

$$\text{Again } 34.12345 = \frac{3412345}{100000} = \frac{682469}{20000} = \frac{682469}{2^5 \times 5^4}$$

Its denominator is of the form $2^m \times 5^n$

$[m = 5, n = 4 \text{ are non} - \text{negative integers}]$

Hence, assertion is true. Since reason gives assertion (a) holds.

4. **Assertion :** When a positive integer a is divided by 3, the values of remainder can be 0, 1 or 2.

Reason : According to Euclid's Division Lemma $a = bq + r$, where $0 \le r < b$ and r is an integer.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Given positive integers A and B, there exists unique integers Q and R satisfying $a = bq + r$, where $0 \le r < b$.

This is known as Euclid's Division Lemma. So, both A and R are correct and R explains A.

5. **Assertion :** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. $= 162$.
Reason : If a, b are two positive integers, then H.C.F. $\times$ L.C.M. $= a \times b$.

Ans : (d) Assertion (A) is false but reason (R) is true.
Here reason is true [standard result]
Assertion is false.

$$\frac{3072}{16} = 192 \neq 162$$

6. **Assertion :** 6^n ends with the digit zero, where n is natural number.
Reason : Any number ends with digit zero, if its prime factor is of the form $2^m \times 5^n$, where m, n are natural numbers.

Ans : (d) Assertion (A) is false but reason (R) is true.
$6^n = (2 \times 3)^n = 2^n \times 3^n$, Its prime factors do not contain 5^n i.e., of the form $2^m \times 5^n$, where m, n are natural numbers. Here assertion is incorrect but reason is correct.

7. **Assertion :** 2 is a rational number.
Reason : The square roots of all positive integers are irrationals.

Ans : (c) Assertion (A) is true but reason (R) is false.
Here reason is not true. $\sqrt{4} = \pm 2$, which is not an irrational number.

8. **Assertion :** $\sqrt{a}$ is an irrational number, where a is a prime number.
Reason : Square root of any prime number is an irrational number.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
As we know that square root of every prime number is an irrational number. So, both A and R are correct and R explains A.

9. **Assertion :** If L.C.M. $\{p, q\} = 30$ and H.C.F. $\{p, q\} = 5$, then $p \cdot q = 150$.
Reason : L.C.M. of $(a, b) \times$ H.C.F. of$(a, b) = a \cdot b$.

Ans : (a) Assertion (A) is true but reason (R) is false.

10. **Assertion :** For any two positive integers a and b, HCF $(a, b) \times$ LCM $(a, b) = a \times b$
Reason : The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

Ans : (c) Assertion (A) is true but reason (R) is false.
We have,

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$
$$\text{LCM} \times 5 = 150$$
$$\text{LCM} = \frac{150}{5} = 30$$
$$\text{LCM} = 30,$$

11. i.e., reason is incorrect and assertion is correct.
Assertion : $n^2 - n$ is divisible by 2 for every positive integer.
Reason : $\sqrt{2}$ is not a rational number.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

12. **Assertion :** $n^2 + n$ is divisible by 2 for every positive integer n.
Reason : If x and y are odd positive integers, from $x^2 + y^2$ is divisible by 4.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).